Reminder: Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor ED discussion, as we will post clarifications of questions there.

Problem 1

Consider the following languages. For each language, determine whether you can use Rice’s Theorem to prove it is undecidable. If so, use Rice’s Theorem to prove it is undecidable. If not, explain why you cannot use Rice’s Theorem, and prove it is undecidable without using Rice’s Theorem.

1. $L_{101} = \{ < M > | M$ is a TM and $101 \in L(M) \}$

2. $L_{SEQ} = \{ < M_1 >, < M_2 > | M_1, M_2$ are TMs and $L(M_1) \subseteq L(M_2) \}$

Problem 2

A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a PDA has any useless states. Formulate this problem as a language and show that it’s decidable.

*Hint:* Remember that determining whether a PDA has an empty language is decidable.