Contraposition

Requirements

1. Statement of the contrapositive
2. Proof of the contrapositive
3. Conclusion that proof of the contrapositive proves the original statement

Example

If a sum of two real numbers is less than 50, then at least one of the numbers is less than 25.

Proof:

The satement will be proven by contraposition. Contrapositive of original statement: If there are two real numbers, neither of which is less than 25, then their sum will be greater than or equal to 50.

The original statement, in symbols

$$\forall x,\ y \in \mathbb{R}\ s.t.\ (x + y < 50) \Rightarrow ((x < 25) \vee (y < 25))$$

The contrapositive, in symbols

$$\forall x,\ y \sim ((x < 25) \vee (y < 25)) \Rightarrow \sim (x + y < 50)$$

Assume for some real numbers a and b, $a \geq 25$ and $b \geq 25$. Then

$$\begin{aligned} a + b &\geq 25 + 25 \\ &\geq 50 \end{aligned}$$

This proves that two numbers, neither of which is less than 25, will sum to a value greater than or equal to 50. By proving the contrapositive, a logically equivalent statement, we have also proved that if the sum of two real number is less than 50, then at least one of the numbers is less than 25.