CS22 Proof Virtues

Proof virtues are the qualities that a mathematical community finds desirable in its proofs. Proofs written for this class should be written with CS22 students as the intended audience, and statements should grounded in concepts and vocabulary that the whole community understands. Arguments need to be *sound:* foundational statements need to be true, and jumps in the argument need to be logical and justified.

In this document, we present some virtues to keep in mind as you write out your proofs. Remember that proofs are **written to be read**, and your ability to communicate ideas is something you will be assessed upon in this class. Overall, proofs should **minimize mental labor** from their audience.

Organization

Break your proof into digestible chunks for the reader, and the organization of the argument should be explained. This helps prevent the reader from getting lost in the proof, and makes your proof easier to understand in general.

Example proof with good organization: Division into Cases.

Signposting

When a proof starts to get technical, or needs to several subclaims, it's often helpful to *signpost*. Signposting is a proof writing technique where the author explicitly writes out where the proof is going. This helps prevent the reader from getting lost or confused in more complicated proofs.

A good example of signposting is here: Bijection. Note how the proof sections out and clearly labels the injectivity and surjectivity arguments so that the reader knows what's being shown and why at every point of the proof.

Focus

Proofs should contain only information relevant to what is being shown: avoid including extraneous information that could distract the reader.

Here's an example of a proof that loses focus:

> Claim: the sum of two even numbers is even.

> Proof: Let m,n be even numbers. Then we know that we can write m=2k, n=2j for some k,j integers. Then, we have that m+n = 2k+2j. Note that since m and n are even, they must be 0, 2, or 4 mod 6 (otherwise they would be odd). Since m+n=2(k+j), we know by closure of integers that k+j is an integer and so m+n is even as claimed.

While this is a bit of an extreme example, note that the modular congruence argument here wasn't really necessary, and doesn't provide much to the proof. The proof would be more focused if this was excluded.

Balancing concision and thoroughness

As a general rule, we don't want to *aim* for extremely short proofs. While there is some elegance in a clever one or two line proof, intentionally trying to write as short of a proof as possible often makes the proof less clear and harder to read. On the other hand, including unnecessary details can make the proof lose focus (see focus above) which can also harm clarity. As a general rule, remember that your proofs are meant to be arguments that can be easily read and understood by your classmates, not professional mathematicians or computers.

Here's a toy example of a proof that's a bit too concise:

> Claim: The square root of 2 is irrational.
>
> Proof: Suppose not. Then we can write $2=a^2/b^2$ where a, b are integers in lowest terms. Then $2b^2=a^2$ so a is even, and so a^2 is divisible by 4. But then b^2 must be even as well, which is a contradiction. So the square root of 2 is irrational.

While everything in this proof is technically correct, a lot of details are excluded and left for the reader to fill in (such as, why do we immediately know that a is even and a^2 is divisible by 4? How does b^2 being even cause a contradiction?) By being too concise, this proof has become hard to read, which is what we want to avoid.

More Examples

We have many more example proofs on the course website, so if you're looking for more samples, check them out here.