SAAs (Before the EPIC Auction Design Recipe)

CSCI 1440/2440

2024-03-06

We begin our study of simultaneous ascending auctions in the multi-parameter setting with arguably the simplest of all combinatorial valuations, additive valuations. In this lecture, we are concerned with establishing incentive guarantees.

1 *SAAs with Additive Valuations: Incentives*

Our goal in this series of notes is to establish that simultaneous ascending auctions (SAAs) yield strong incentive guarantees. To proceed, we must articulate these incentive goals, in the context of indirect mechanisms. We thus begin by putting forth an analog of truthful bidding in indirect mechanisms.

Recall that indirect auctions can invoke value queries or demand queries, the former asking bidders for a cardinal value (i.e., a bid above the current ask price), and the latter asking them for a set (i.e., their preferred bundle). These two types of queries give rise to two analogs of truthfulness, both of which we call sincere.

Definition 1.1. In an ascending auction that repeatedly queries bidders, a bidder's strategy is called **sincere** if they reply to all queries, and do so in a way that truthfully reflects their valuations.

For example, a sincere bidder in an ascending auction for a single good would answer "yes" to the demand query "Would you like the good for $10?" iff her value for the good were at least $10. Similarly, a sincere bidder in an analogous auction, but with value queries, would bid at least $10 in response to the query "Do I hear $10?"

Definition 1.2. An indirect mechanism is DSIC if sincere bidding is a dominant strategy. Likewise, an indirect mechanism is EPIC or BIC, if sincere bidding is an EPNE or BNE, respectively.

Recall that DSIC is a particularly strong property of an auction. Indeed, the English auction is not DSIC. Here is a counterexample that establishes this claim.

Example 1.3. We show by counterexample that sincere bidding is not a dominant-strategy equilibrium in the English auction. In this example, a bidder can obtain slightly more (but less than ϵ) expected utility by deviating from sincere bidding, even when the others bid sincerely. Thus, sincere bidding is not a best-response to sincere bidding, so that sincere bidding is not even an EPNE or BNE.

Suppose two bidders i and j are participating in an auction for one good, with values are $v_i = 3.5$ and $v_j = 2.5$. Suppose further, $p^0 = 0$ and $\epsilon = 1$. If bidder i bids sincerely, they drop out at round 3, winning the good at price 3, which yields utility 0.5. If, on the other hand, bidder i drops out at round 2 (i.e., at the same time as bidder j), bidder i wins the good with probability 1/2, in which case her utility is $3.5 - 2$, and her expected utility is 0.75. Hence, dropping out early yields greater expected utility for bidder i than bidding sincerely in this example, even when bidder j bids sincerely.

Example 1.3 establishes that the English auction is not DSIC. It does not, however, establish that the English auction is not DSIC "up to (the price increment) ϵ," because the benefit of bidding insincerely is less than $1, the price increment in this example.

Definition 1.4. A strategy is ϵ-dominated (or dominated up to ϵ) if it is still dominated even with an additive boost of size ϵ. More formally, a strategy $s_i \in S_i$ is ϵ-dominated by another strategy $s_i' \in S_i$ if there exists $\mathbf{s}_{-i} \in S_{-i}$ s.t. $u_i(s_i', s_{-i}) > u_i(s_i', s_{-i}) + \epsilon$. A strategy is undominated up to ϵ if no other strategy ϵ-dominates it.

The English auction is also not DSIC up to ϵ.[1] The Japanese auction, however, which poses demand queries instead of value queries, and further imposes an activity rule, *is* DSIC up to ϵ[2] (though it is also not DSIC[3]). The homework exercises ask you to establish these facts, as indicated by the sidenotes.

[1] Problem 1, Part 1

[2] Problem 2, Part 2

[3] Problem 2, Part 1

Knowing that the Japanese auction is DSIC up to ϵ, we might hope that k parallel Japanese auctions might likewise be DSIC up to $O(\epsilon)$ (e.g., $k\epsilon$), assuming additive bidder valuations (and, in general, heterogenous goods), so that the bidders' interests across auctions are decoupled. But alas they are not. Let's see why.

For starters, let's investigate k parallel Vickrey auctions, since we know that a single Vickrey auction is DSIC.

