Second-Price Sealed-Bid Auctions

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We introduce the second-price sealed-bid auction, and describe the strategic consequences of this payment rule.

1 *The Second-Price, Sealed-Bid Auction*

The second-price sealed-bid auction is also called the Vickrey auction, named after Nobel laureate William Vickrey. In this auction format, whoever submits the highest bid is the winner, and she pays the second-highest bid. Ties are broken randomly: if multiple bidders submit the highest bid, exactly one of them is chosen as the winner.

2 *A Dominant Strategy for this Auction*

We begin with the second-price auction, because reasoning about it is easier than reasoning about the first-price auction.

Theorem 2.1. *Bidding one's value is a dominant strategy in the second-price, sealed-bid auction, regardless of the other bidders' values.*

Proof. We plot utility as a function of one's bid, given one's value v and the highest other-agent bid p, in Figures 1 and 2. If a bidder bids above p, she wins the auction, and earn utility $v - p$. When $v \geq p$, bidding v yields as high utility as any other bid greater than p; likewise, when $v \leq p$, bidding v yields as high utility as any other bid less than p. In both cases, bidding one's value yields all the utility one could hope for. As greater utility cannot be achieved by bidding other than v, we conclude that bidding truthfully is optimal. □

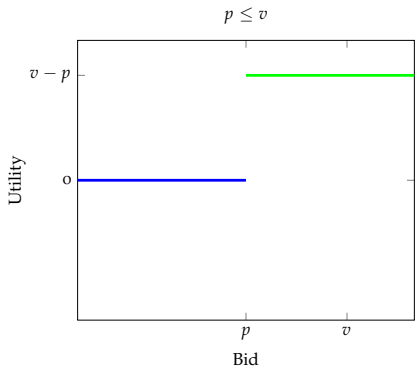

Figure 1: Utility as a function of bid, when the highest other-agent bid p lower bounds v: i.e., $p \leq v$.

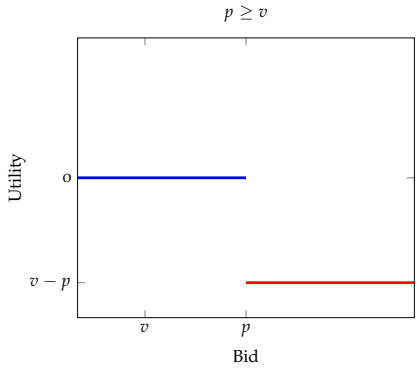

Figure 2: Utility as a function of bid, when the highest other-agent bid p upper bounds v: i.e., $p \geq v$.

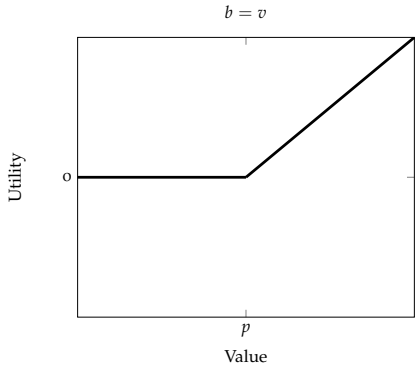

Figure 3: Utility as a function of value, when bidding truthfully, assuming the highest other-agent bid is fixed at p.