Propositional Logic

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Overview

Truth tables

Truth tables help convey how to interpret logical statements.

P	NOT(P)
F	**T**
T	**F**

P	Q	P AND Q
F	**F**	**F**
F	**T**	**F**
T	**F**	**F**
T	**T**	**T**

Like a multiplication table, but flattened (so easier to apply to more inputs.)

OR

P	Q	P OR Q
F	**F**	**F**
F	**T**	**T**
T	**F**	**T**
T	**T**	**T**

In English, often rendered as "P or Q or both". Inclusive or.

P	Q	P XOR Q
F	**F**	**F**
F	**T**	**T**
T	**F**	**T**
T	**T**	**F**

Exclusive or, too.

OR examples

"Someday I want to visit London (X) or Paris (Y)."

X	Y	sentence
F	**F**	**F**
F	**T**	**T**
T	**F**	**T**
T	**T**	**T**

X OR Y

"With seatbelts, it's click it (X) or ticket (Y)."

X	Y	sentence
F	**F**	**F**
F	**T**	**T**
T	**F**	**T**
T	**T**	**F**

X XOR Y

More OR Examples

"The recipe calls for milk chocolate (X) or peanut (Y) M&Ms."

X	Y	sentence
F	**F**	**F**
F	**T**	**F**
T	**F**	**T**
T	**T**	**T**

X

"You may select the chicken (X) or the beef (Y)."

X	Y	sentence
F	**F**	**T**
F	**T**	**T**
T	**F**	**T**
T	**T**	**F**

NOT(X AND Y) aka X NAND Y

Implication

Perhaps counterintuitive.

X	Y	X IMPLIES Y
F	**F**	**T**
F	**T**	**T**
T	**F**	**F**
T	**T**	**T**

Claim: If factoring can be performed easily, then $(2x)^2$ is even for all integers x.

True or false? True. X IMPLIES Y is guaranteed to be true if Y is true.

Definition: In "X IMPLIES Y", X is called the *hypothesis* and Y the *conclusion*.

X IMPLIES Y is the same as Y OR NOT(X).

Truth table for iff

X	Y	X IFF Y
F	**F**	**T**
F	**T**	**F**
T	**F**	**F**
T	**T**	**T**

Example: A number is divisible by three *if and only if* the sum of its digits is divisible by three.

Definitions have this form. Can also be written NOT(X XOR Y). Or (X AND Y) or (NOT(X) AND NOT(Y)).