09-25

CS 53, Fall 2017

Due September 27 at 2:59 pm

Problem 1: Let C ={'#','$','%'}. Define four C-vectors $\boldsymbol{a}_0, \boldsymbol{a}_1\boldsymbol{a}_2, \boldsymbol{a}_3$, according to the following table:

	#	$	%
$\boldsymbol{a}_0$	2	3	4
$\boldsymbol{a}_1$	1	0	-1
$\boldsymbol{a}_2$	3	4	5
$\boldsymbol{a}_3$	8	4	2

1. Form Vecs representing the vectors $\boldsymbol{a}_0, \boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$.
2. Write a procedure that, given a C-vector $\boldsymbol{x}$, returns the list $[\boldsymbol{a}_0 \cdot \boldsymbol{x}, \boldsymbol{a}_1 \cdot \boldsymbol{x}, \boldsymbol{a}_2 \cdot \boldsymbol{x}, \boldsymbol{a}_3 \cdot \boldsymbol{x}]$.

Problem 2: In this problem, we will represent a row-matrix as a list of C-vectors represented as Vecs. Write a one-line procedure `row_matrix_times_vec(M, x)` that, given a row-matrix M thus represented and given a C-vector $\boldsymbol{x}$ represented as a Vec, returns the product of the row-matrix and the vector $\boldsymbol{x}$. The output should be represented as a list of scalars. Recall that a row-matrix is intended to represent a dot-product function, and that multiplying a row-matrix by a vector is equivalent to applying the corresponding function to the vector.

Problem 3: Let C ={'#','$','%'}. Define four vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$, all with domain C, according to the following table:

	#	$	%
$\boldsymbol{a}$	2	3	4
$\boldsymbol{b}$	1	0	-1
$\boldsymbol{c}$	3	4	5
$\boldsymbol{d}$	8	4	2

1. Form Vecs representing the vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}$.
2. Write a procedure that, given a C-vector $\boldsymbol{x}$, returns the {'a','b','c','d'}-vector, represented as a Vec, that maps 'a' to $\boldsymbol{a} \cdot \boldsymbol{x}$, maps 'b' to $\boldsymbol{b} \cdot \boldsymbol{x}$, maps 'c' to $\boldsymbol{c} \cdot \boldsymbol{x}$, and maps 'd' to $\boldsymbol{d} \cdot \boldsymbol{x}$.

Problem 4: Let R ={'a', 'b', 'c', 'd'}. Define three R-vectors $\boldsymbol{v}_0, \boldsymbol{v}_1, \boldsymbol{v}_2$, according to the following table:

	'a'	'b'	'c'	'd'
$\boldsymbol{v}_0$	2	1	3	8
$\boldsymbol{v}_1$	3	0	4	4
$\boldsymbol{v}_2$	4	-1	5	2

1. Form Vecs representing the vectors $\boldsymbol{v}_0, \boldsymbol{v}_1, \boldsymbol{v}_2$.
2. Write a procedure that, given a three-element list $[\alpha_0, \alpha_1, \alpha_2]$, returns the R-vector that is the value of the linear combination $\alpha_0 \boldsymbol{v}_0 + \alpha_1 \boldsymbol{v}_1 + \alpha_2 \boldsymbol{v}_2$. The output should represented as an Vec.

Problem 5: In this problem, we will represent a column-matrix as a list of R-vectors represented as Vecs. Write a one-line procedure `col_matrix_times_vec(M, x)` that, given a column-matrix M thus represented and given a vector $\boldsymbol{x}$ represented as a list of scalars, returns the R-vector that is the product of the column-matrix M and the vector $\boldsymbol{x}$. Recall that a column-matrix is intended to represent a linear-combinations function,and that multiplying a row-matrix by a vector is equivalent to applying the corresponding function to the vector.

Problem 6: Find a set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ of vectors (n is up to you), each represented as a list, whose affine hull equals

$$[4, 5, 6] + \text{Span } \{[-3, -2, -1], [7, 8, 0]\}$$

Problem 7: Find a vector $\boldsymbol{u}$ and a set $\{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ of vectors (n is up to you), each represented by a list, such that $\boldsymbol{u} + \text{Span } \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n\}$ equals the affine hull of

$$\{[256, 512, 1024], [3.14159, 2.718281828, 53], [1, 10, 100]\}$$