Projective Geometry and Camera Models

Computer Vision

CS 143

Brown

James Hays

Slides from Derek Hoiem, Alexei Efros, Steve Seitz, and David Forsyth

Administrative Stuff

- Textbook
- Matlab Tutorial
- Office hours
  - James: Monday and Wednesday, 1pm to 2pm
  - Geoff, Monday 7-9pm
  - Paul, Tuesday 7-9pm
  - Sam, Wednesday 7-9pm
  - Evan, Thursday 7-9pm
- Project 1 is out

Last class: intro

- Overview of vision, examples of state of art

- Computer Graphics: Models to Images
- Comp. Photography: Images to Images
- Computer Vision: Images to Models

What do you need to make a camera from scratch?

Today's class

Mapping between image and world coordinates

- Pinhole camera model
- Projective geometry
  - Vanishing points and lines
- Projection matrix

Today's class: Camera and World Geometry

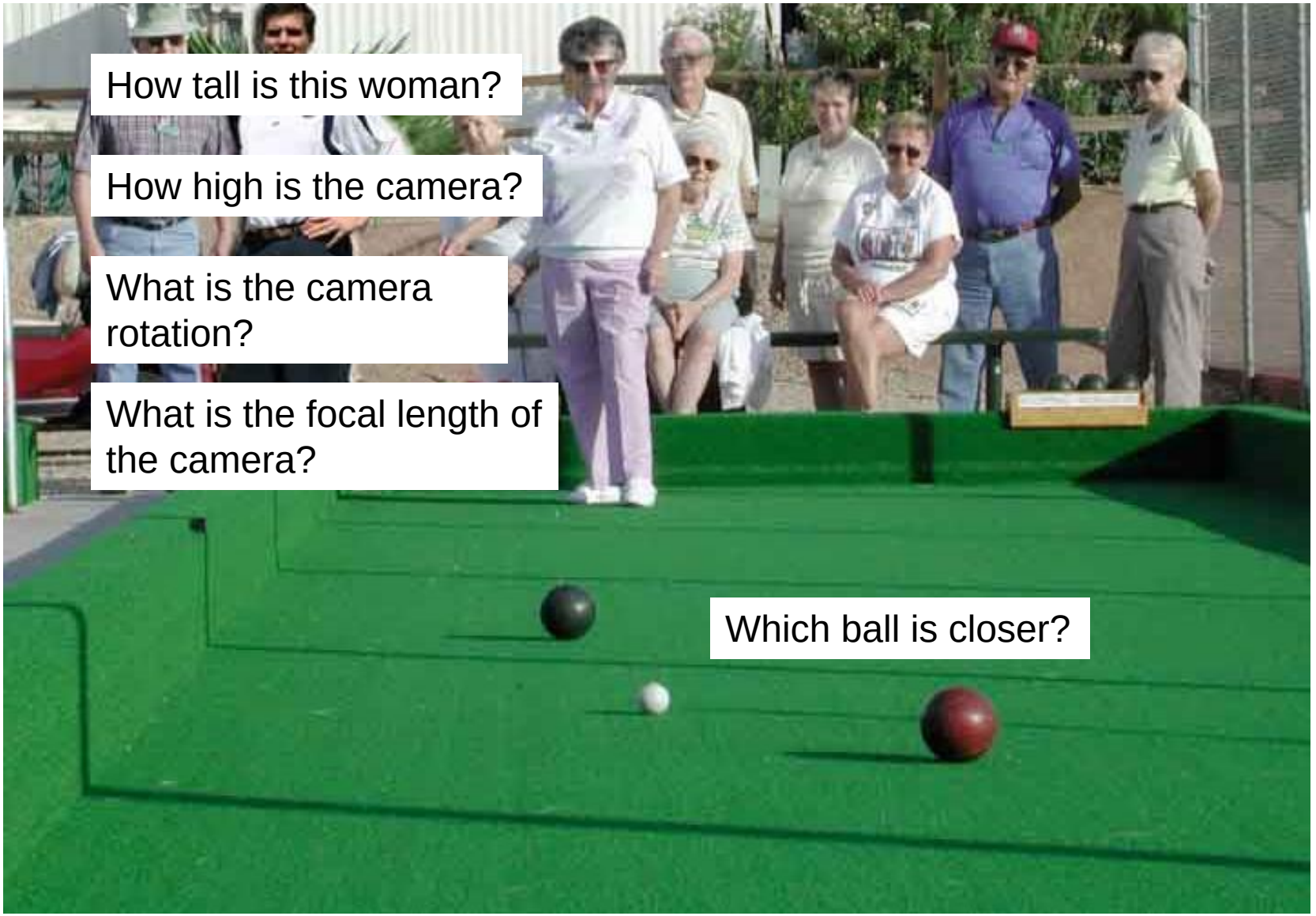
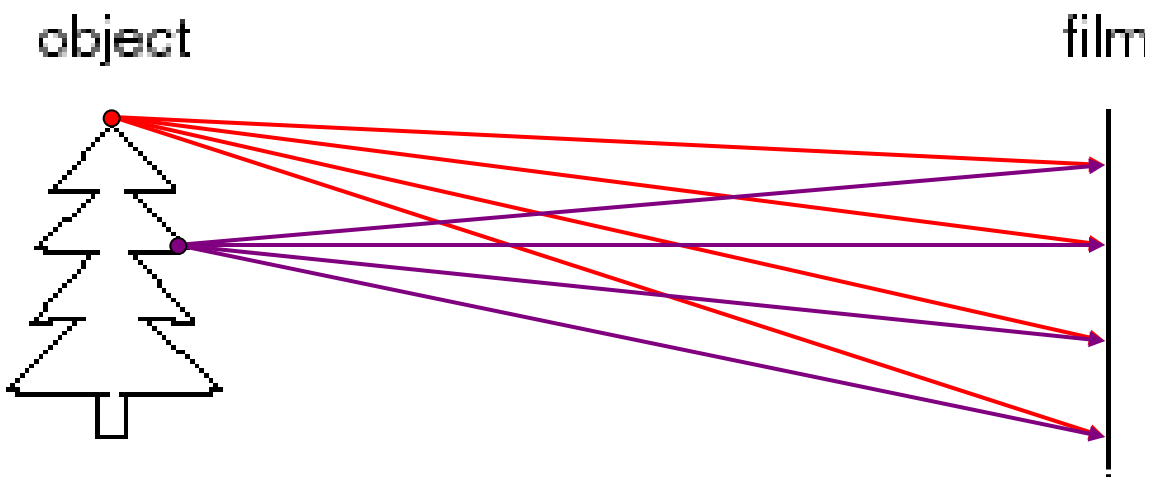


Image formation

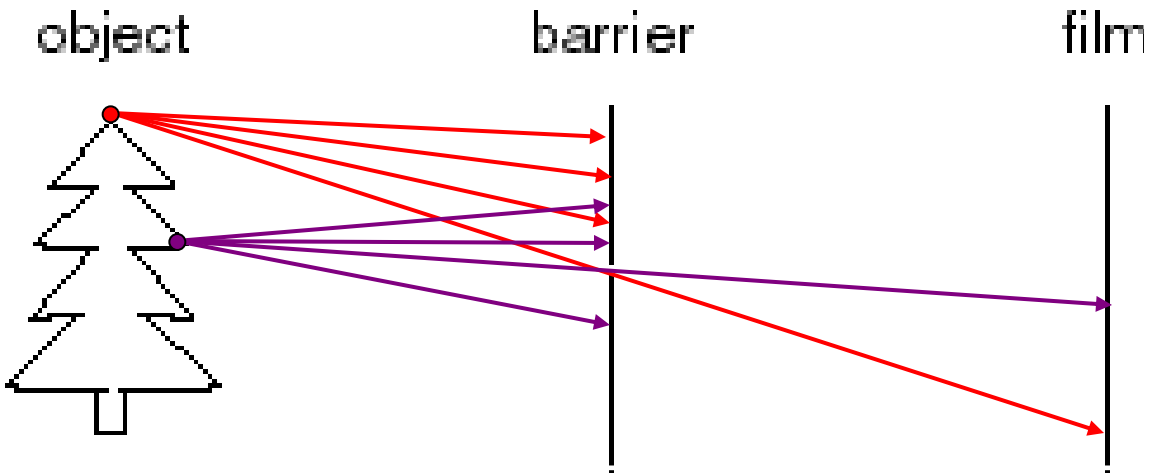


Let's design a camera

- Idea 1: put a piece of film in front of an object
- Do we get a reasonable image?

