Course Intro

Michael L. Littman

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Overview

Course Overview

Sample Proofs

Website

- Class goals.
- Course outline.
- Meet the UTAs!
- Collaboration policy.
- Assignments, dates and deadlines.
- Attendence policy.
- TA hours.

Other sites

- Piazza: Best way to get quick answers. Key announcements there, too.
- Gradescope: Handins, homework grading.
- Overleaf (optional): LaTeX without installation.

Show odd times odd is odd

- How approach a problem like this one?
- Check a few cases to see if you believe it.
  $3 \times 5 = 15$, $7 \times 3 = 21$. One times anything is the same, so, if it was odd, it stays odd. So far so good.
- Go to definitions. What does odd actually mean, mathematically? A number is *odd* if it can be written $2k + 1$ for an integer k.
- Use definitions to express the problem.
  We have two odd numbers: $2k_1 + 1$, $2k_2 + 1$.
  What can we say about their product?

$$
\begin{aligned}
(2k_1 + 1)(2k_2 + 1) &= 4k_1k_2 + 2k_1 + 2k_2 + 1 \\
&= 2(2k_1k_2 + k_1 + k_2) + 1 \\
&= 2k_3 + 1,
\end{aligned}
$$

Since $k_3 = 2k_1k_2 + k_1 + k_2$ is an integer, the product is odd.

Bad "proof"

Each step must be done carefully to avoid going off the rails.

Pick any y and let $x = 2y$	x	$=$	$2y$
Multiply by $-x$	$-x^2$	$=$	$-2xy$
Add $2x^2$	x^2	$=$	$2x^2 - 2xy$
Subtract $2xy$	$x^2 - 2xy$	$=$	$2x^2 - 4xy$
Factor	$x(x - 2y)$	$=$	$2x(x - 2y)$
Cancel common terms	1	$=$	2

Conclusion: Math is over. If we can conclude $1 = 2$, we can conclude *anything*.

Proof by contradiction

- If n^2 is even, then n is even.

  We are given that n^2 is even. Let's *assume* that n is odd. Earlier in the lecture, we showed that the product of two odds is odd. That implies that n^2 must be odd. But, n^2 can't be both odd and even, so that's a *contradiction*. That means the negation of our most recent assumption *must* be true. In other words, n must have actually been even.

The basic idea is that if *any time a* happens then b must happen *and* we know b didn't happen, well, then a couldn't have happened either. After all, if *a had* happened, it would have made b happen. But, b didn't happen, so a couldn't have happened.

Deeper proof by contradiction

- $\sqrt{2}$ is irrational.

  Let's *assume* it is rational. That means $\sqrt{2} = p/q$ where p and q are integers. Furthermore, we can assume p/q is in lowest terms so they have no factors in common. Squaring both sides, we get $2 = p^2/q^2$ or $2q^2 = p^2$. Since q^2 is an integer, and p^2 is an integer times 2, p^2 is even. By the argument above, that means p must be even. If p is even, p^2 must be divisible by 4. Since $2q^2$ is divisible by 4, q^2 must be divisible by 2. That means both p and q are even. But, then p/q is not in lowest terms, which we had already concluded. Because we've reached a *contradiction*, we can conclude that our most recent assumption must be false. That is, $\sqrt{2}$ must *not* be rational.