HW2

September 28, 2023 at 10:20 am

Reminder: Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor ED discussion, as we will post clarifications of questions there.

Problem 1

Given the following Regular Expression and NFA, convert the regular expression into an automaton and automaton into a regular expression. Show the steps of the transformation.

1. a(abb)*∪ b

2.

![Diagram](image)

Problem 2

1. Show that if $M$ is a DFA that recognizes language $B$, swapping the accept and non-accept states in $M$ yields a new DFA that recognizes the complement of $B$. Conclude that the class of regular languages is closed under complement.
2. Show by giving an example that if $M$ is an NFA that recognizes language $C$, swapping the accept and non-accept states in $M$ doesn’t necessarily yield a new NFA that recognizes the complement of $C$.

3. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

**Problem 3**

1. Give a regular expression generating the language. $L_1 = \{ w \ | \ w$ contains at least 2 1s $\}$ over $\Sigma = \{0,1\}$.

2. Give a regular expression generating the language. $L_2 = \{ w \ | \ w$ contains no substring '010' $\}$ over $\Sigma = \{0,1\}$.

**Problem 4**

A homomorphism is a function $h : \Sigma^* \rightarrow \Delta^*$, where $\Delta$ is a finite alphabet defined as follows:

- $h(\epsilon)$;
- For any $a \in \Sigma$, $h(a)$ is any string in $\Delta^*$;
- For $a \in \Sigma^*$ such that $a = a_1 a_2 \ldots a_n$ with $n \geq 2$, $h(a) = h(a_1)h(a_2) \ldots h(a_n)$.
- A homomorphism is a function from strings to strings that “respects” concatenation: for any $x, y \in \Sigma^*$, $h(xy) = h(x)h(y)$.

**Example** $h : \{0,1\} \rightarrow \{a,b\}^*$ where $h(0) = ab$ and $h(1) = ba$. Then $h(011) = ababbaba$.

Given a homomorphism: $h : \Sigma^* \rightarrow \Delta^*$ and a language $L \subseteq \Sigma^*$, define $h(L) = \{ h(w) | w \in L \} \subseteq \Delta^*$.

Prove that the set of regular languages is closed under homomorphism.