Project 2 Results

Results

Common Problems

Interest Points: Corners

Computer Vision

CS 143, Brown

James Hays

Slides from Rick Szeliski, Svetlana Lazebnik, and Kristin Grauman

Feature extraction: Corners

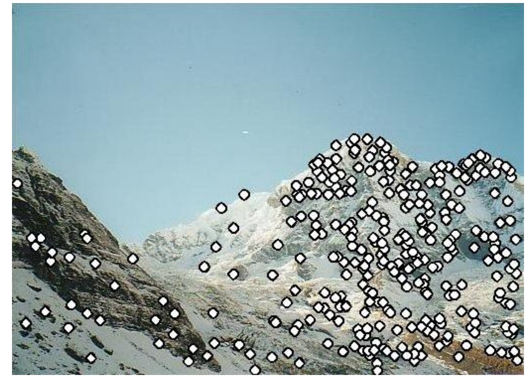
Slides from Rick Szeliski, Svetlana Lazebnik, and Kristin Grauman

Why extract features?

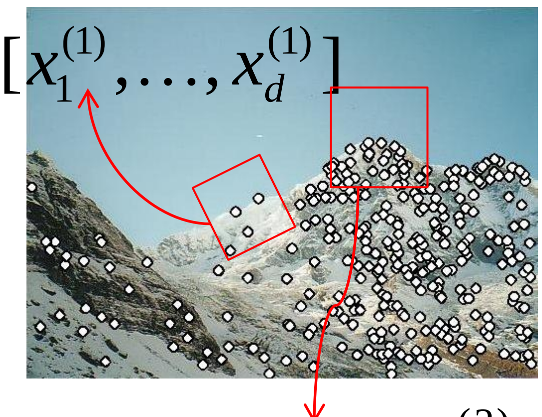
- Motivation: panorama stitching
  - We have two images – how do we combine them?

Local features: main components

1) Detection: Identify the interest points



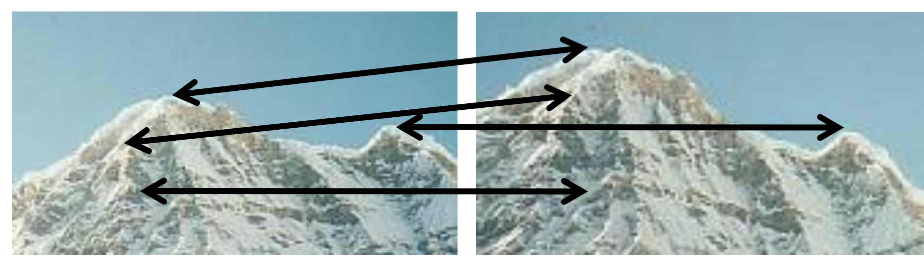
2) Description: Extract vector feature descriptor surrounding each interest point.



$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$

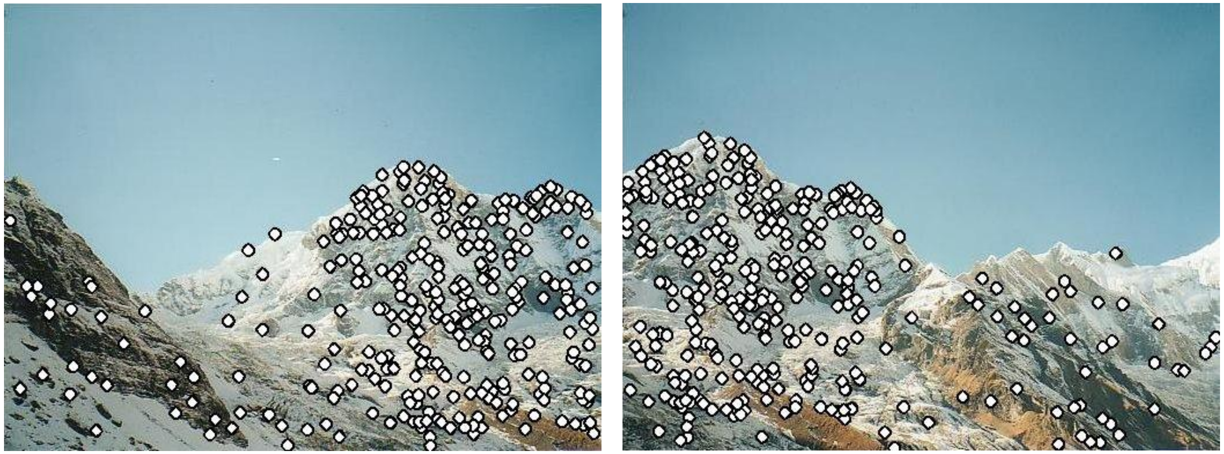
$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) Matching: Determine correspondence between descriptors in two views



Kristen Grauman

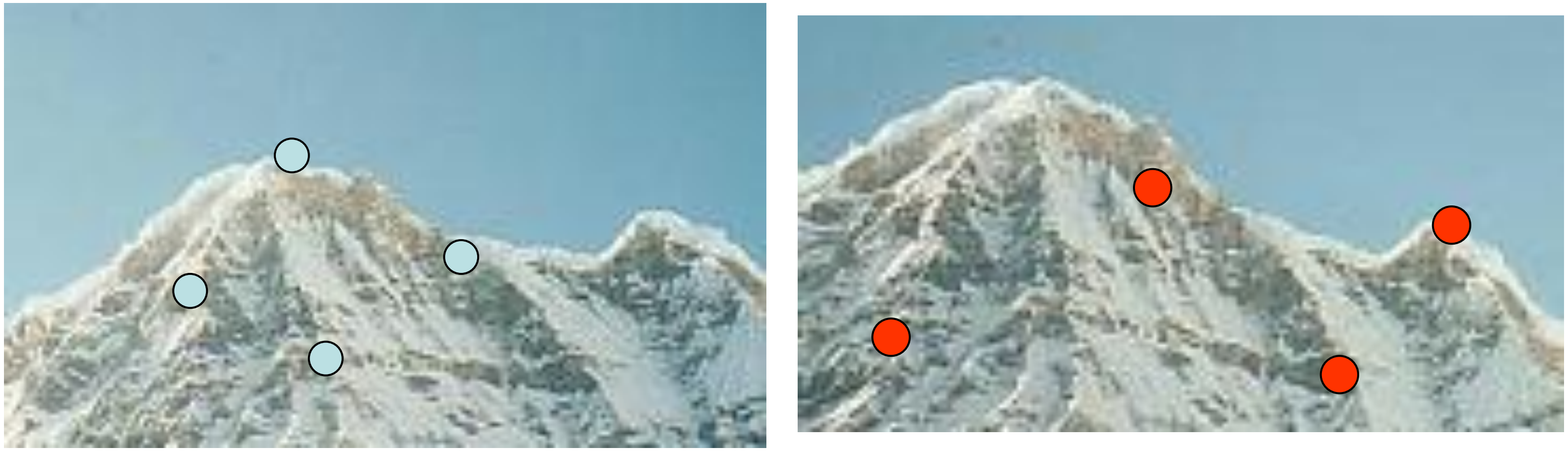
Characteristics of good features



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

Goal: interest operator repeatability

- We want to detect (at least some of) the same points in both images.



No chance to find true matches!

- Yet we have to be able to run the detection procedure *independently* per image.

