10–23

CS 53, Fall 2017

Due October 25 at 2:59 pm

Do the following problems, then do Problem 3 in `hw10-18.pdf` provided earlier. The following problems use the stencil `exchange_lemma_practice.py`. The third problem uses the stencil `The_Basis_other_problems.py`

Exchange Lemma for vectors over $\mathbb{R}$

Problem 1: Let $S = \{[1,0,0,0,0], [0,1,0,0,0], [0,0,1,0,0], [0,0,0,1,0], [0,0,0,0,1]\}$, and let $A = \{[1,0,0,0,0], [0,1,0,0,0]\}$. For each of the following vectors z, find a vector w in $S - A$ such that Span $S =$ Span $(S \cup \{z\} - \{w\})$.

(a) $z = [1,1,1,1,1]$

(b) $z = [0,1,0,1,0]$

(c) $z = [1,0,1,0,1]$

Exchange Lemma for vectors over $GF(2)$

Problem 2: We refer in this problem to the vectors over $GF(2)$ specified in Problem 5.14.4.

Let $S = \{v_1, v_2, v_3, v_4\}$. Each of the following parts specifies a subset A of S and a vector z such that $A \cup \{z\}$ is linearly independent. For each part, specify a vector w in $S - A$ such that Span $S =$ Span $(S \cup \{z\} - \{w\})$. (Hint: Drawing subgraphs of the graph will help.)

(a) $A = \{v_1, v_4\}$ and z is

a	b	c	d	e	f	g	h
		1	1				

(b) $A = \{v_2, v_3\}$ and z is

a	b	c	d	e	f	g	h
	1	1					

(c) $A = \{v_2, v_3\}$ and z is

a	b	c	d	e	f	g	h
1				1			

Finally, remember to do Problem 3.