Slide source: Seitz

Pinhole camera

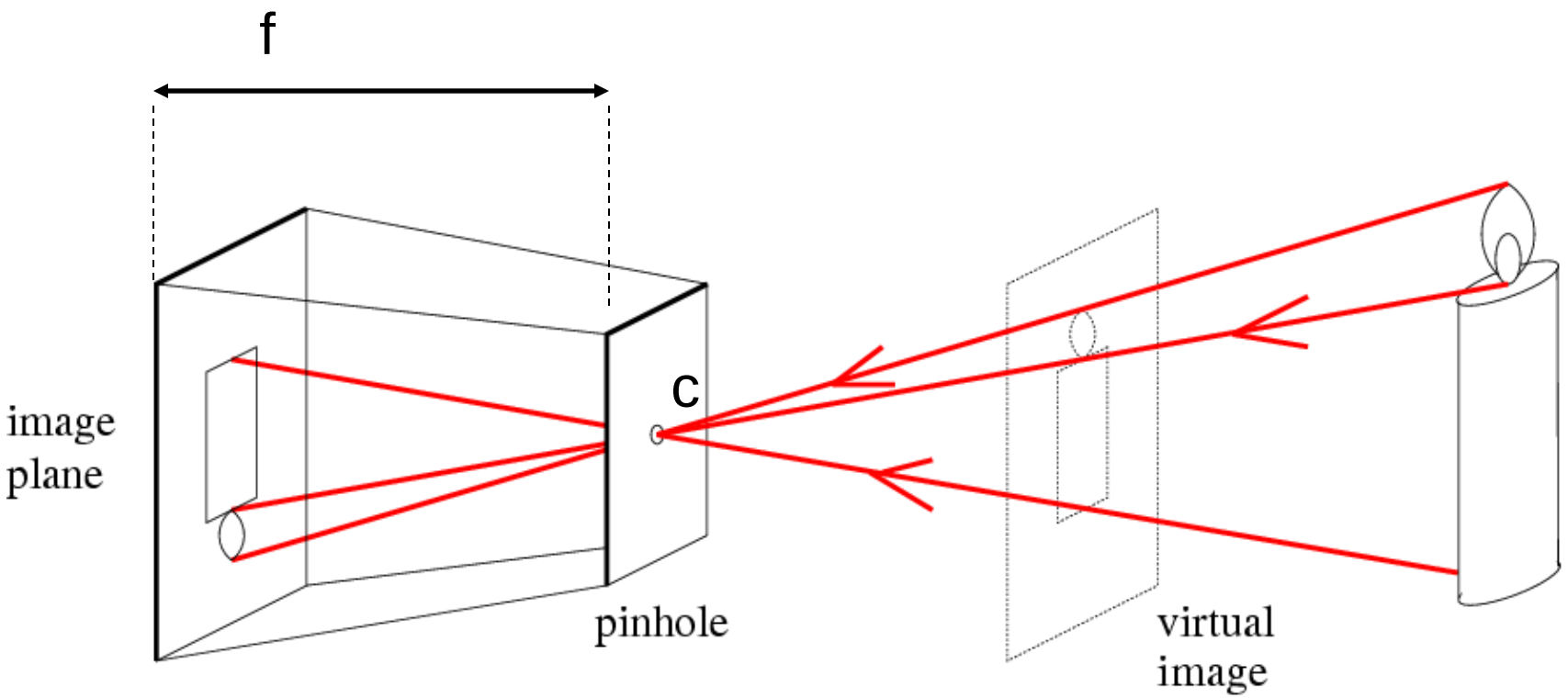


Idea 2: add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**

Slide source: Seitz

Pinhole camera



f = focal length
c = center of the camera

Figure from Forsyth

Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mo-Ti, China, 470BC to 390BC)

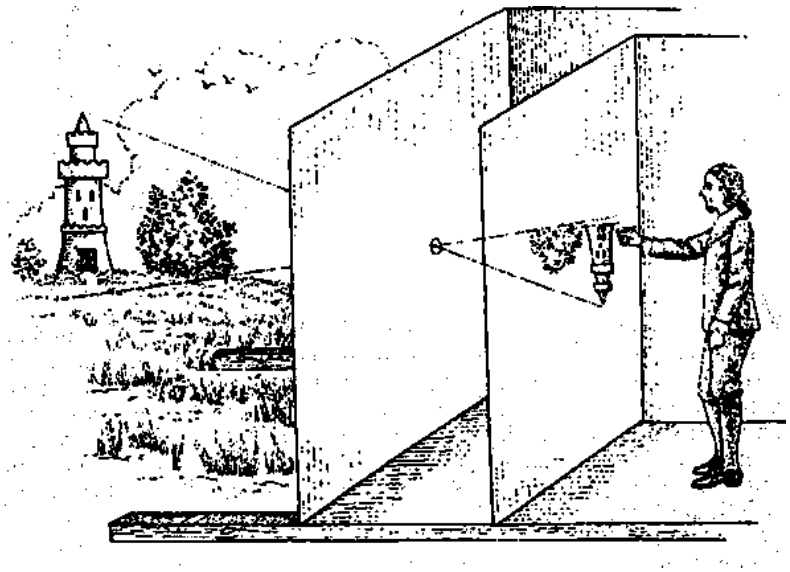
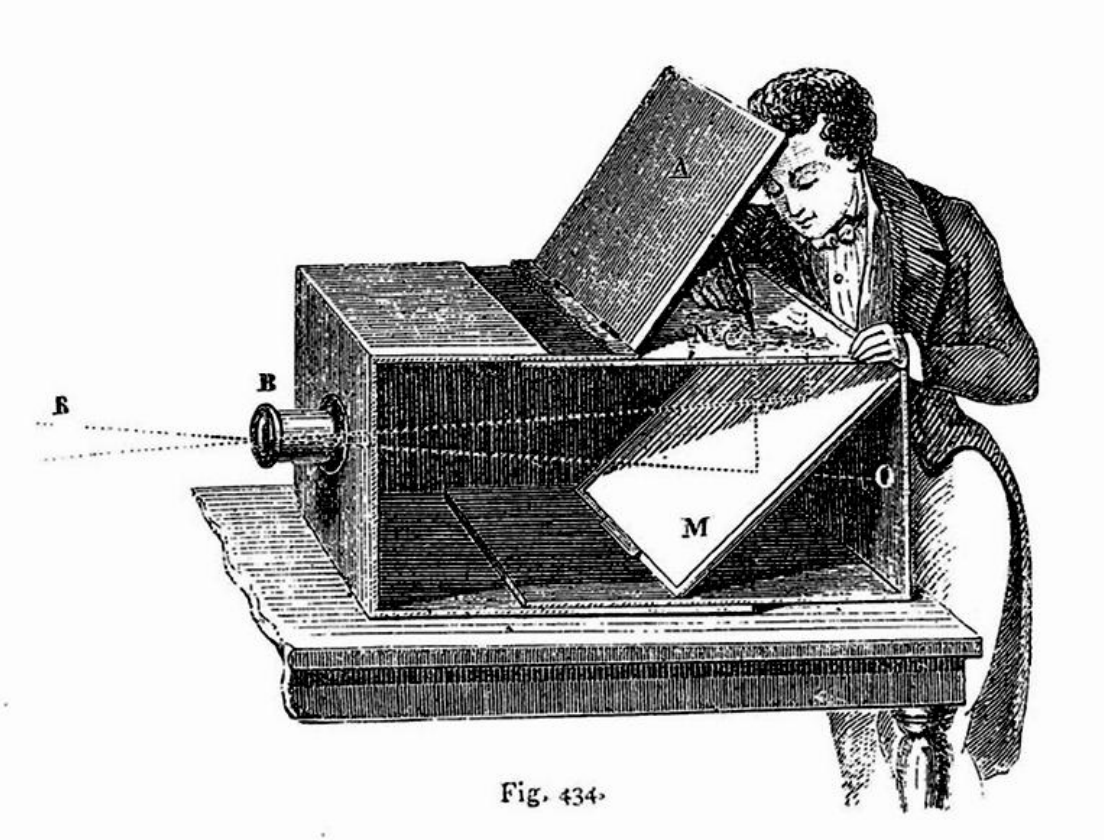


Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys

Camera Obscura used for Tracing



Lens Based Camera Obscura, 1568

First Photograph

Oldest surviving photograph

- Took 8 hours on pewter plate

Joseph Niepce, 1826

Photograph of the first photograph

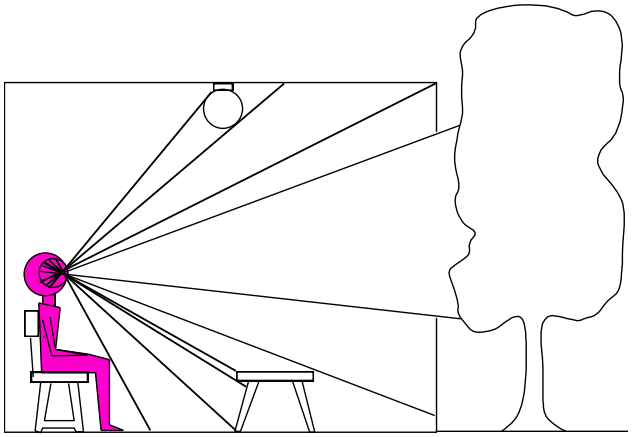
Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes

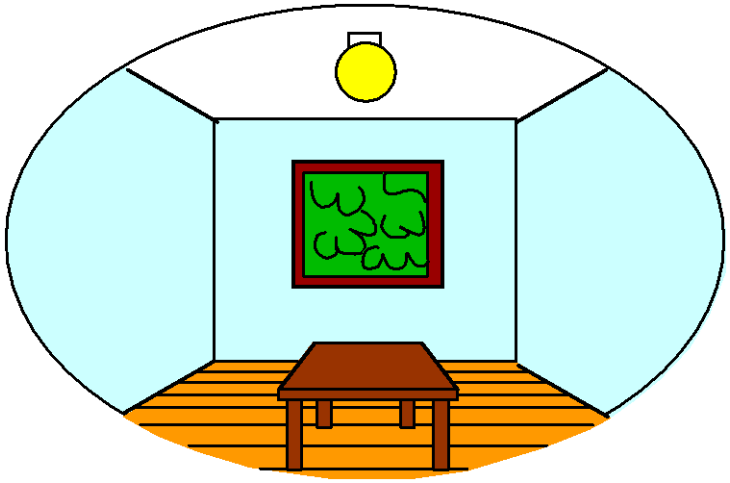
Dimensionality Reduction Machine (3D to 2D)