Example 1.5. Suppose goods a and b are being auctioned off in two parallel Vickrey auctions, and assume two bidders, i and j. Let $v_i(a) = v_i(b) = 2$, and $v_j(a) = v_j(b) = 1$. Suppose bidder j declares that he will bid $1 million on both goods unless bidder i forfeits a, in which case he will forfeit b (which wasn't rightly his to win, anyway). Given this opposing strategy, it is in bidder i's interest to (non-truthfully) make no attempt to procure a so that she will at least win b, obtaining a utility of 1, rather than 0.

Hence, we observe that k parallel Vickrey auctions (i.e., a direct mechanism, assuming additive valuations) are not DSIC. The reason for this is that bidders' strategies (bidder j's strategy in this example) can create dependencies across auctions, even in the very special case of additive valuations, where they are independent.

Perhaps surprisingly, this observation about a direct mechanism precludes the existence of an *indirect* DSIC mechanism comprising k parallel ascending auctions. If there were such a mechanism, then running it through the Revelation Principle would yield k parallel Vickrey auctions; furthermore, as the Revelation Principle is outcome preserving, this resulting mechanism would be DSIC. But we just argued that k parallel Vickrey auctions are *not* DSIC. Hence, no such indirect mechanism can be DSIC either.

At this point, we might decide that it is time to throw in the towel. But as it happens there are other fish in the sea: i.e., there are other equilibrium concepts besides DSIC that are worthy of our attention. Specifically, we now set our sights on designing EPIC auctions, another worthy goal, which we can (at least sometimes) achieve.

Remark 1.6. Though weaker than DSIC, EPIC is still a worthwhile goal. Recall, for example, that we have derived only a BNE for first-price auctions, not an EPNE. In particular, while we know that the equilibrium of first-price auctions assuming uniformly distributed valuations involves shading bids by a known constant ($^{n-1}/_{n}$), this equilibrium is not an EPNE. In expectation over all other-bidder values, this strategy is optimal (i.e., symmetrically, a best response). But it is clearly not optimal for an agent to shade in this way *ex-post*; given a particular choice of other-bidder values, the optimal bid is the second-highest value plus a small increment.

Having moved the goalpost, we can repeat the question: are k parallel Vickrey auctions EPIC, assuming additive valuations? That is, if everyone else is bidding truthfully, should bidder i likewise bid truthfully? In this case, it cannot hurt bidder i to do so, just as it cannot hurt her to do so in a single Vickrey auction, assuming everyone else is bidding truthfully.[4]

[4] As DSIC is in general stronger than EPIC, a single Vickrey auction is EPIC as well as DSIC. The two concepts actually coincide in direct mechanisms.

Taking another look at the Revelation Principle, it does not tell us much in this case. It just tells use that there is hope. If we search, we may well find an EPIC indirect mechanism that "solves" (i.e., yields sincere bidding as an EPNE) combinatorial auctions assuming additive valuations, since k parallel Vickrey auctions are a direct mechanism that likewise solves this special case.

Remark 1.7. Since the notions of DSIC and EPIC coincide in direct mechanisms, searching for indirect mechanisms as potential inputs to Revelation Principle that yield the Vickrey auction, a DSIC/EPIC mechanism, we can look to either DSIC or EPIC indirect mechanisms. Indeed, we find that the Japanese auction is DSIC and the English auction is EPIC, both up to ϵ.[5]

[5] Problems 1 and 2, Part 2

Seek and you shall find. Indeed, k parallel English auctions are EPIC up to $O(\epsilon)$, assuming additive valuations. We lay the groundwork for a rigorous proof of this claim in the next lecture,[6] after presenting a recipe for the design of EPIC ascending auctions.

[6] Grad problem

To summarize:

- The English auction is not DSIC.[7] The English auction is *not* even DSIC up to ϵ.[8]
- The Japanese auction, which forbids re-entry (once a bidder stops bidding, they can never bid again), is also *not* DSIC.[9] The Japanese auction is, however, DSIC up to ϵ.[10]
- The English auction is *not* EPIC (or even BIC).[11] It is, however, EPIC up to ϵ.[12]
- Assuming additive valuations, neither k parallel English nor Japanese auctions for heterogeneous goods, are DSIC, even up to $O(\epsilon)$.[13]
- Assuming additive valuations, k parallel English and k parallel Japanese auctions for heterogeneous goods are EPIC up to $O(\epsilon)$.[14]

[7] Example 1.3

[8] Problem 1, Part 1

[9] Problem 2, Part 1

[10] Problem 2, Part 2

[11] Example 1.3

[12] Problem 1, Part 2

[13] Example 1.5 plus the Revelation Principle

[14] Grad problem