Homework 7: EPIC Auctions

CSCI 1440/2440

2025-11-05

Due Date: Tuesday, November 11, 2025. 11:59 PM.

We encourage you to work in groups of size two. Each group need only submit one solution. Your submission must be typeset using LaTeX. Please submit via Gradescope with you and your partner's Banner ID's and which course you are taking.

For 1000-level credit, you need only solve the first three problems. For 2000-level credit, you should solve all four problems. (The grad problem is not due until November 18.)

Preliminaries

We say that an ascending auction is DSIC up to ϵ iff no bidder who deviates from sincere bidding can improve upon sincere bidding by more than ϵ, where ϵ is the price increment of the auction. Likewise, we can define the notion of any equilibrium "up to ϵ," where by deviating from the equilibrium strategy, no bidder can improve their expected utility by more than ϵ, assuming the other players are

1 *English Auctions*

1. Show by counterexample that the English auction is not DSIC, even up to ϵ. Keep in mind that the other bidders need not bid sincerely, and that strategies in the English auction can be outright bizarre: i.e., behavior from one round to the next need not be at all sensible.

2. Prove that the English auction is EPIC, up to ϵ.: i.e., sincere bidding is an *ex-post* Nash equilibrium (EPNE), up to ϵ.

3. Apply the Revelation Principle to transform the English auction, in which sincere bidding is an ϵ-EPNE, into a direct mechanism, which is DSIC/EPIC up to ϵ. Briefly describe how the Revelation Principle closes the loopholes in the English auction design.

2 *Japanese Auctions*

This problem concerns Japanese auctions, a variant of the classic English auction that poses demand queries rather than value queries, and that forbids bidders from re-entering after exiting (i.e., skipping even one round of bidding in) the auction.

A k-good Japanese auction for k homogeneous goods can be formally specified as follows: Given a fixed price increment ϵ,

- Initialize the set of bidders $S_0 = [n]$ and the price $p_0 = 0$.
- At each round $i = 1, 2, \ldots$,
  - Let price $p_i = i\epsilon$.
  - Let S_i be the bidders in S_{i-1} who remains interested at price p_i. **N.B.** No bidder who expressed disinterest earlier can re-express their interest at some later round.
- Increment i until $|S_i| \leq k$. Call the final round t.
  - Give $|S_t|$ of the k goods to the bidders in S_t at price $t\epsilon$.
  - Give the remaining $k - |S_t|$ of the goods (if any) to random bidders in $S_{t-1} \setminus S_t$ at price $(t-1)\epsilon$.

Bidders with unit demand valuations do not demand more than one good. More specifically, their value for any bundle they might be allocated is simply their maximum value across all individual goods: i.e., a bidder i's valuation is **unit demand** if their value for a bundle of goods $X \subseteq G$ is given by

$$v_i(X) = \max_{j \in X} v_i(j),$$

where $v_i(j)$ denotes i's value for good j.

Assuming unit demand valuations, show the following:

1. Like the English auction, the k-good Japanese auction is not DSIC.

   Hint: It suffices to exhibit a counterexample in the single-good case: i.e., when $k = 1$.

2. The k-good Japanese auction is DSIC up to ϵ.

3 *Online Auctions*

Online auctions are big business: eBay has reported revenues of roughly $10 billion per annum since 2020!

Online auctions usually involve **proxy bidders**, bots that bid on behalf of users. Proxy bidding works as follows: a user, i reports a "value," say v_i, which can be understood as the maximum price i is willing to pay for a good. Such a report is **valid** if it is at least some fixed increment ϵ greater than the current asking price for the good, say p (which is initialized to a reserve price).

Whenever i enters a valid report, i's proxy bidder bids $p + \epsilon$ on i's behalf in the auction. The auction then tentatively allocates the good to i, as i's proxy bid is now the highest. Assuming i is not the only bidder who submitted a valid report, proxy bidding continues, with proxy bidders bidding on behalf of their users until the price is ϵ above the second-highest valid report, at which point the user with the highest report becomes the tentative winner, and bidding temporarily ceases—until one of the proxy bidders receives a new valid report. This process continues until an auction termination condition (such as a time limit) is met.

Once again: If j is the only user who has submitted a valid report, then j is tentatively allocated the good at the reserve price plus ϵ. If bidder i submits a bid b_i that exceeds b_j, then the asking price rises to $b_j + \epsilon$, and i is tentatively allocated the good. Or, if b_i lies between $p + \epsilon$ and b_j, then the price rises to $b_i + \epsilon$, and the good remains (tentatively) in j's hands.

Although eBay auctions are the most prevalent online auctions in use today, Amazon also ran auctions back in the day.[1] The primary difference between eBay and Amazon auctions is that eBay auctions end at a prespecified time, while Amazon auctions would continue until quiescence: i.e., until a grace period of, say, 24 hours had passed without any active bidding.

[1] Amazon lost the battle for this market, but no matter; their revenue in 2020 was nearly $400 billion!

In this question, we assume proxy bidding, and restrict our attention to the following four types of auction participants:

1. "Truthful" bidders, namely those who submit a report as if the auction were sealed-bid—they enter their value once and only once, when the auction starts.

2. Incremental bidders, who bid on eBay as if it were an English auction—they never enter their value; they merely increment their report so long as the asking price is below their value.

3. Jump bidders, who again bid when the asking price is below their value, submitting a report that is below their value, but more than just the minimal acceptable increment above the current price.

4. Snipers (only relevant on eBay, not Amazon), who submit reports no higher than their value once and only once just a few seconds before the auction ends.

Answer the following questions by providing a proof sketch or a counterexample. To provide a counterexample, choose a set of values for the bidders and a set of other-agent bidding strategies.

1. Is truthful bidding a DSE up to ϵ in an eBay auction, as eBay's bidding advice once suggested it was?[2]

[2] That's a hint!

2. What about in Amazon's now defunct auctions? Was truthful bidding a DSE up to ϵ in those auctions—assuming independent private values (IPV).

3. Which of the four bidding strategies comprise a symmetric EPNE up to ϵ in an eBay auction? What about in Amazon-style auctions?

4. Beyond a possible first-mover advantage, speculate about why eBay auctions might have survived, while Amazon-style auctions are mostly defunct.

5. Give at least one disadvantage of eBay's auction design from the point of view of revenue maximization, and another from the point of view of welfare maximization.

6. **Extra credit:** Devise a scenario in which jump bidding is a best response on eBay. *Hint:* Use the fact that jump bidding sends a signal about a bidder's interest in the good to the other bidders.

4 *k Parallel English Auctions are EPIC*

By following step 3 of the EPIC auction design recipe, complete the proof that k parallel English auctions are EPIC up to $O(k\epsilon)$, assuming additive valuations. That is, show that for every inconsistent bidding strategy, there exists a consistent bidding strategy that generates at least as much utility.

You may assume *integer* valuations and $\epsilon = 1$, so that there are no small additive discretization errors.

You may also make use of the following theorem, which we will discuss in class before the end of the semester.

Theorem: Assuming bidders with additive valuations, if sincere bidding in k parallel English auctions yields outcome $(\boldsymbol{y}, \boldsymbol{q})$, then $\boldsymbol{y}$ is $k\epsilon$-efficient (i.e., $k\epsilon$-welfare-maximizing) and for all goods $j \in [k]$, $q_j \in [p_j - k\epsilon, p_j + k\epsilon]$, where $\boldsymbol{p}$ are the Vickrey prices.