3D world

2D image



Point of observation



Slide source: Seitz

Projection can be tricky…

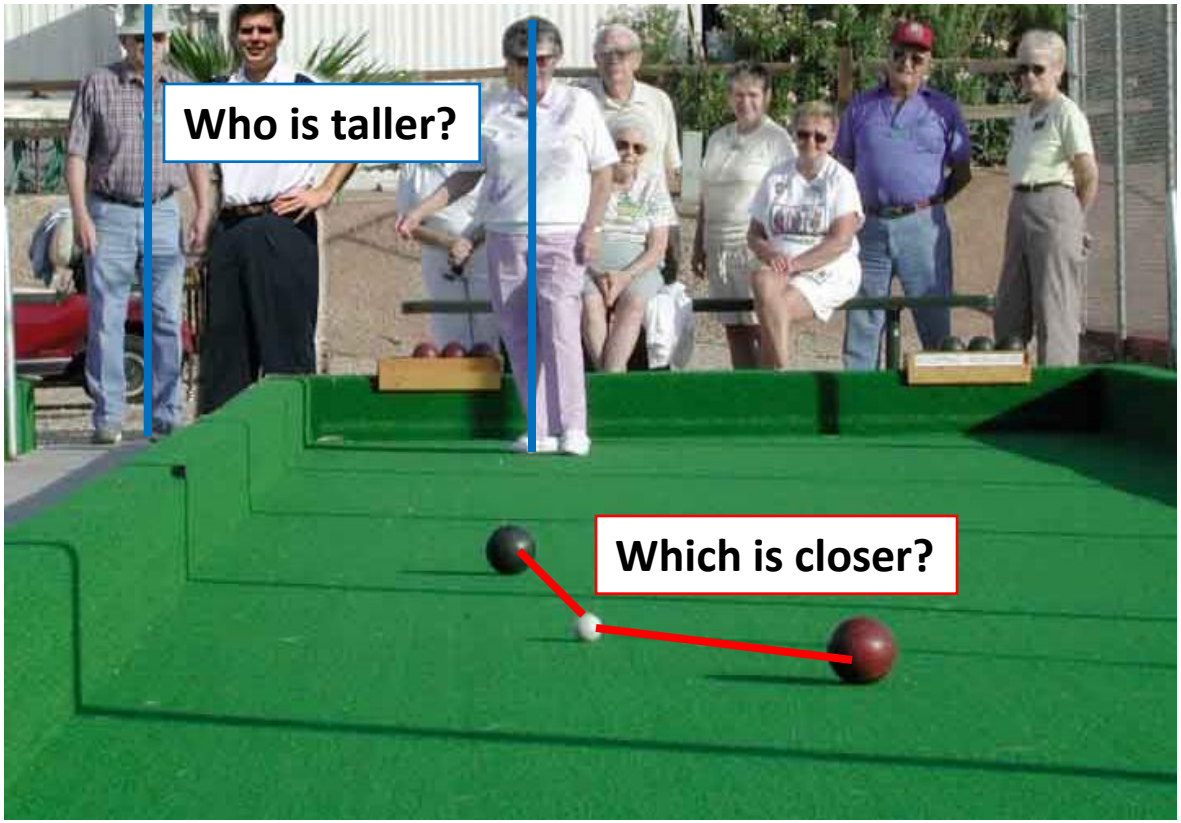
Slide source: Seitz

Projection can be tricky…

Projective Geometry

What is lost?

- Length



Length is not preserved

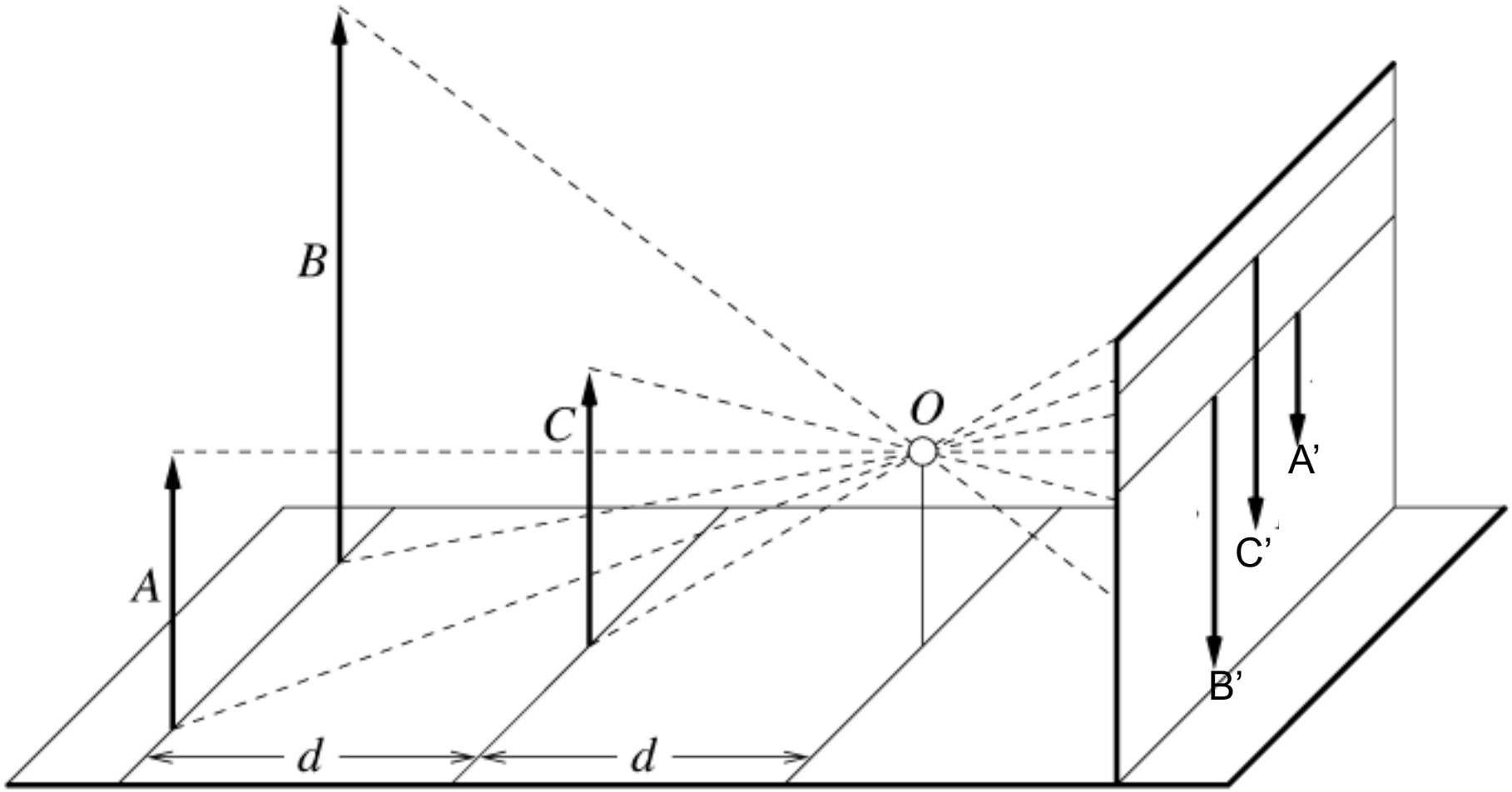
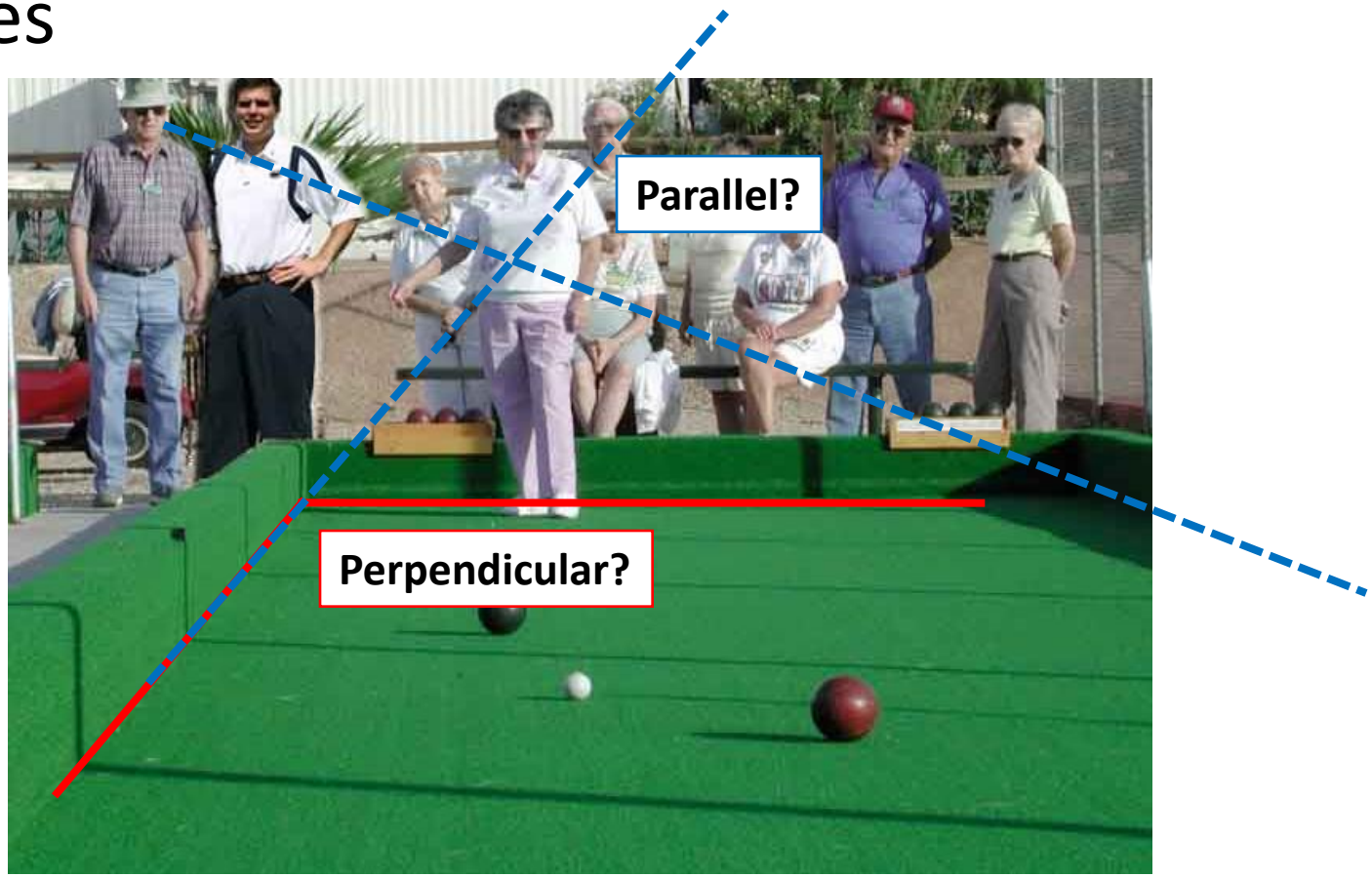