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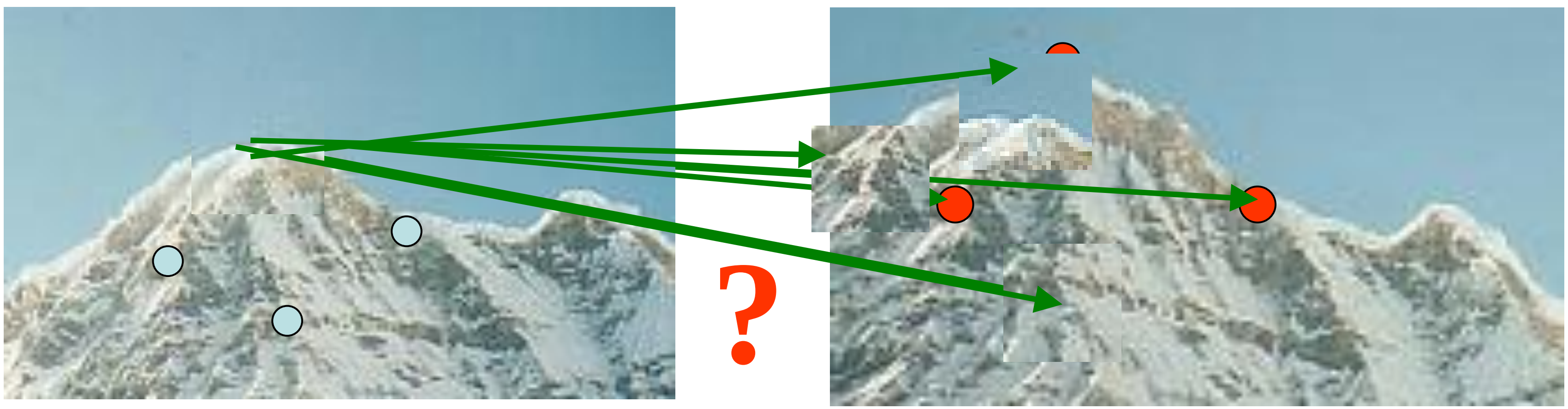
Goal: descriptor distinctiveness

- We want to be able to reliably determine which point goes with which.



- Must provide some invariance to geometric and photometric differences between the two views.

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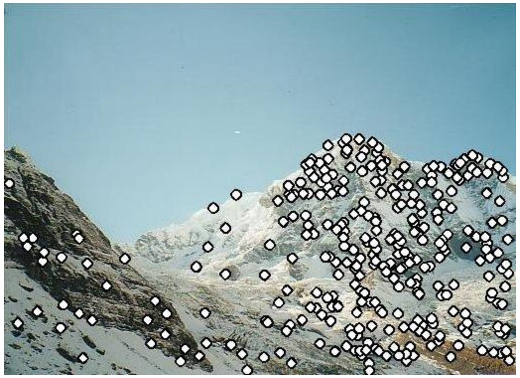
Applications

Feature points are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Indexing and database retrieval
- Object recognition

Local features: main components



1) Detection: Identify the interest points

2) Description: Extract vector feature descriptor surrounding each interest point.

3) Matching: Determine correspondence between descriptors in two views

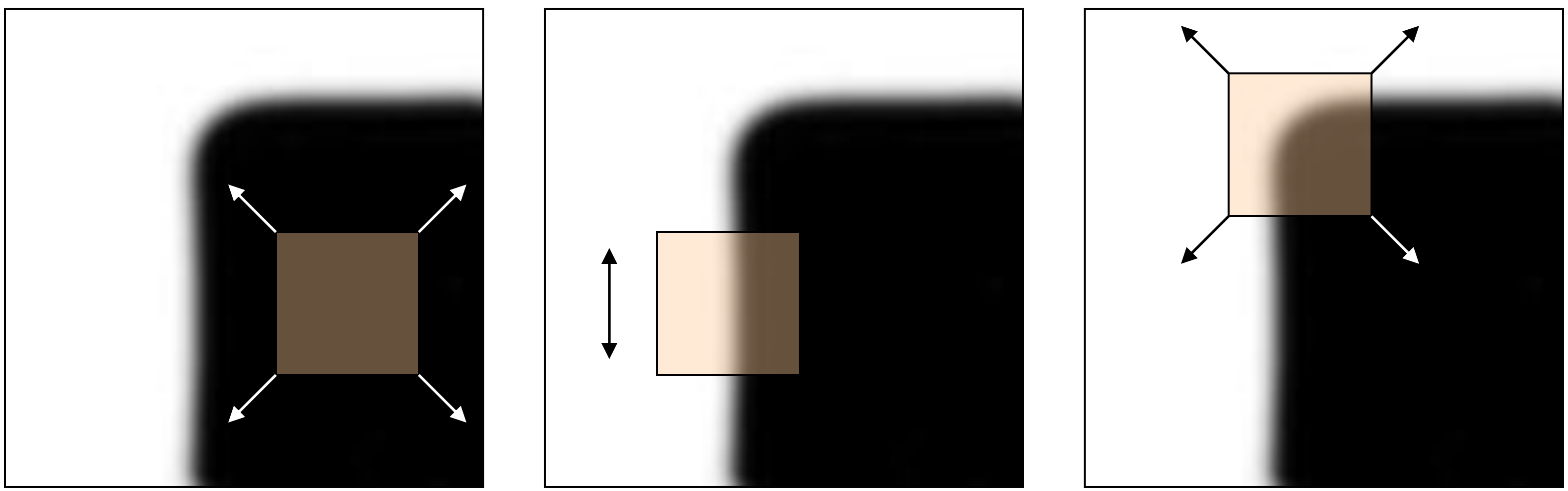
Many Existing Detectors Available

Hessian & Harris	[Beaudet ‘78], [Harris ‘88]
Laplacian, DoG	[Lindeberg ‘98], [Lowe 1999]
Harris-/Hessian-Laplace	[Mikolajczyk & Schmid ‘01]
Harris-/Hessian-Affine	[Mikolajczyk & Schmid ‘04]
EBR and IBR	[Tuytelaars & Van Gool ‘04]
MSER	[Matas ‘02]
Salient Regions	[Kadir & Brady ‘01]
Others…	

- What points would you choose?

Corner Detection: Basic Idea

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region: no change in all directions

"edge": no change along the edge direction

"corner": significant change in all directions

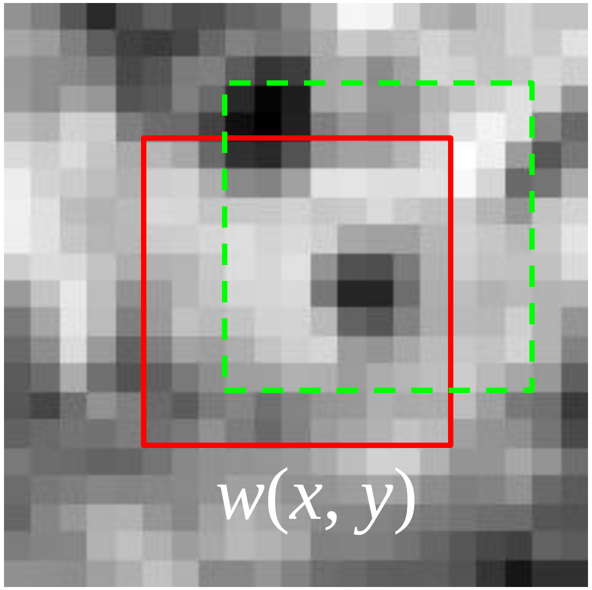
Source: A. Efros

Corner Detection: Mathematics

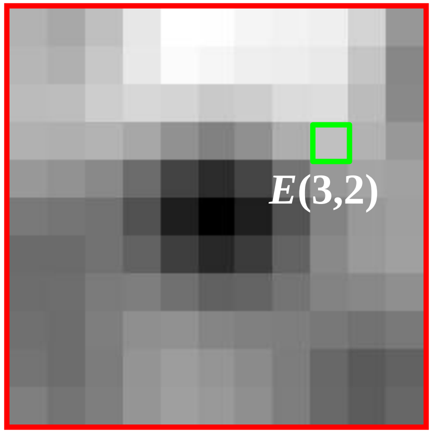
Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

