Posted-Price Mechanisms: Bidder Behavior

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We describe bidder[1] behavior in posted-price mechanisms.

[1] Buyer, I suppose, but we'll stick with bidder.

1 *Simple vs. Optimal Auctions*

While Myerson's analysis yields an elegant solution to the optimal auction design problem, his auction is not widely used in practice. There are likely multiple reasons why not, such as its complexity, and its heavy reliance on distributional knowledge. On the other hand, posted-price mechanisms *are* widely used in practice. So we might be interested to know, what makes for an *optimal* posted-price mechanism? We can perhaps answer this question in a single-parameter environment by spelling out the revenue function, taking derivatives, etc., but as this calculation may be complex, we take an alternative approach in this lecture. We analyze a simple, randomized posted-price mechanism. Perhaps surprisingly, the guarantees we derive do not rely on (much) distributional knowledge.

2 *Posted-Price Mechanisms*

In a **posted-price** mechanism, the seller announces (i.e., posts) the price π at which she is willing to sell the good, after which any bidder who indicates that he is willing to pay the posted price is uniformly eligible to win the good.[2] The winner is then charged the posted price π, and all others pay nothing.

[2] Sort of like how when we go out for coffee, we pay whatever price the establishment has decided on, irrespective of possible competition from other people. (Well, aside from the long lines.)

Given a posted price mechanism with price π, we can ask:

- How should bidders behave: i.e., what should they "bid?"
- Assuming they behave as predicted, how good is the outcome?

"Good" in the second question implies that we are measuring something. In this lecture, that something is total expected revenue, which is the sum of the total expected payments.

3 *Bidder Behavior*

In a posted-price mechanism, the price a bidder pays is independent of her "bid" (i.e., her private value). As a result, the analysis to determine what strategy a bidder should use in a posted-price mechanism is similar to that of the second-price auction.

The (familiar) case analysis is described graphically in Figures 1 and 2, and summarized in Figure 3. We see that the posted-price mechanism for one good is DSIC: regardless of what any other bidder does, "bidding" (i.e., behaving consistently with) one's true value is a dominant strategy.

Because payments for the winner are pre-determined by the seller, a bid in this mechanism is in fact a binary signal. Placing a bid $b_i \geq \pi$ is telling the seller "I am willing to purchase the good at posted price π." Placing a bid $b_i < \pi$ is telling the seller "I am not willing to purchase the good at posted price π."

As a result, the following bids also comprise a dominant strategy:

$$b_i \in \begin{cases} [\pi, \infty), & \text{if } v_i \geq \pi \\ (-\infty, \pi), & \text{otherwise.} \end{cases}$$

In other words, there are multiple dominant strategies in this mechanism. For example, the following two strategies would each do just as well as bidding truthfully in a posted-price mechanism:

$$b_i = \begin{cases} \pi, & \text{if } v_i \geq \pi \\ 0, & \text{otherwise} \end{cases}$$

$$b_i = \begin{cases} \infty, & \text{if } v_i \geq \pi \\ -\infty, & \text{otherwise.} \end{cases}$$

Because there are myriad dominant strategies, the posted-price mechanism differs from the second-price auction in an important way. *Bidders need not bid truthfully!* In particular, bidders need not divulge their private information in order to maximize their utilty.

Remark 3.1. A bidder in a second-price auction would also have myriad dominant strategies if the highest other-agent bid, b^*, were known. Truthful bidding, however, remains dominant, when b^* is unknown: i.e., regardless of the value of b^* that is realized.

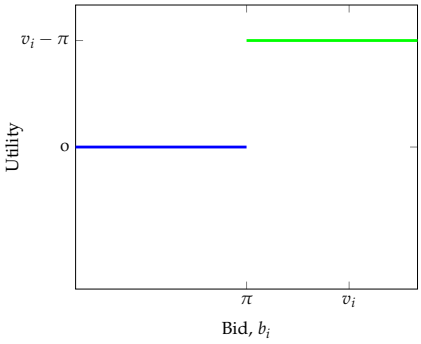

Figure 1: The utility of bidder i if the posted price is smaller than v_i, as a function of how she behaves.

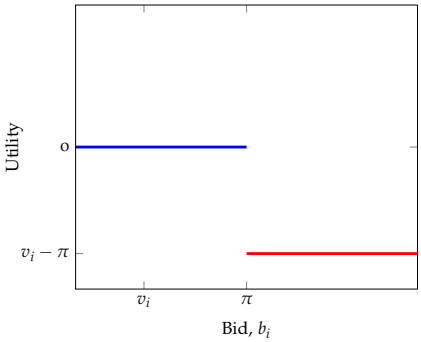

Figure 2: The utility of bidder i if the posted price is larger than v_i, as a function of how she behaves.

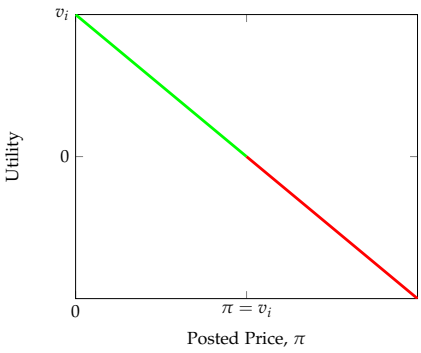

Figure 3: The utility of bidder i if she wins, as a function of the posted price.