Figure by David Forsyth

Projective Geometry

What is lost?

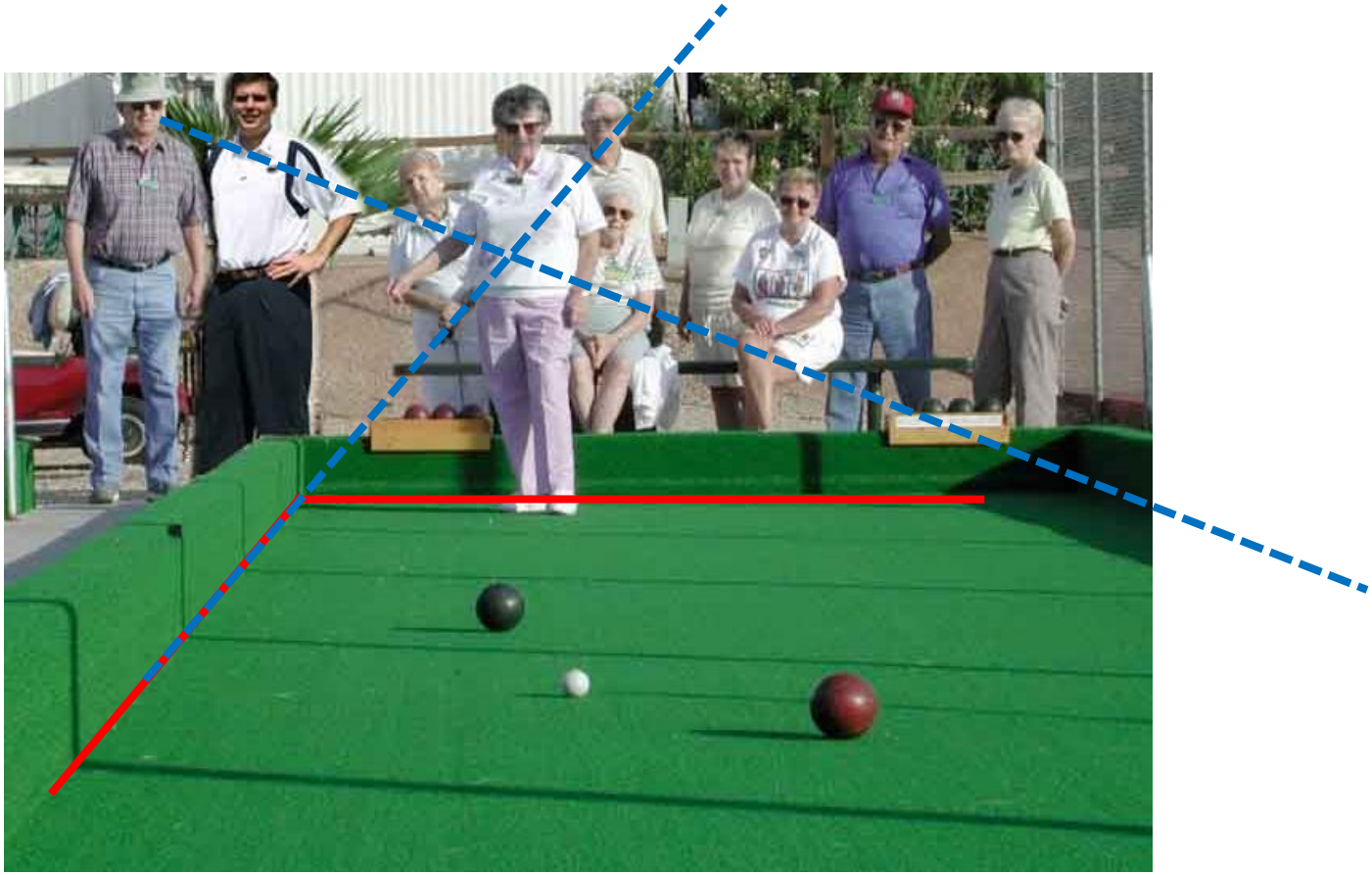
- Length
- Angles



Projective Geometry

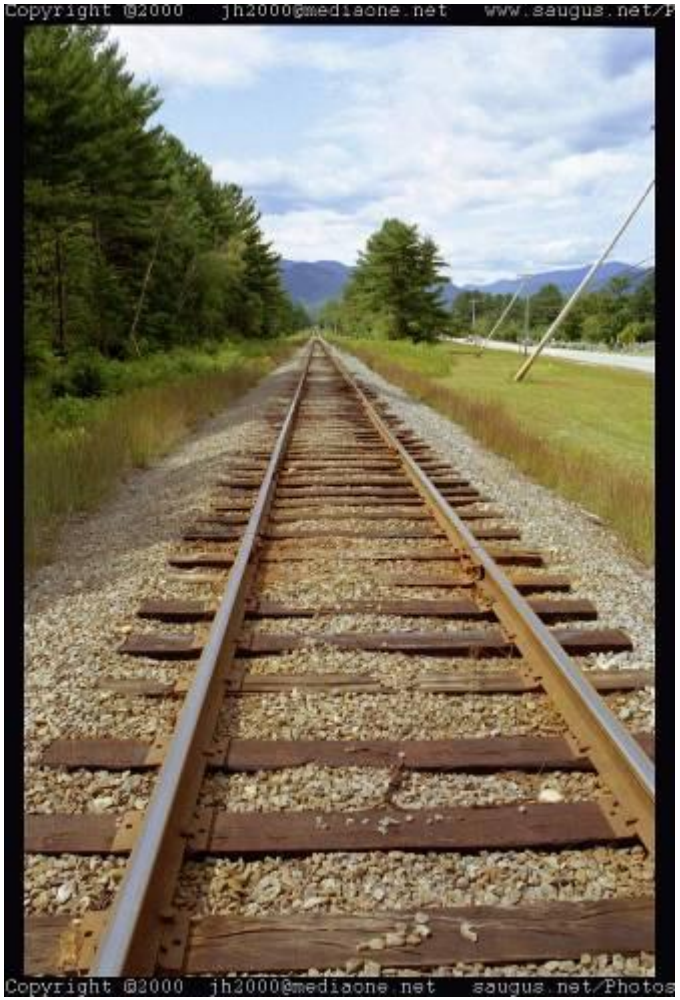
What is preserved?