$I(x, y)$



$E(u, v)$

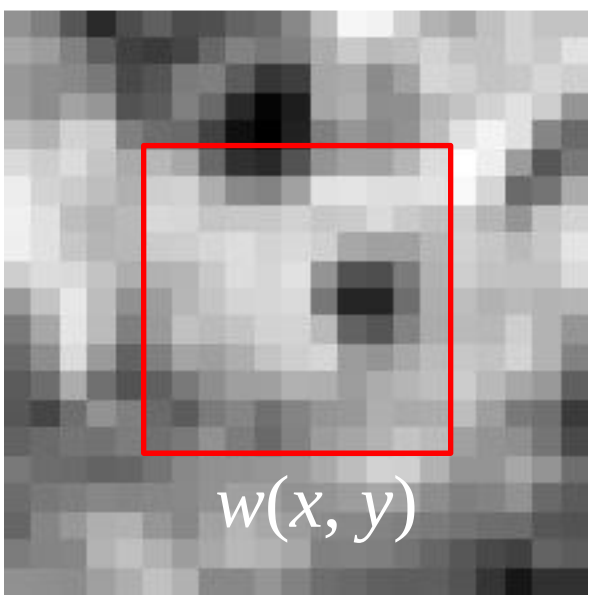


Corner Detection: Mathematics

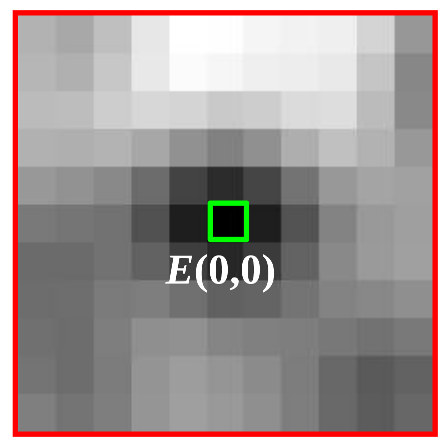
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

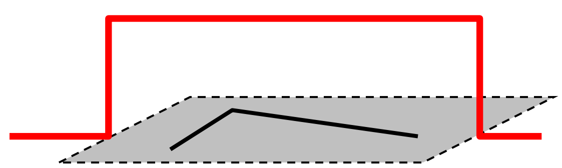
$$E(u,v)=\sum_{x,y} w(x,y)\;\; I(x+u,y+v)-I(x,y)\;^{2}$$

Window function

Shifted intensity

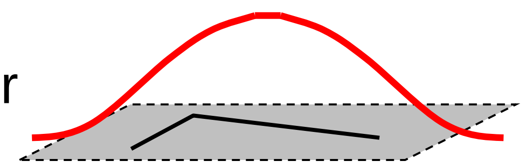
Intensity

Window function $w(x,y)$ =



1 in window, 0 outside

or



Gaussian

Source: R. Szeliski

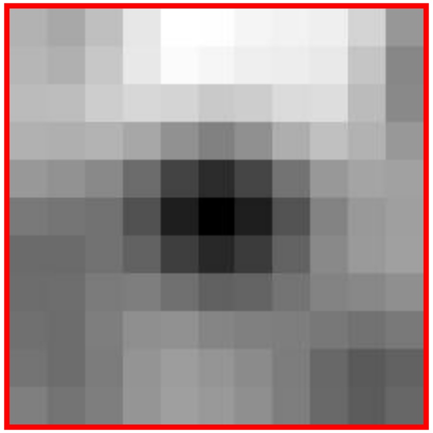
Corner Detection: Mathematics

Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v)=\sum_{x,y} w(x,y)\ \ I(x+u,y+v)-I(x,y)\ ^{2}$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Corner Detection: Mathematics

Change in appearance of window $w(x,y)$ for the shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

We want to find out how this function behaves for small shifts

Local quadratic approximation of $E(u,v)$ in the neighborhood of (0,0) is given by the *second-order Taylor expansion*:

$$E(u,v) \approx E(0,0) + [u \;\; v] \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2} [u \;\; v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Corner Detection: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

Second-order Taylor expansion of *E*(*u*,*v*) about (0,0):

$$E(u,v) \approx E(0,0) + [u \quad v] \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2} [u \quad v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E_u(u,v) = \sum_{x,y} 2w(x,y) \left[I(x+u, y+v) - I(x,y) \right] I_x(x+u, y+v)$$

$$E_{uu}(u,v) = \sum_{x,y} 2w(x,y) I_x(x+u, y+v) I_x(x+u, y+v)$$

$$+ \sum_{x,y} 2w(x,y) \left[I(x+u, y+v) - I(x,y) \right] I_{xx}(x+u, y+v)$$

$$E_{uv}(u,v) = \sum_{x,y} 2w(x,y) I_y(x+u, y+v) I_x(x+u, y+v)$$

$$+ \sum_{x,y} 2w(x,y) \left[I(x+u, y+v) - I(x,y) \right] I_{xy}(x+u, y+v)$$

Corner Detection: Mathematics

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v) \approx E(0,0) + [u \quad v] \begin{bmatrix} E_u(0,0) \\ E_v(0,0) \end{bmatrix} + \frac{1}{2} [u \quad v] \begin{bmatrix} E_{uu}(0,0) & E_{uv}(0,0) \\ E_{uv}(0,0) & E_{vv}(0,0) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$E(0,0) = 0$$

$$E_u(0,0) = 0$$

$$E_v(0,0) = 0$$

$$E_{uu}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_x(x,y)$$

$$E_{vv}(0,0) = \sum_{x,y} 2w(x,y) I_y(x,y) I_y(x,y)$$

$$E_{uv}(0,0) = \sum_{x,y} 2w(x,y) I_x(x,y) I_y(x,y)$$

Corner Detection: Mathematics

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Second-order Taylor expansion of $E(u,v)$ about (0,0):

$$E(u,v)\approx[u \;\; v]\begin{bmatrix}\sum_{x,y} w(x,y)I_x^2(x,y) & \sum_{x,y} w(x,y)I_x(x,y)I_y(x,y)\\ \sum_{x,y} w(x,y)I_x(x,y)I_y(x,y) & \sum_{x,y} w(x,y)I_y^2(x,y)\end{bmatrix}\begin{bmatrix}u\\ v\end{bmatrix}$$

$$E(0,0)=0$$

$$E_u(0,0)=0$$

$$E_v(0,0)=0$$

$$E_{uu}(0,0)=\sum_{x,y} 2w(x,y)I_x(x,y)I_x(x,y)$$

$$E_{vv}(0,0)=\sum_{x,y} 2w(x,y)I_y(x,y)I_y(x,y)$$

$$E_{uv}(0,0)=\sum_{x,y} 2w(x,y)I_x(x,y)I_y(x,y)$$

Corner Detection: Mathematics

The quadratic approximation simplifies to

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a *second moment matrix* computed from image derivatives:

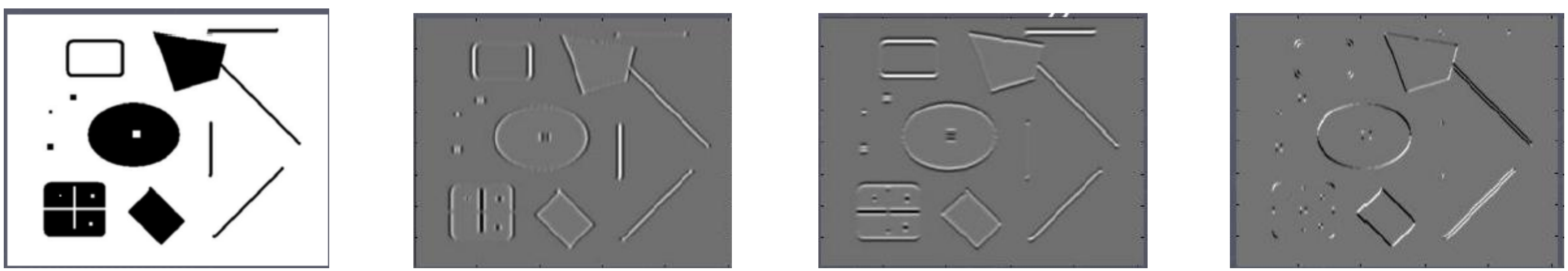
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y] = \sum \nabla I (\nabla I)^T$$

Corners as distinctive interest points

$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives (averaged in neighborhood of a point).



