EPIC Ascending Auctions

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Just as direct mechanisms must charge Groves payments to be DSIC, indirect mechanisms must charge Groves payments to be EPIC. We investigate the converse: are indirect mechanisms that charge Groves payments necessarily EPIC? The answer to this question gives rise to a recipe for designing EPIC indirect mechanisms.

1 *VCG Payments are Necessary*

The "VCG uniqueness" theorem states that the VCG mechanism is the unique DSIC and IR mechanism among all direct, mechanisms.[1,2] In particular, no direct means of allocating goods to bidders can be DSIC and IR, unless it imposes VCG payments.

[1] up to an additive constant

[2] assuming connected type spaces

We define a **VCG outcome** of a direct mechanism as any welfare-maximizing allocation together with VCG payments. We then pose an analogous proposition, as it pertains to indirect mechanisms, namely any welfare-maximizing indirect mechanism that is both DSIC and IR imposes VCG payments:

Proposition 1.1. *If an indirect mechanism is DSIC and IR, and if it is also welfare-maximizing, then it must impose VCG payments.*[3,4,5]

[3] up to an additive constant

[4] assuming connected type spaces

[5] Hereafter, we drop these qualifiers, but they continue to apply throughout.

Proof Sketch. Consider an indirect mechanism that is DSIC, IR, and welfare maximizing. Apply the revelation principle. The result is a DSIC, IR outcome-preserving (hence, welfare-maximizing) direct mechanism. But the unique direct DSIC, IR, and welfare-maximizing mechanism is the VCG mechanism. Hence, the indirect mechanism must likewise impose VCG payments. □

This proposition gives rise to a strategy for designing indirect mechanisms that yield a VCG outcome: Simply check that the auction is DSIC, IR, and welfare maximizing, and then *boom!*—VCG payments, and hence a VCG outcome, come for free.

But DSIC is a strong condition. Assuming additive valuations, SAAs are not DSIC, even up to $O(\epsilon)$. So does this mean their outcome is not necessarily a VCG outcome? Not so fast! Since the notions of EPIC and DSIC coincide in direct mechanisms, we can relax the antecedent in the previous proposition from DSIC to EPIC.

Proposition 1.2. *If an indirect mechanism is EPIC and IR, and if it is also welfare maximizing, then the outcome must be a VCG outcome.*

Proof Sketch. Consider an indirect mechanism that is EPIC, IR, and welfare maximizing. Apply the revelation principle. The result is an EPIC, IR outcome-preserving (hence, welfare-maximizing) direct mechanism. But EPIC and DSIC coincide in direct mechanisms, and the unique direct DSIC (hence, EPIC), IR, and welfare-maximizing mechanism is the VCG mechanism. Hence, the indirect mechanism likewise imposes VCG payments. □

In sum, to prove an indirect mechanism yields a VCG outcome, it suffices to show that it is EPIC, IR, and welfare maximizing and then *boom!*—VCG payments, and hence a VCG outcome, come for free.

2 *VCG Payments are Sufficient*

While it is of course nice to know that EPIC, IR, and welfare-maximizing indirect mechanisms recover a VCG outcome, what we would really like to know is *how to design indirect mechanisms, specifically ascending auctions, with incentive guarantees*. Hence, we are driven to investigate the converse: if sincere bidding in an indirect mechanism yields a VCG outcome, is the mechanism necessarily EPIC?

The next example shows that this is too tall an order. In fact, it shows that it is not straightforward to design an EPIC indirect mechanism for bidders with combinatorial valuations.

Consider the usual English auction with the following funky (or perhaps ridiculous) modification:

> If a select bidder does not bid when the price is q, but does bid when the price is $q + \epsilon$, the auction ends immediately, and that bidder wins the good at the price q.

In this funky English auction, sincere bidding yields a VCG outcome (up to ϵ). However, if all other bidders are sincere, it is better for the select bidder to sit out the first round, and then win the good in the second round at price 0. Therefore, for an auction to be EPIC, it is not enough for sincere bidding to yield a VCG outcome.

We will resolve this issue by supplying an additional condition, beyond a mere VCG outcome, which guarantees EPIC. Before doing so, however, we restrict the space of bidding strategies to restore equivalence between DSIC and EPIC in ascending auctions.[6]

[6] Recall that these two properties are equivalent in direct mechanisms.

Definition 2.1. A strategy s_i for bidder i is called **consistent** iff there exists a type $v_i \in T_i$ for which $s_i(v_i)$ is sincere. We denote the space of bidder i's consistent strategies by C_i, with select element c_i.

By definition, the bid space in a direct mechanism is precisely the type space. Moreover, each type gives rise to a consistent strategy, namely bidding sincerely w.r.t. that type; likewise each consistent strategy derives from a type. Therefore, *the space of consistent strategies* *in an indirect mechanism and the bid space in the corresponding direct mechanism are in 1-to-1 correspondence.*

Theorem 2.2. *If sincere bidding in an indirect mechanism yields a VCG outcome, then it is EPIC, as long as there does not exist a bidder for whom an inconsistent strategy yields more utility than sincere bidding, assuming all others bid sincerely.*

Proof Sketch. Consider an indirect mechanism that yields a VCG outcome under sincere bidding. Apply the revelation principle to this mechanism. The result is a direct mechanism in which truthful bidding yields a VCG outcome: i.e., the ensuing direct mechanism is VCG. But VCG is DSIC/EPIC, so that truthful bidding in the direct mechanism is a DSE/EPNE. But then back in the indirect mechanism, the corresponding sincere strategies must have likewise been an EPNE. In particular, sincere bidding for bidder i must have been a best response relative to i's other *consistent* strategies, assuming all the other bidders likewise employ sincere strategies. We cannot, however, say anything about how effective i's strategy may have been relative to i's own inconsistent strategies. □

Theorem 2.2 yields a design recipe for EPIC ascending auctions:

1. Design an allocation rule that is welfare maximizing, assuming sincere bidding.
2. Show that sincere bidding yields VCG payments.
3. Argue that no inconsistent strategy is a profitable deviation (i.e., none yields greater utility than sincere bidding for any bidder) when others bid sincerely.

We apply this recipe in upcoming lectures to design EPIC ascending auctions for different valuation classes (specifically, additive, unit demand, and diminishing marginal values).

Remark 2.3. Theorem 2.2 also suggests an analogous design recipe for DSIC ascending auctions. In the DSIC case, however, the recipe does not have much bite. Step 3 tends to fail, because it is not generally possible to argue that no inconsistent strategy is a profitable deviation when others have the flexibility to bid arbitrarily.