- Straight lines are still straight

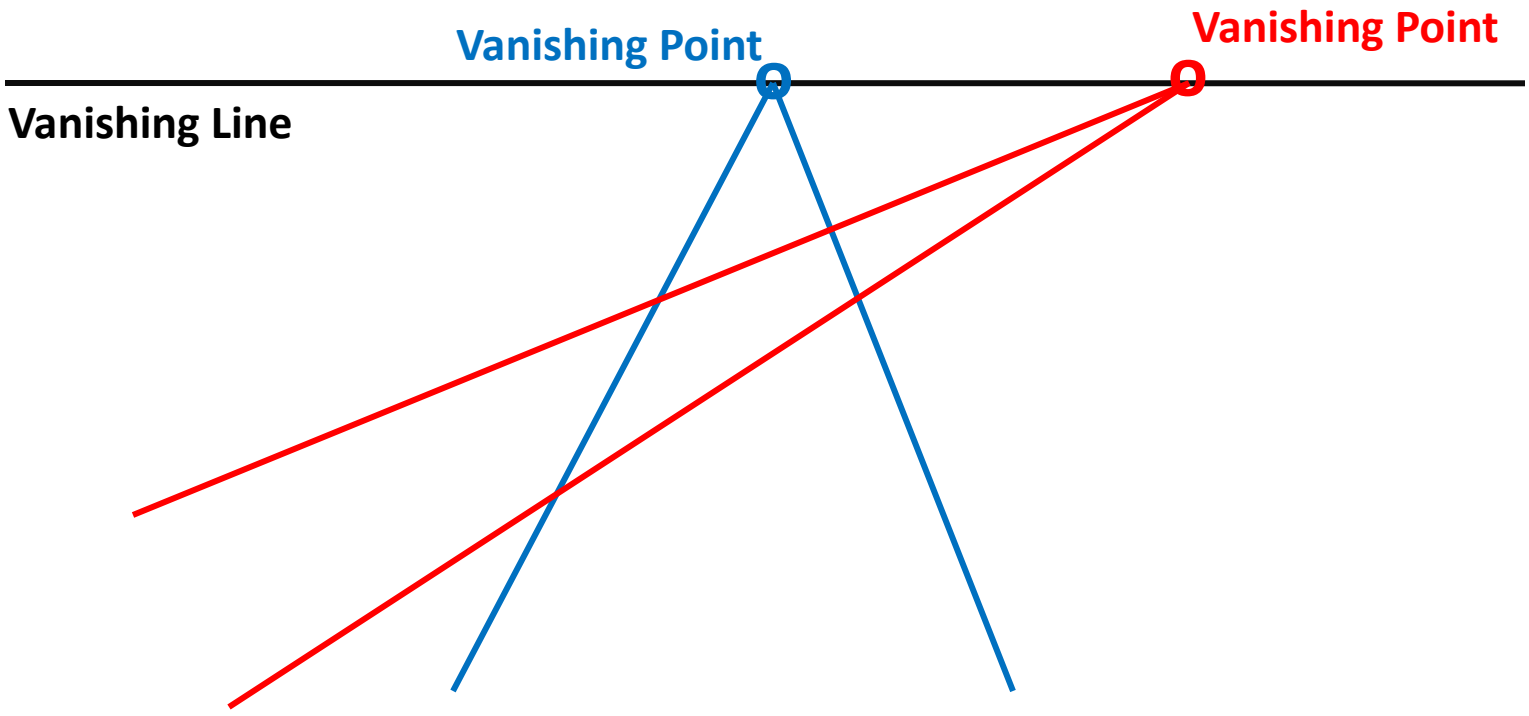


Vanishing points and lines

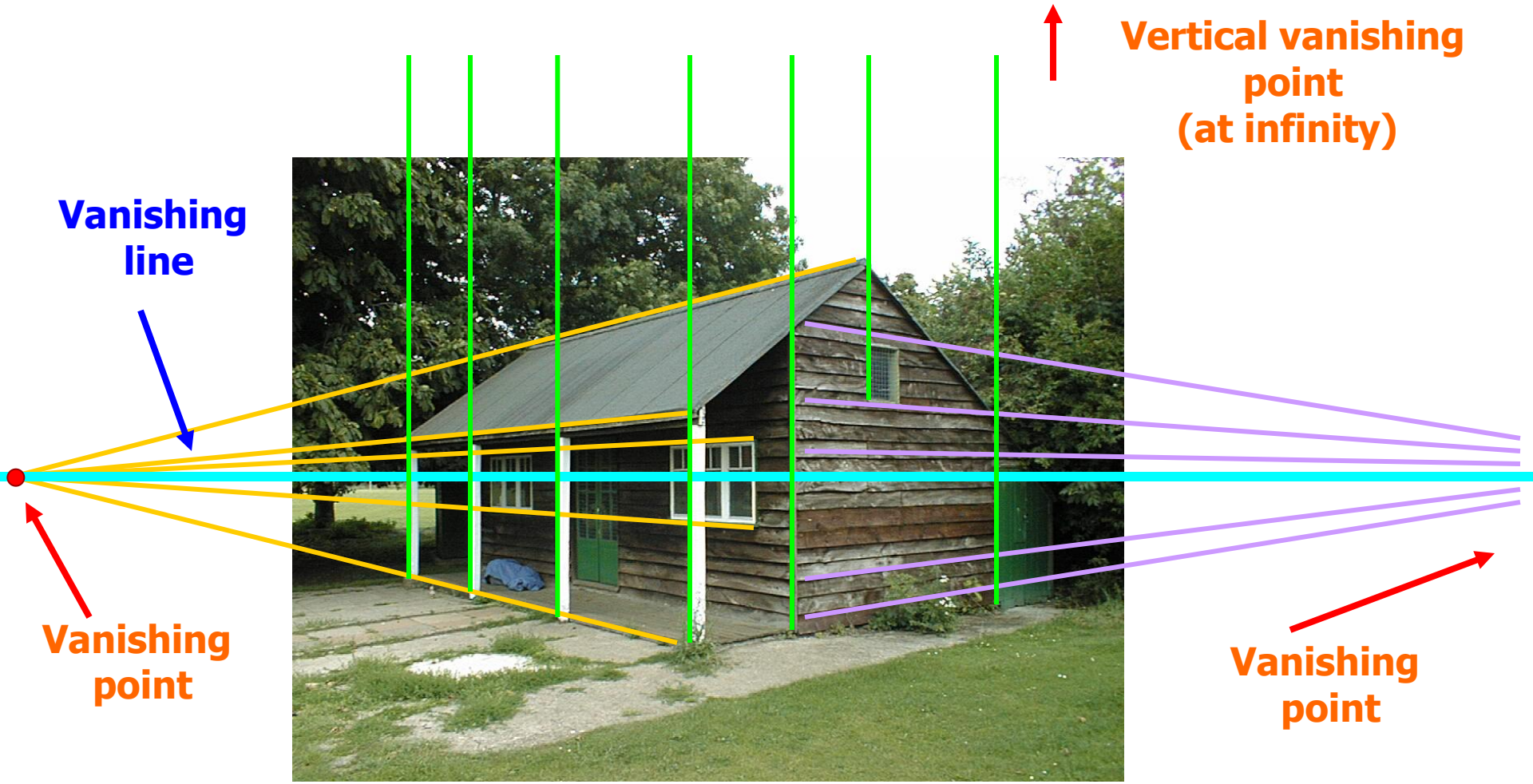
Parallel lines in the world intersect in the image at a “vanishing point”