Notation:

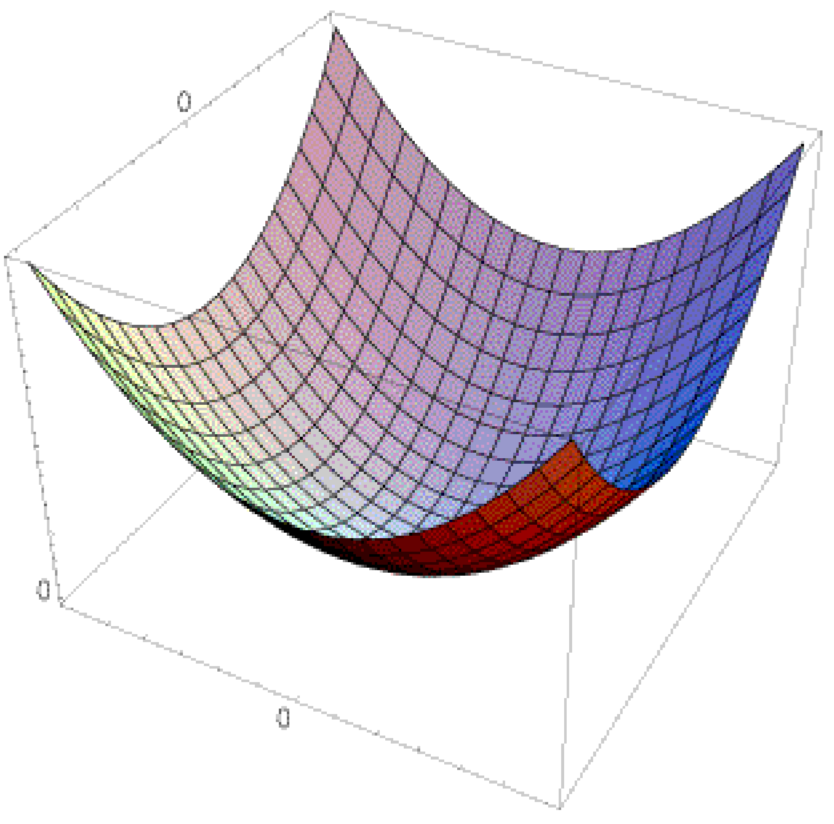
$$I_x \Leftrightarrow \frac{\partial I}{\partial x} \qquad I_y \Leftrightarrow \frac{\partial I}{\partial y} \qquad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$$

Interpreting the second moment matrix

The surface *E*(*u*,*v*) is locally approximated by a quadratic form. Let's try to understand its shape.

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

First, consider the axis-aligned case
(gradients are either horizontal or vertical)

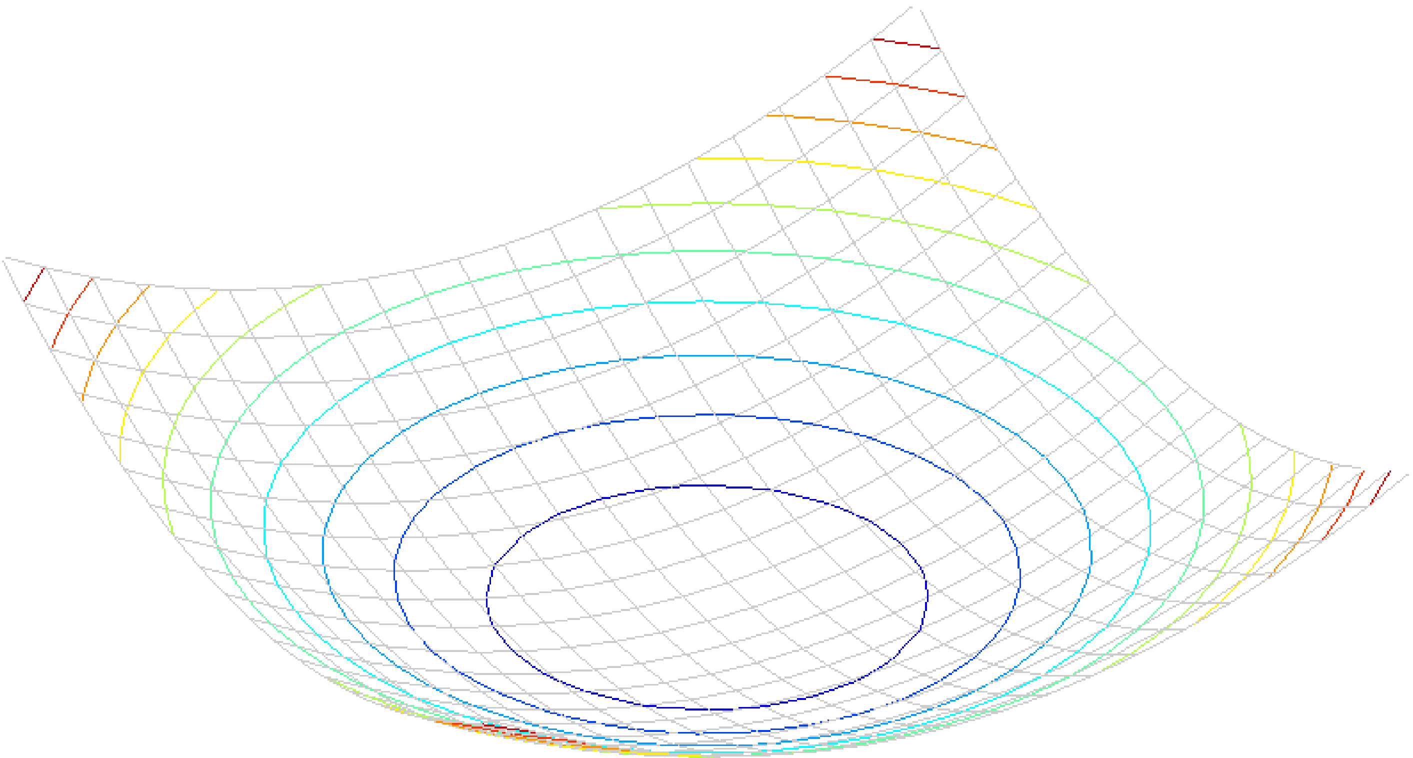
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

If either λ is close to 0, then this is **not** a corner, so look for locations where both are large.

Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $$[u\ \ v]\ M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

This is the equation of an ellipse.



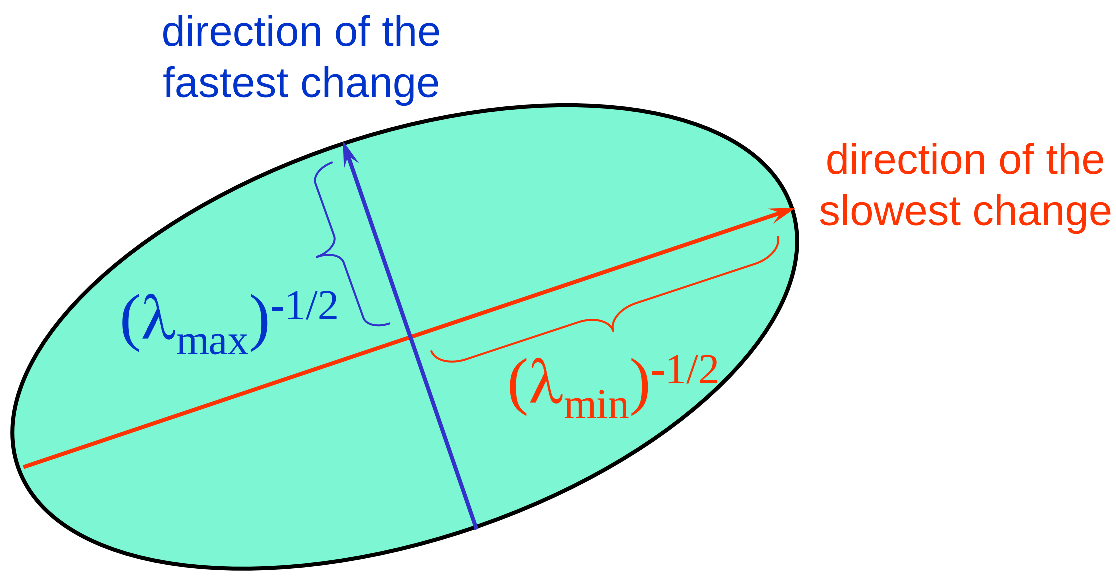
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $$[u \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

This is the equation of an ellipse.

