Relations

Michael L. Littman

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Overview

Sequences (4.2)

Binary Relations (4.4)

Definition

Definition: A *sequence* is an ordered list of items.

Like a set: Collection of items.

Unlike a set: Order matters, can have repeats.

Simple example: Cartesian coordinates like (x, y).

Definition: The *empty sequence* is a sequence of length 0 and is denoted λ.

Operation on sets: Product

Definition: The product of two sets A and B is written $A \times B$ and is the set of all length-two sequences where the first element of the sequence comes from A and second from B.

Example:

- $X = \{1, 2\}$
- $Y = \{0, 2, 4\}$
- $X \times Y = \{(1, 0), (1, 2), (1, 4), (2, 0), (2, 2), (2, 4)\}$
- $|X| \times |Y| = |X \times Y| = 6$

Extends to chains of products: $S_1 \times S_2 \times S_3 \times \cdots S_k$.

If $S_1 = S_2 = S_3 = \cdots = S_k = S$, can write simply S^k.

Binary relations, intuitively

Binary here just means "two". Relation means we're talking about the relationship between two items.
Examples:

- "less than", $a < b$ defines a relationship between two items. The relationship *holds* if and only if a represents a smaller number than b. $5 < 11$, $2020 \not< 1984$.
- "subset", $A \subseteq B$.
- "is the square of".
- "is a factor of".

Binary relations, formally

Definition. A *binary relation*, R, consists of a set, A, called the domain of R, a set, B, called the codomain of R, and a subset of $A \times B$ called the graph of R.

Relation notation

$R : A \to B$. We say R maps A to B.

Kind of like a two-place predicate.
If the relation is from a set A to itself, we say it's a relation "on A".

Infix notation is common: $a\ R\ b$, meaning (a, b) is in the graph of R.

Binary relation examples

Binary relations essentially are functions that take in two items and return a Boolean.

- superset ($\supset$)?. Yes.
- or? Sure, you could see it that way.
- add ($+$)? No, binary, but returns a number.
- equals ($=$)? Yes.
- is divisible by? Yes.
- element of ($\in$)? Yes. Domain and codomain can be different.
- is a perfect square? No, only takes one input. It's a predicate.
- is the perfect square of? Yes.
- greatest common divisor? No, returns a number.
- greatest common divisor is one? Yes, also called "relatively prime" or "co prime".