

## Statements that can be used without proof

You may use without proof any statements that were proved in class or in the corresponding sections of the book. You may also use the following statements. When in doubt, ask!

Let  $x$ ,  $y$  and  $z$  be real numbers.

1. Closure under addition:  $x + y$  is real. If  $x$  and  $y$  are integers, then  $x + y$  is an integer. If  $x$  and  $y$  are rational, then  $x + y$  is rational.
2. Commutativity of addition:  $x + y = y + x$
3. Associativity of addition:  $x + (y + z) = (x + y) + z$
4. Additive identity:  $x + 0 = x$
5. Additive inverses: There exists a unique real number  $-x$  such that  $x + (-x) = 0$ . If  $x$  is an integer, then  $-x$  is an integer. If  $x$  is rational, then  $-x$  is rational. We also have that  $-(-x) = x$ .
6. Closure under multiplication:  $xy$  is real. If  $x$  and  $y$  are integers, then  $xy$  is an integer. If  $x$  and  $y$  are rational, then  $xy$  is rational.
7. Commutativity of multiplication:  $xy = yx$
8. Associativity of multiplication:  $x(yz) = (xy)z$
9. Multiplicative identity:  $1x = x$
10. Multiplicative inverses: If  $x$  is nonzero, then there exists a unique real number  $x^{-1}$  such that  $xx^{-1} = 1$ . If  $x$  is rational, then  $x^{-1}$  is rational. We also have that  $(x^{-1})^{-1} = x$ .
11. Distributivity:  $x(y + z) = xy + xz$
12.  $0x = 0$
13. If  $x$  and  $y$  are both positive, then  $x + y$  is positive. If  $x$  and  $y$  are both negative, then  $x + y$  is negative.
14. If  $x$  and  $y$  are both positive or both negative, then  $xy$  is positive. If one of  $x$  and  $y$  is positive and the other is negative, then  $xy$  is negative.