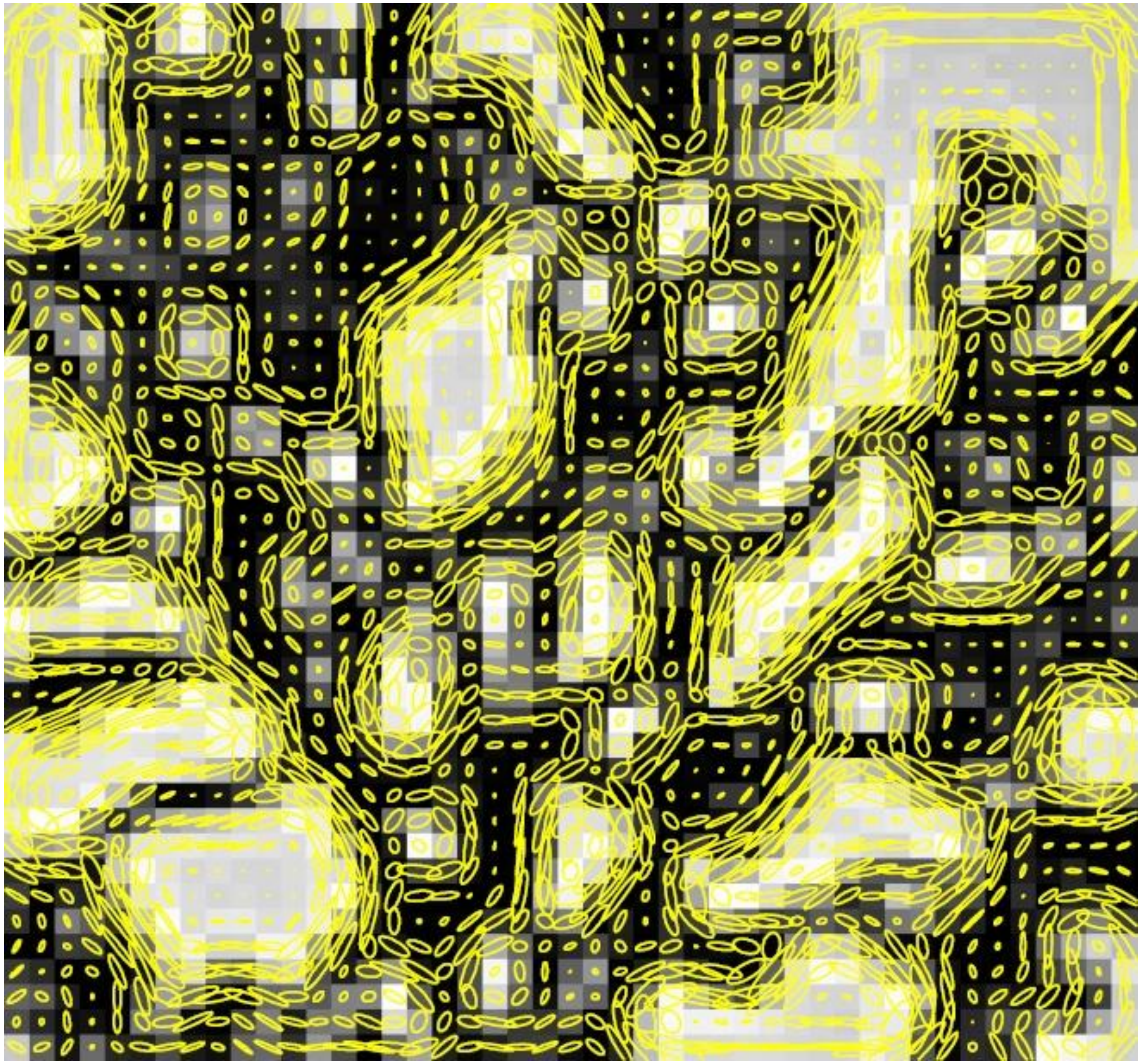
Diagonalization of M: $$M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by *R*