Vanishing points and lines



Vanishing points and lines

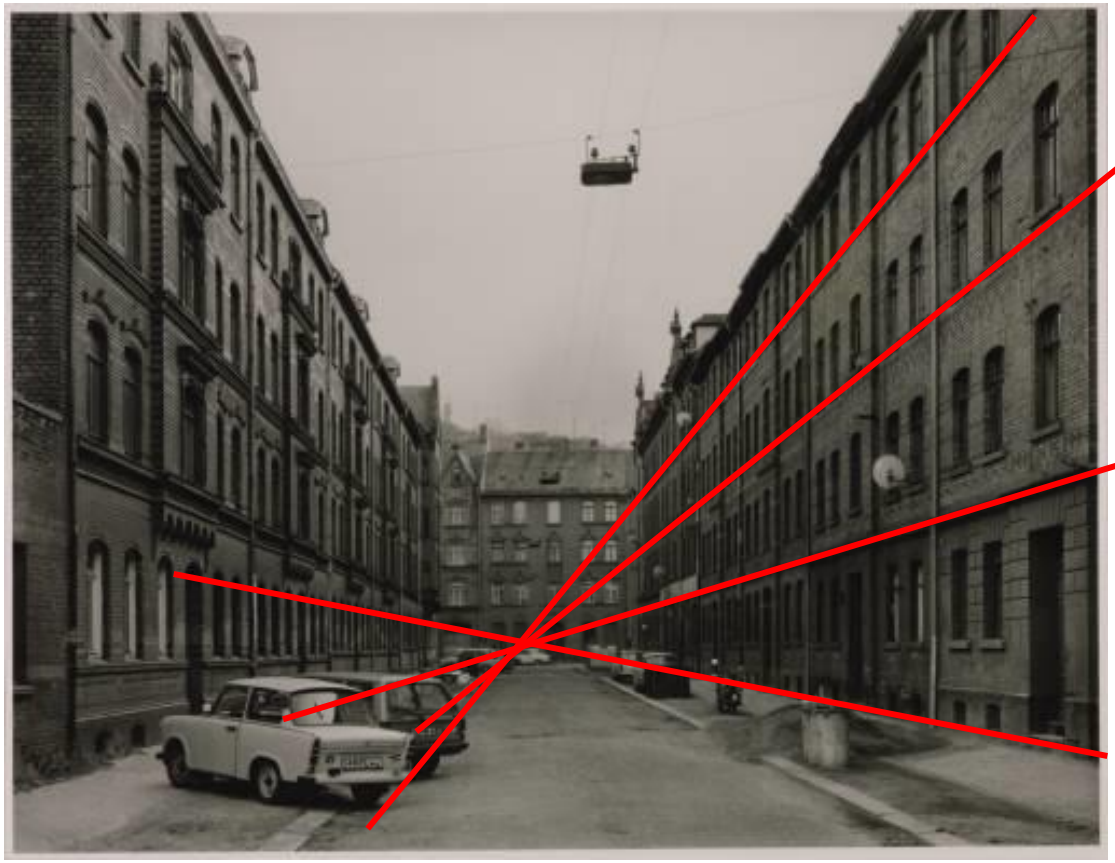


Slide from Efros, Photo from Criminisi

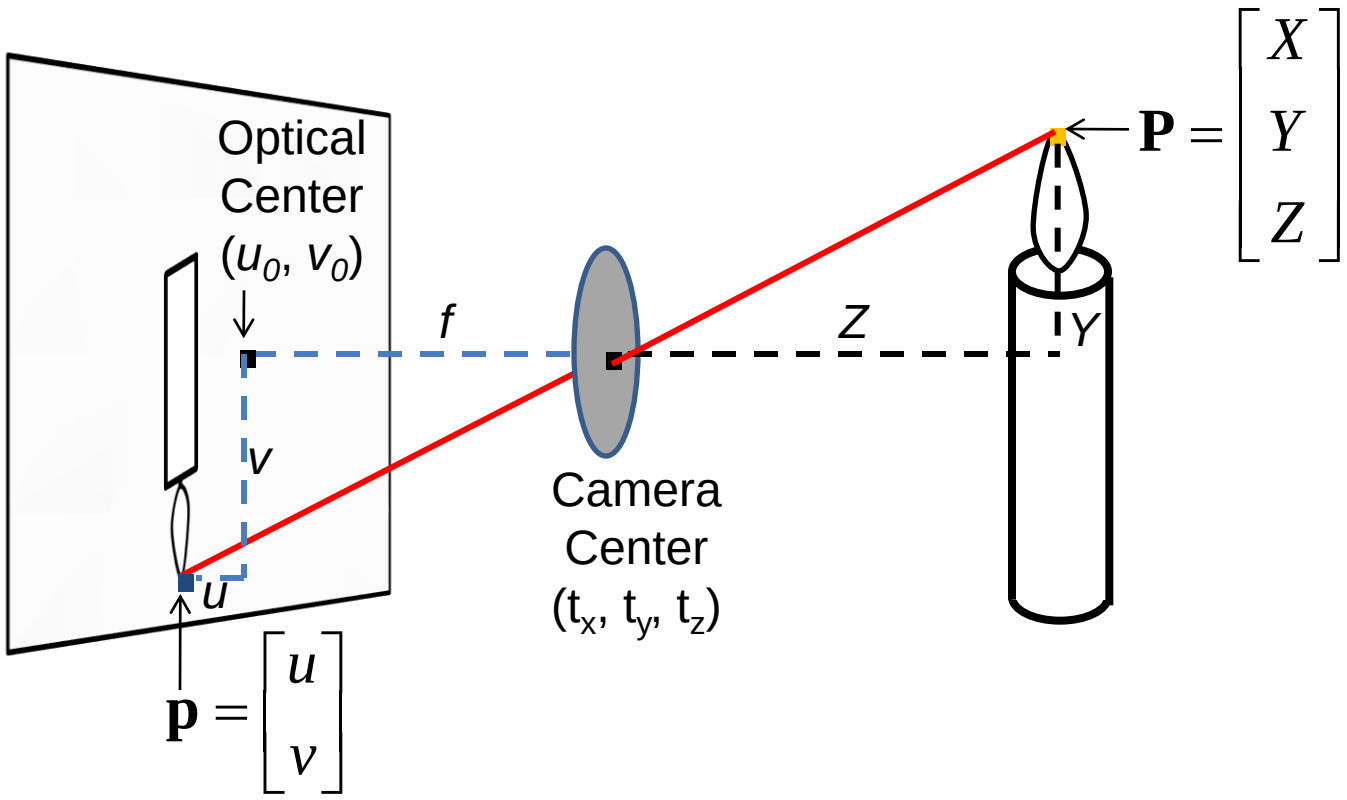
Vanishing points and lines

Photo from online Tate collection

Note on estimating vanishing points



Projection: world coordinates→image coordinates



Homogeneous coordinates

Conversion

Converting to *homogeneous* coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates

Invariant to scaling

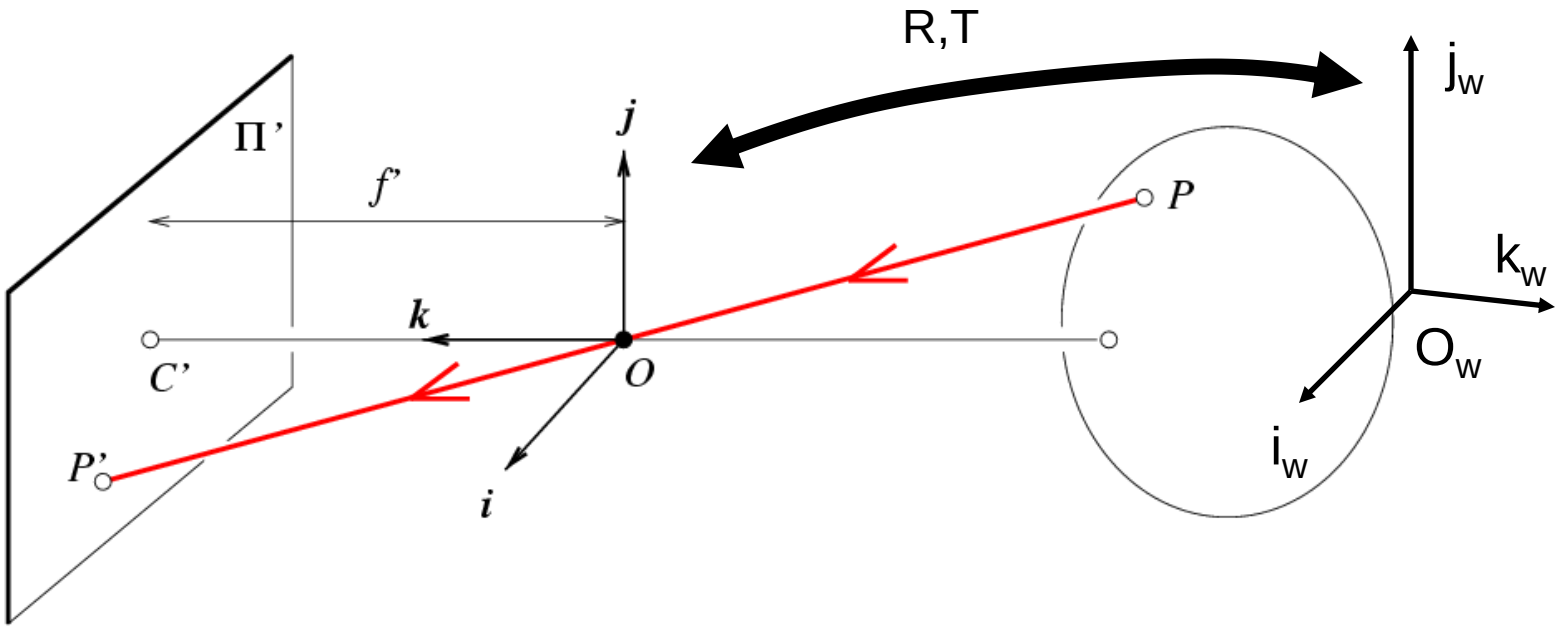
$$k\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} kx \\ ky \\ kw \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{kx}{kw} \\ \frac{ky}{kw} \end{bmatrix} = \begin{bmatrix} \frac{x}{w} \\ \frac{y}{w} \end{bmatrix}$$

Homogeneous Coordinates

Cartesian Coordinates

Point in Cartesian is ray in Homogeneous

Projection matrix

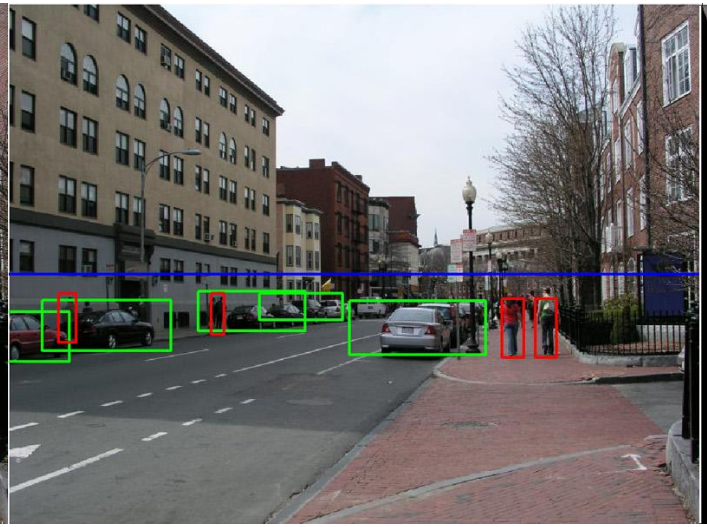
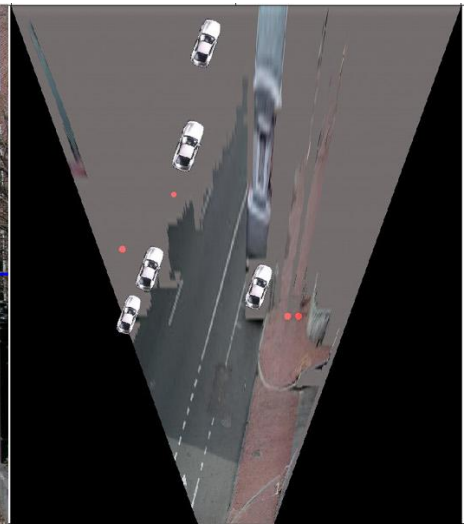


$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

x: Image Coordinates: (u,v,1)
K: Intrinsic Matrix (3x3)
R: Rotation (3x3)
t: Translation (3x1)
X: World Coordinates: (X,Y,Z,1)

Interlude: why does this matter?

