Independent Private Values

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We describe perhaps the simplest way in which values can be modeled.

Recall that an auction is a Bayesian game (i.e., a game of incomplete information), and hence players/bidders are endowed with private information in the form of (possibly correlated) types $\mathbf{t}$ drawn from some common prior distribution F.

Additionally, a **valuation** is a function from bundles of goods to values: that is, a valuation assigns a value to every subset of the goods on offer—one value in first- and second-price auctions, but 2^{98} values in the Canadian spectrum auction.

A valuation, in general, is a function of *all* the bidders' types: i.e., for each bidder $i \in [n]$, $v_i : T \to \mathbb{R}$. In the **independent private values (IPV)** auction model, however, it is not. On the contrary, this model makes the following two simplifying assumptions:

1. any one bidder's *type* is independent of any other bidder's type
2. bidder's *valuations* are private, meaning they are not influenced by other bidders' types (i.e., their information)

The first assumption decorrelates bidders' types: when a bidder learns her own type, she does not learn anything about other bidders' types. The mathematical implication of this assumption is that we can draw bidder i's type $t_i \in T_i$ from a distribution F_i, which is independent of F_j, for all other bidders $j \neq i \in [n]$:

$$t_i \sim F_i, \quad \forall i \in [n]. \tag{1}$$

The second assumption allows us to simplify valuations as follows:

$$v_i(t_i, \mathbf{t}_{-i}) = v_i(t_i), \quad \forall i \in [n], \forall \mathbf{t}_{-i} \in T_{-i}. \tag{2}$$

Since each bidder's valuation is a function of only her own type, we can draw valuations rather than types from distributions:

$$v_i \sim F_i, \quad \forall i \in [n]. \tag{3}$$

The IPV model is applicable in auction settings where goods do not have any resale value, or where reselling is infeasible. If goods cannot be resold, then how other bidders value goods is irrelevant.