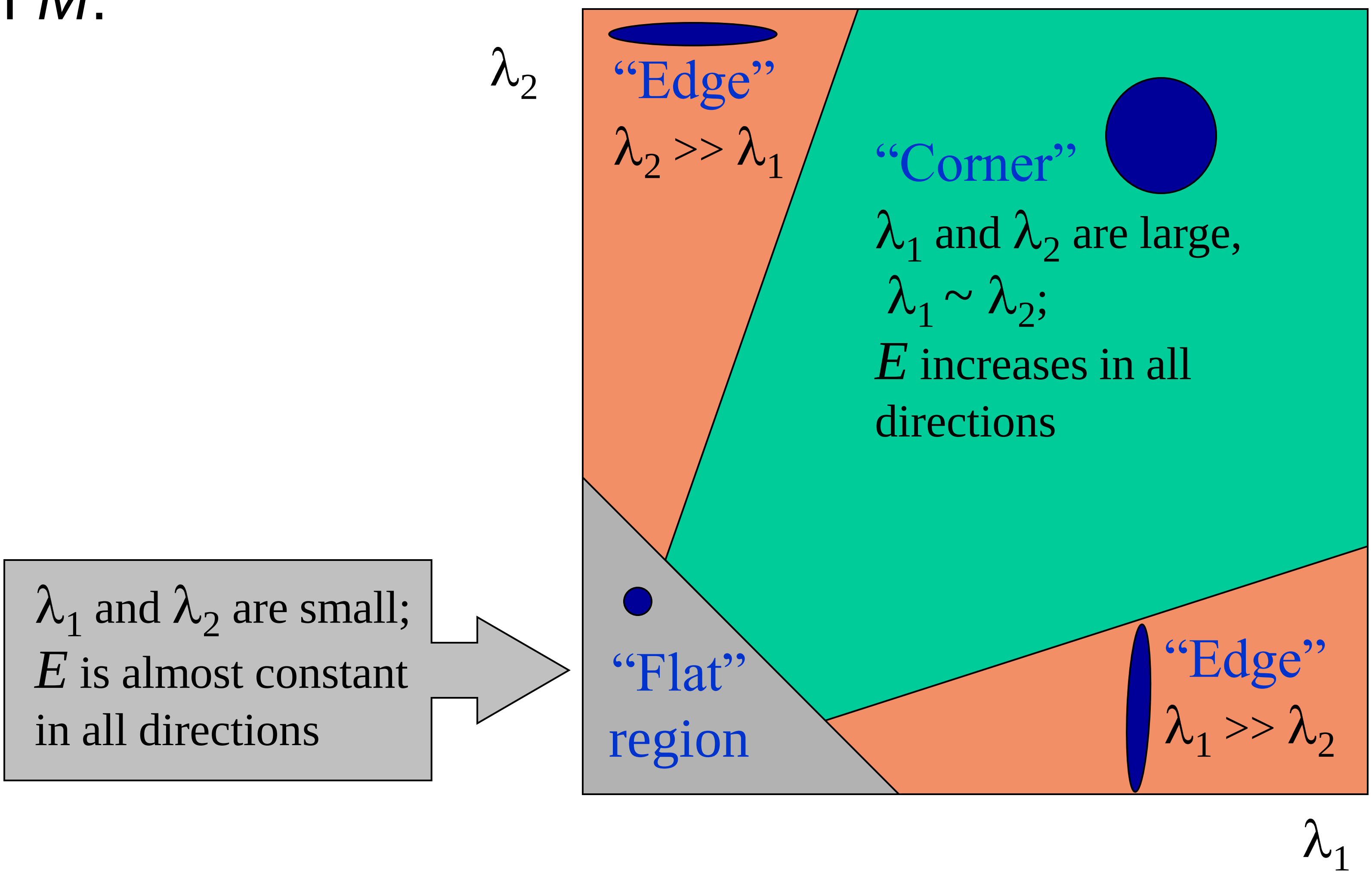
Visualization of second moment matrices

Visualization of second moment matrices



Interpreting the eigenvalues

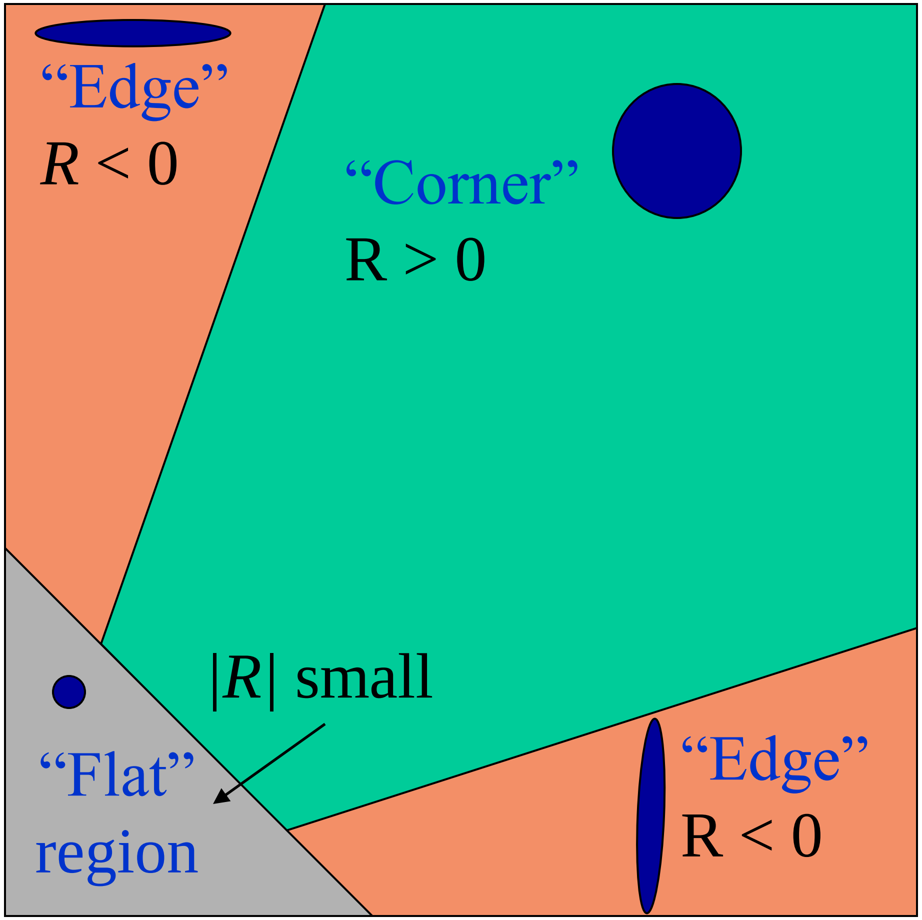
Classification of image points using eigenvalues of *M*:



Corner response function

$$R = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Harris corner detector

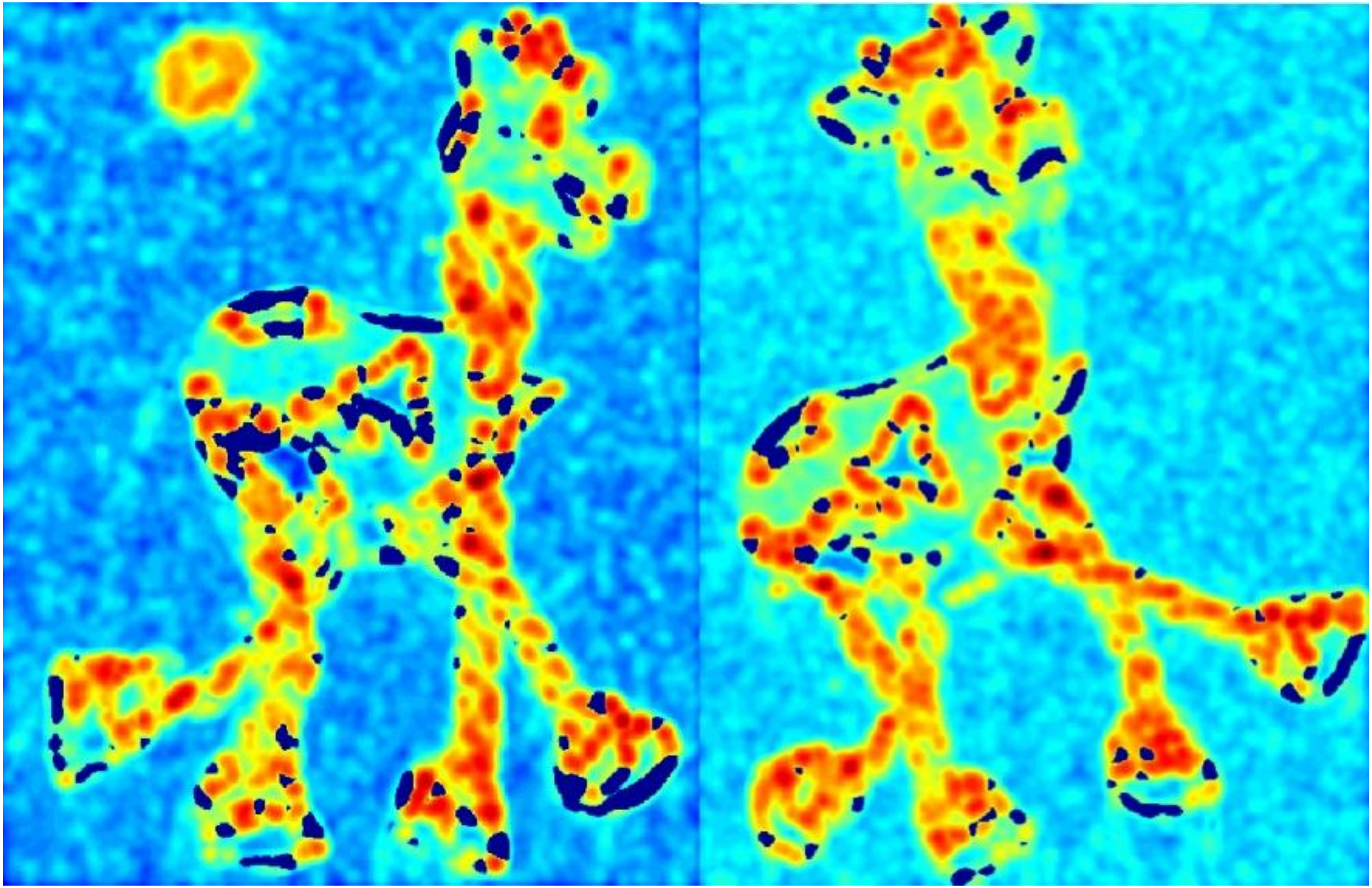
1) Compute *M* matrix for each image window to get their *cornerness* scores.
2) Find points whose surrounding window gave large corner response (*f*> threshold)
3) Take the points of local maxima, i.e., perform non-maximum suppression