Object Recognition (CVPR 2006)

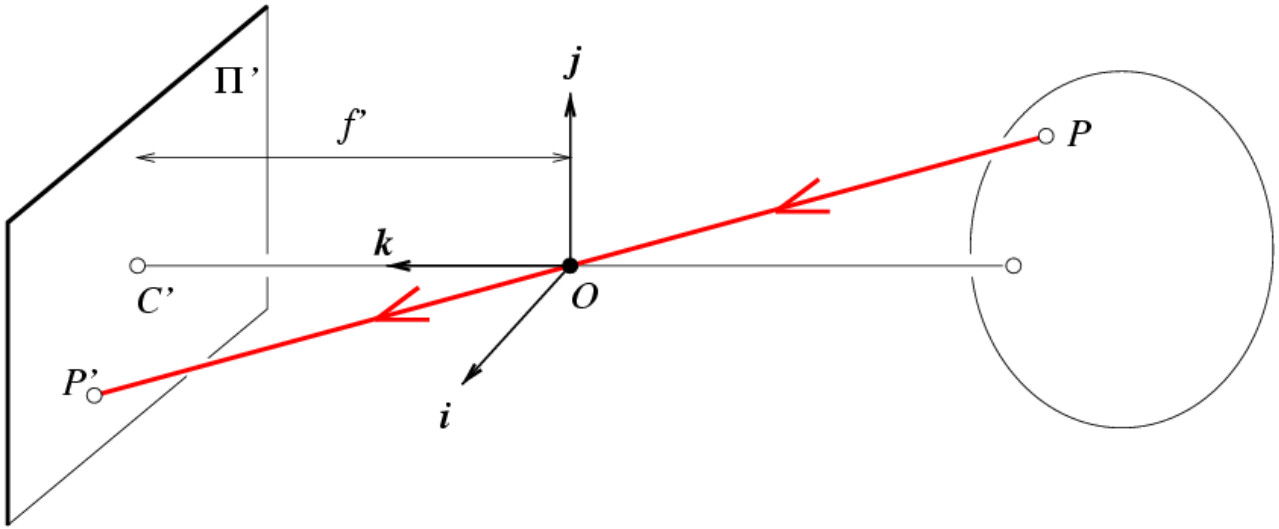




Inserting photographed objects into images (SIGGRAPH 2007)

Original

Created

Projection matrix



Intrinsic Assumptions

- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions

- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\v\\1\end{bmatrix} = \overbrace{\begin{bmatrix}f & 0 & 0\\0 & f & 0\\0 & 0 & 1\end{bmatrix}}^{\mathbf{K}}\begin{bmatrix}0\\0\\0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Slide Credit: Saverese

Remove assumption: known optical center

Intrinsic Assumptions
- Unit aspect ratio
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \begin{bmatrix}f & 0 & u_0 & 0\\ 0 & f & v_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

Remove assumption: square pixels

Intrinsic Assumptions
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \begin{bmatrix}\alpha & 0 & u_0 & 0\\ 0 & \beta & v_0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

Remove assumption: non-skewed pixels

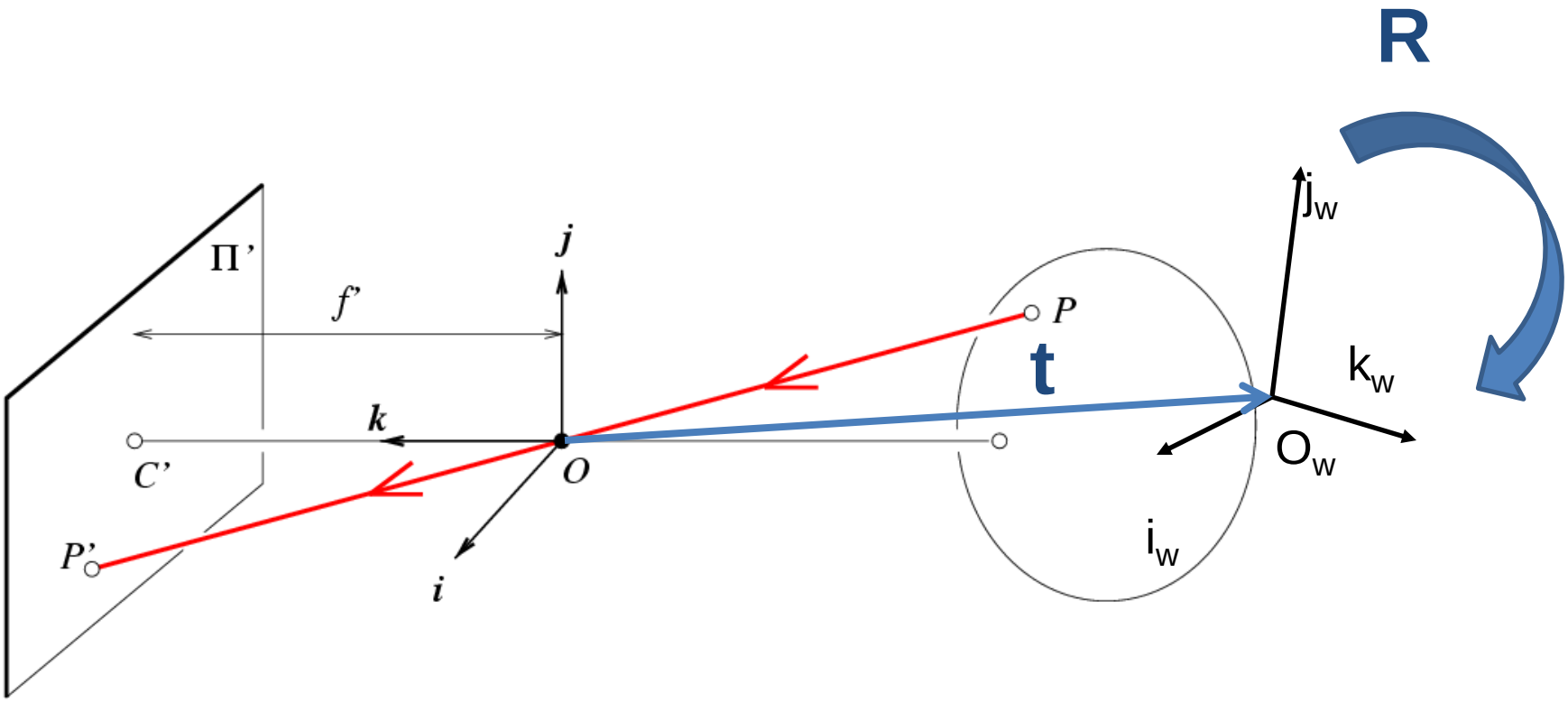
Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Note: different books use different notation for parameters

Oriented and Translated Camera



Allow camera translation

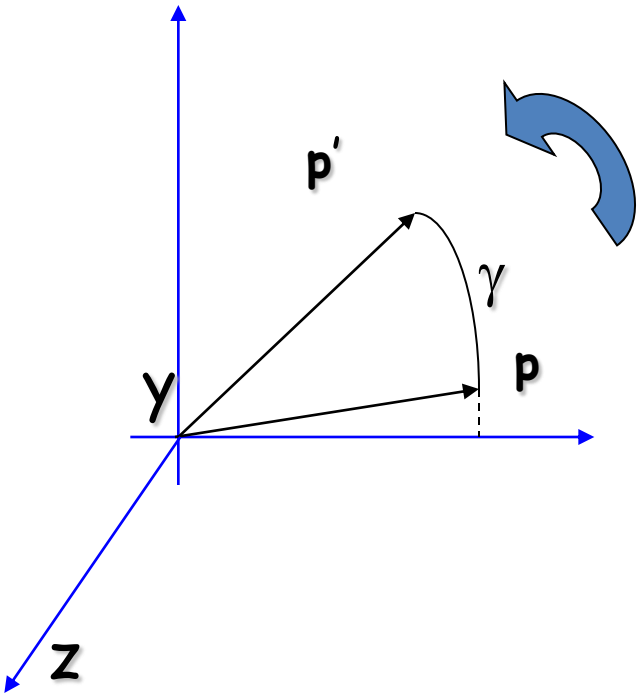
Intrinsic Assumptions Extrinsic Assumptions

- No rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{t}\end{bmatrix}\mathbf{X} \Rightarrow w\begin{bmatrix}u\\ v\\ 1\end{bmatrix} = \begin{bmatrix}\alpha & 0 & u_0\\ 0 & \beta & v_0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & t_x\\ 0 & 1 & 0 & t_y\\ 0 & 0 & 1 & t_z\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ 1\end{bmatrix}$$

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:



