

Logical Equivalences

Given propositions p , q , and r , a tautology \mathbf{t} , and a contradiction \mathbf{c} , the following logical equivalences hold.

1. *Commutative Laws:*

$$p \wedge q \equiv q \wedge p$$

$$p \vee q \equiv q \vee p$$

2. *Associative Laws:*

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

3. *Distributive Laws:*

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

4. *Identity Laws:*

$$p \wedge \mathbf{t} \equiv p$$

$$p \vee \mathbf{c} \equiv p$$

5. *Negation Laws:*

$$p \vee \neg p \equiv \mathbf{t}$$

$$p \wedge \neg p \equiv \mathbf{c}$$

6. *Double Negative Law:*

$$\neg \neg p \equiv p$$

7. *Idempotent Laws:*

$$p \wedge p \equiv p$$

$$p \vee p \equiv p$$

8. *Universal Bound Laws:*

$$p \vee \mathbf{t} \equiv \mathbf{t}$$

$$p \wedge \mathbf{c} \equiv \mathbf{c}$$

9. *De Morgan's Laws:*

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

10. *Absorption Laws:*

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$

11. *Negation of **t** and **c**:*

$$\neg \mathbf{t} \equiv \mathbf{c}$$

$$\neg \mathbf{c} \equiv \mathbf{t}$$

12. *Definition of Conditional:*

$$p \implies q \equiv \neg p \vee q$$

13. *Definition of Biconditional:*

$$p \iff q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$$