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Harris Detector: Steps

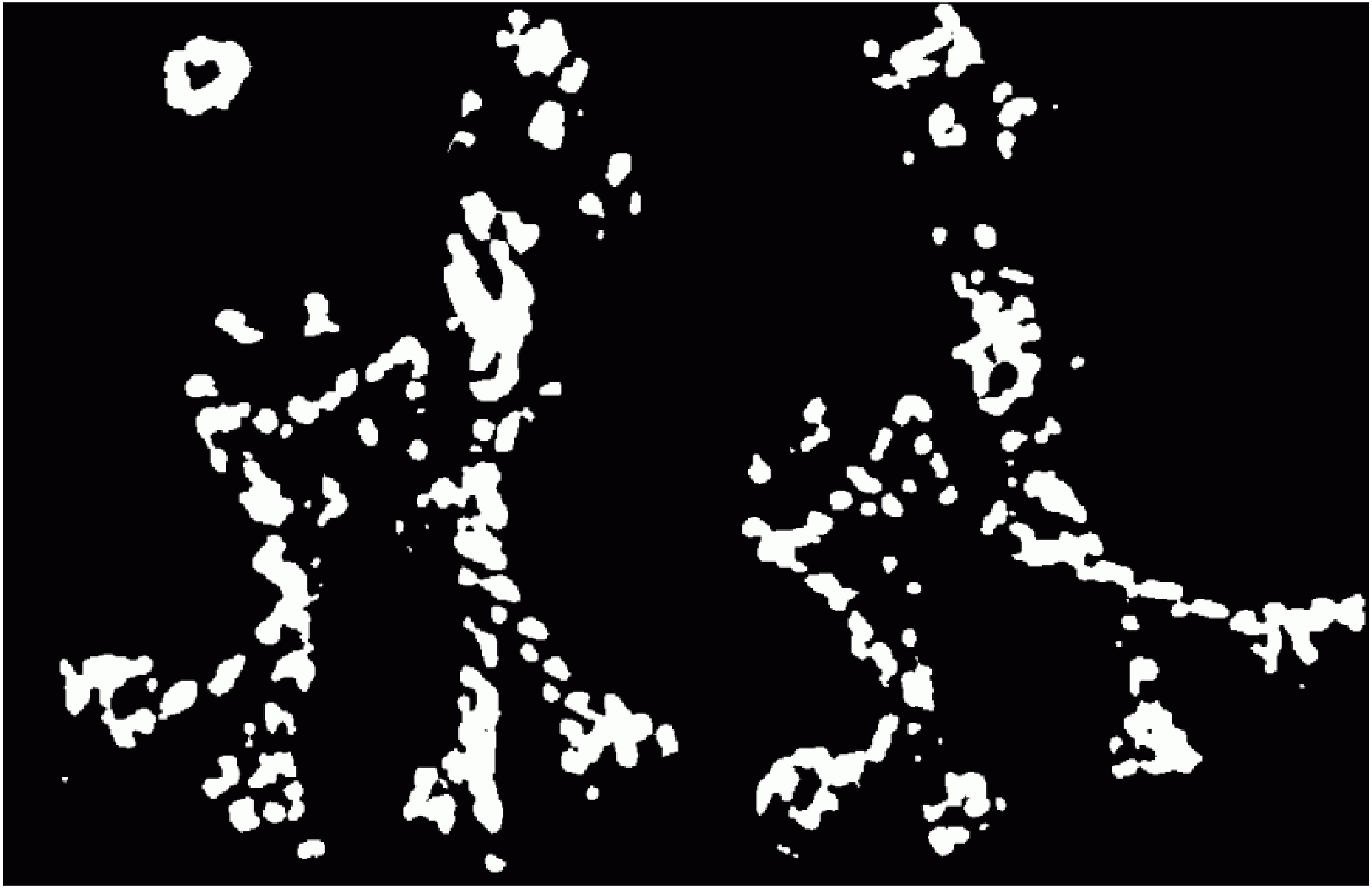
Harris Detector: Steps

Compute corner response R



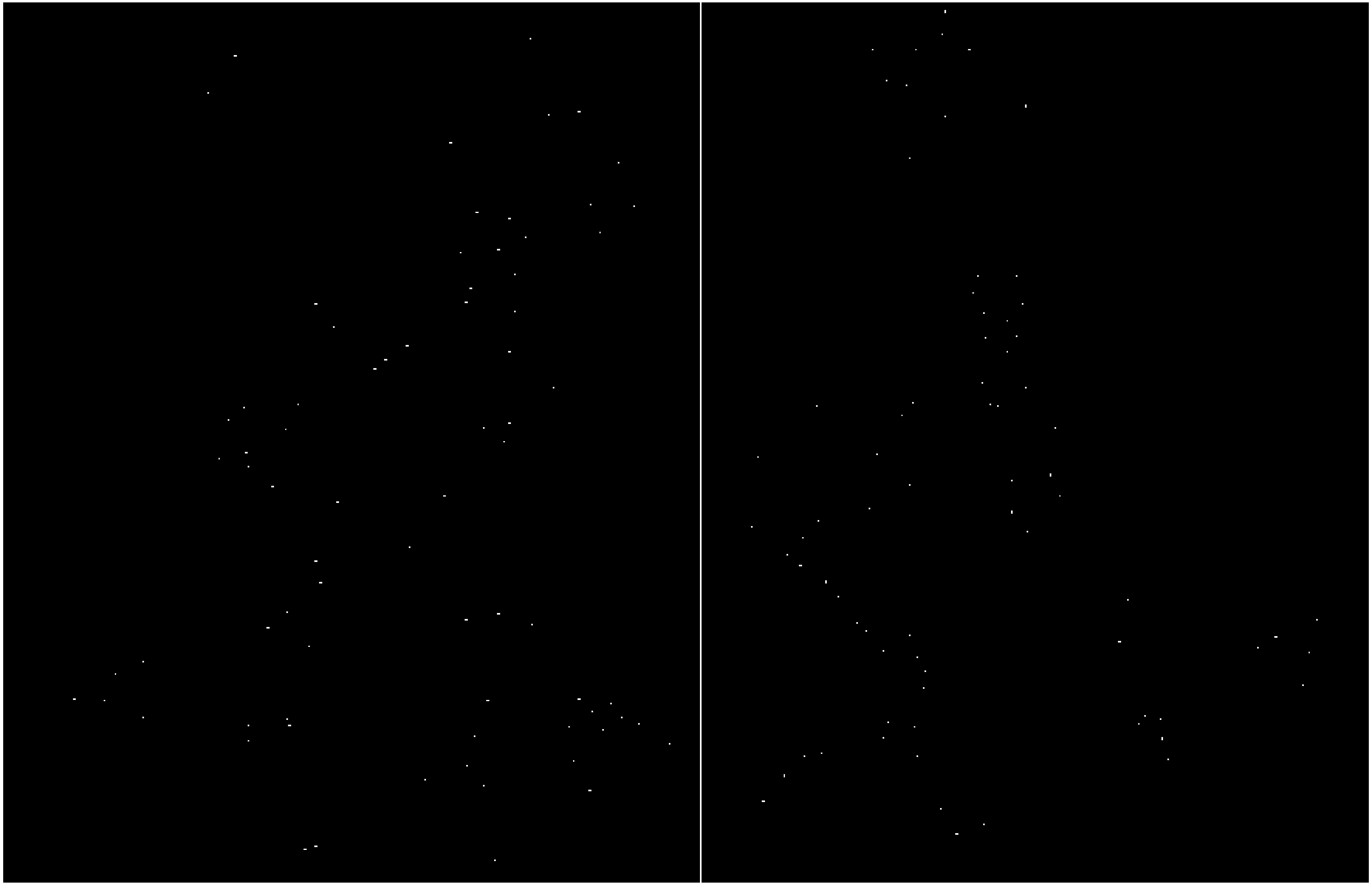
Harris Detector: Steps

Find points with large corner response: R>threshold



Harris Detector: Steps

Take only the points of local maxima of R

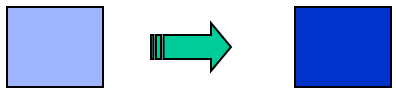


Harris Detector: Steps

Invariance and covariance

- We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations
  - **Invariance:** image is transformed and corner locations do not change
  - **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