$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Allow camera rotation

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Degrees of freedom

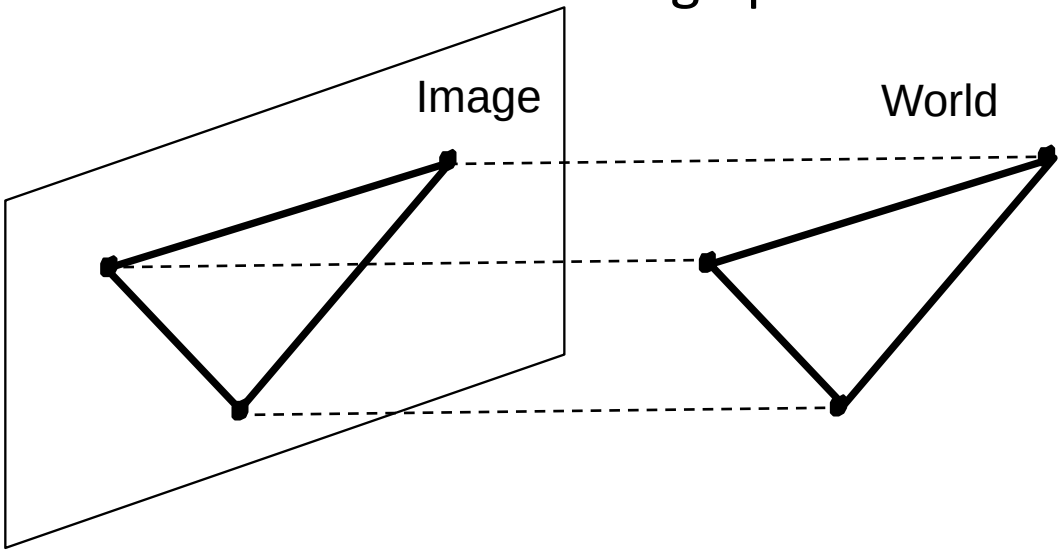
$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

5 6

$$w\begin{bmatrix}u\\v\\1\end{bmatrix} = \begin{bmatrix}\alpha & s & u_0\\0 & \beta & v_0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\r_{21} & r_{22} & r_{23} & t_y\\r_{31} & r_{32} & r_{33} & t_z\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix}$$

Orthographic Projection

- Special case of perspective projection
  - Distance from the COP to the image plane is infinite



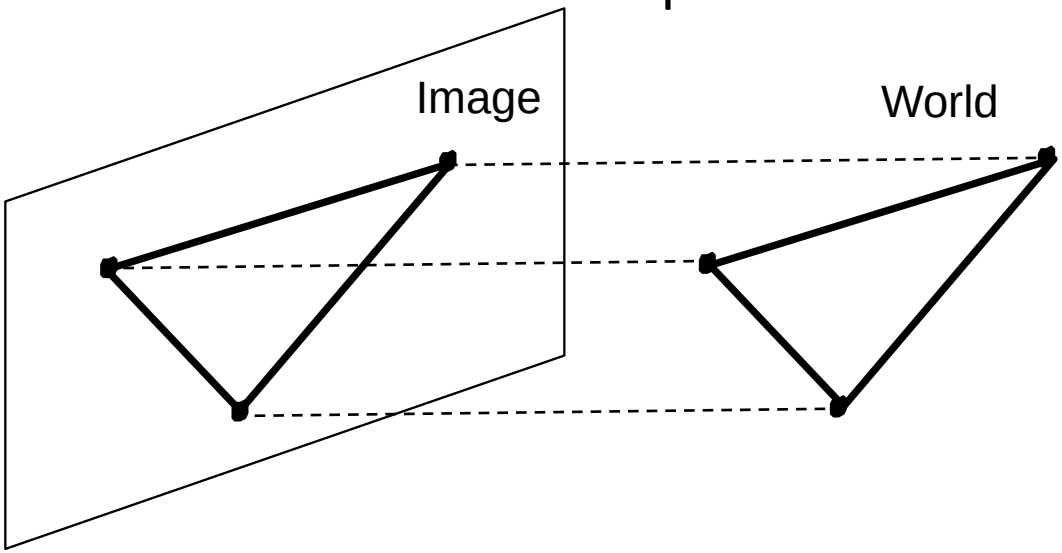
  - Also called “parallel projection”
  - What’s the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide by Steve Seitz

Scaled Orthographic Projection

- Special case of perspective projection
  - Object dimensions are small compared to distance to camera

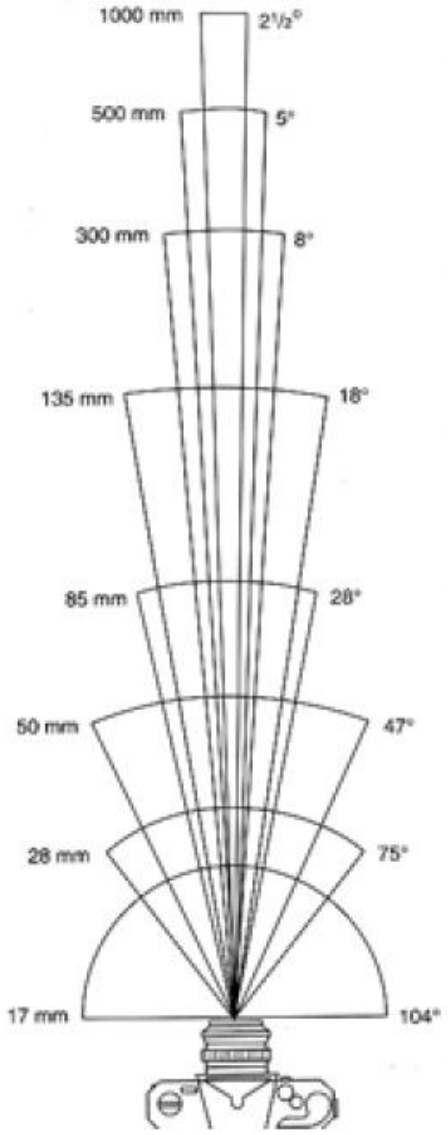


  - Also called “weak perspective”
  - What’s the projection matrix?

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 0 & s \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Slide by Steve Seitz

Field of View (Zoom)



17mm

28mm

50mm

85mm

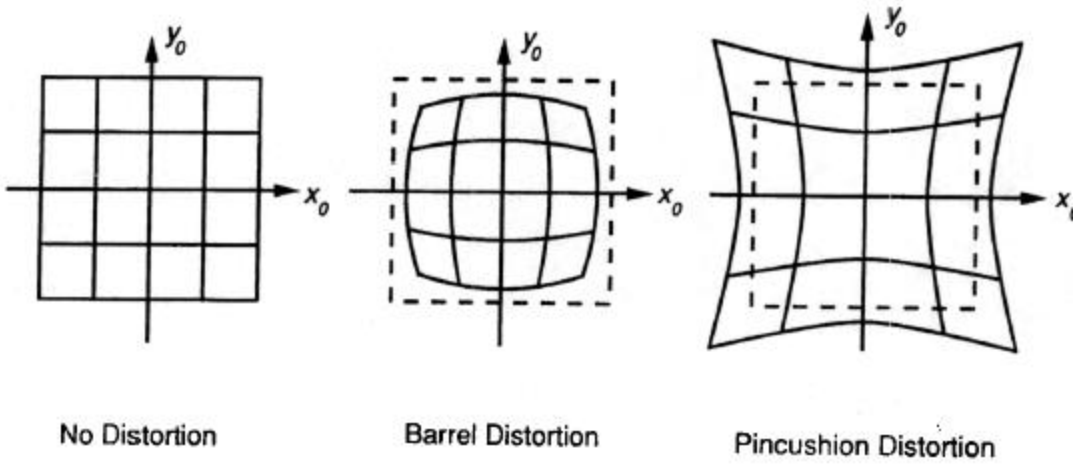
From London and Upton

Suppose we have two 3D cubes on the ground facing the viewer, one near, one far.

1. What would they look like in perspective?
2. What would they look like in weak perspective?

Photo credit: GazetteLive.co.uk

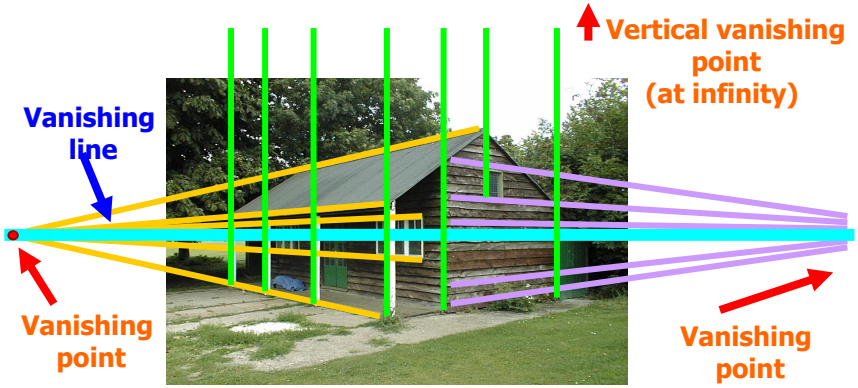
Beyond Pinholes: Radial Distortion



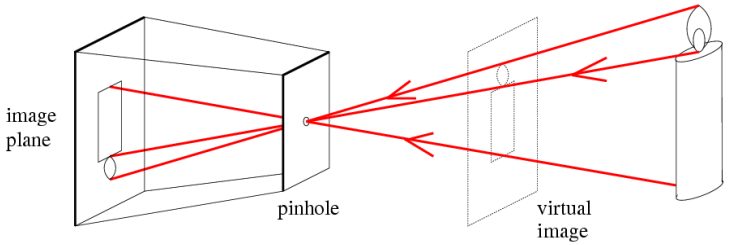
Corrected Barrel Distortion

Image from Martin Habbecke

Things to remember



- Vanishing points and vanishing lines



- Pinhole camera model and camera projection matrix

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

- Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$