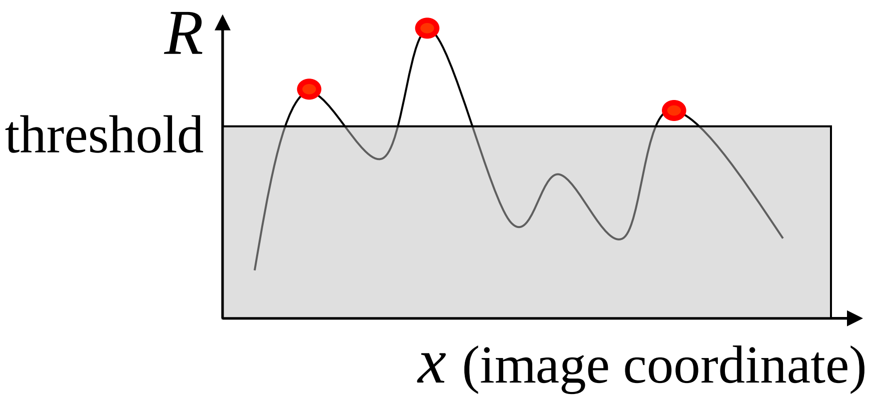
Affine intensity change

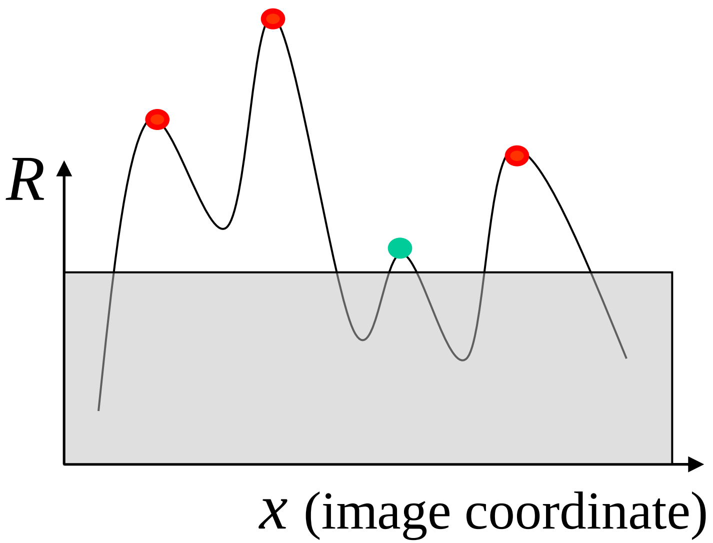


$$I \rightarrow a\,I + b$$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

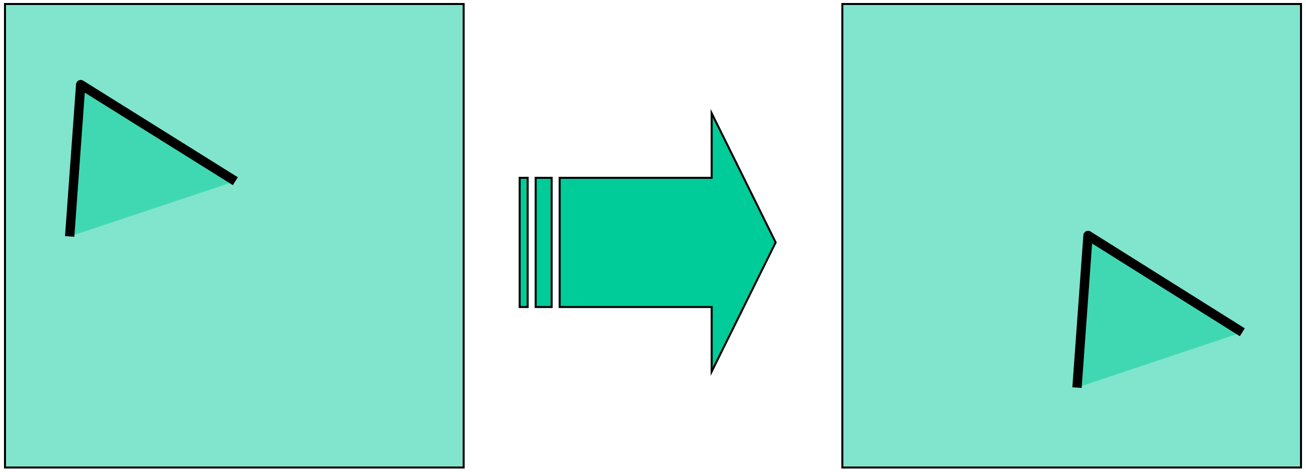
- Intensity scaling: $I \rightarrow a\,I$





Partially invariant to affine intensity change

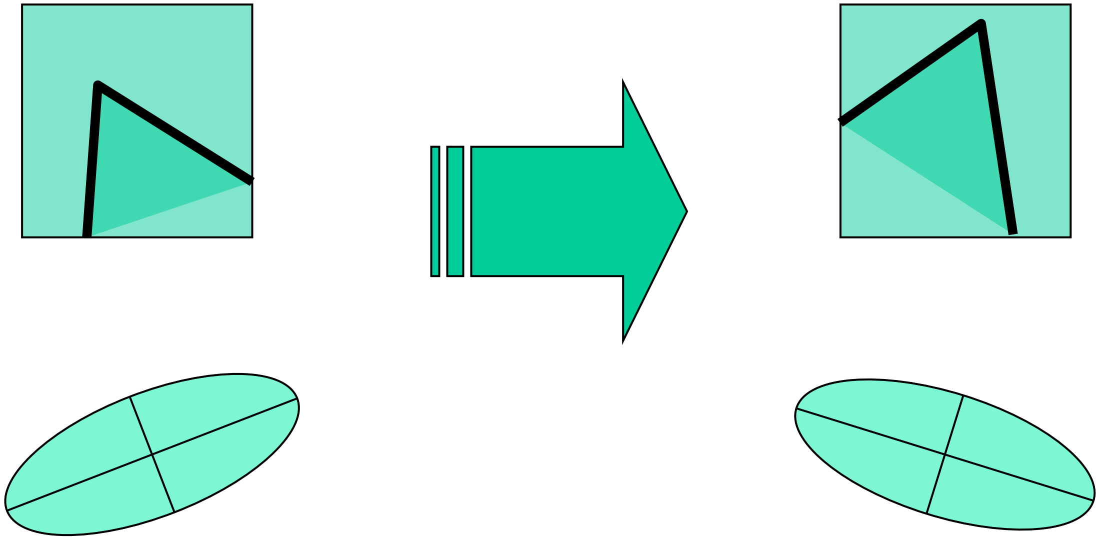
Image translation



- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

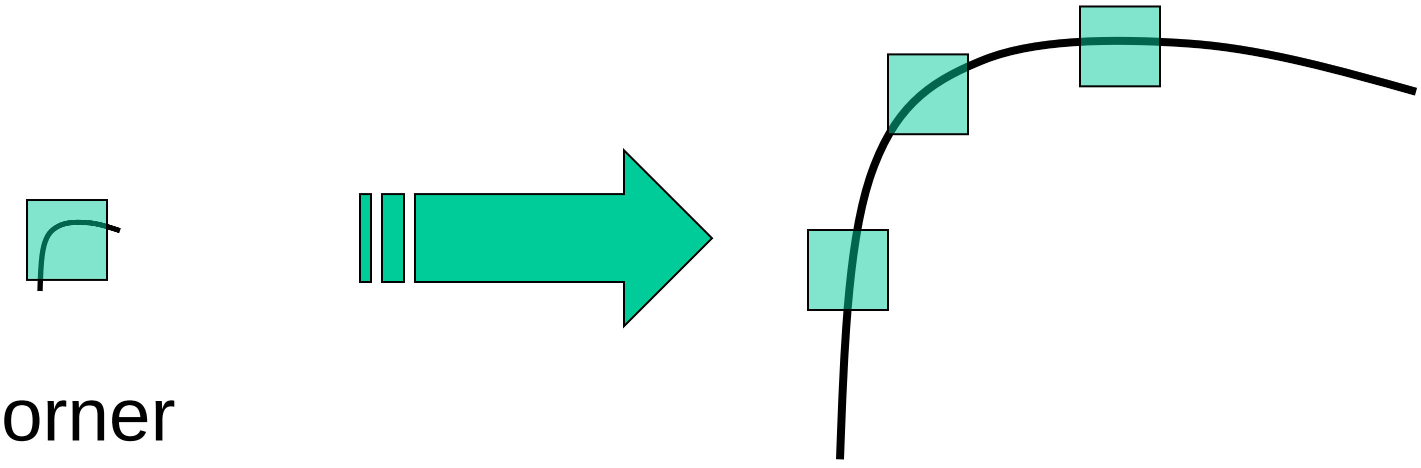
Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



All points will be classified as edges

Corner location is not covariant to scaling!