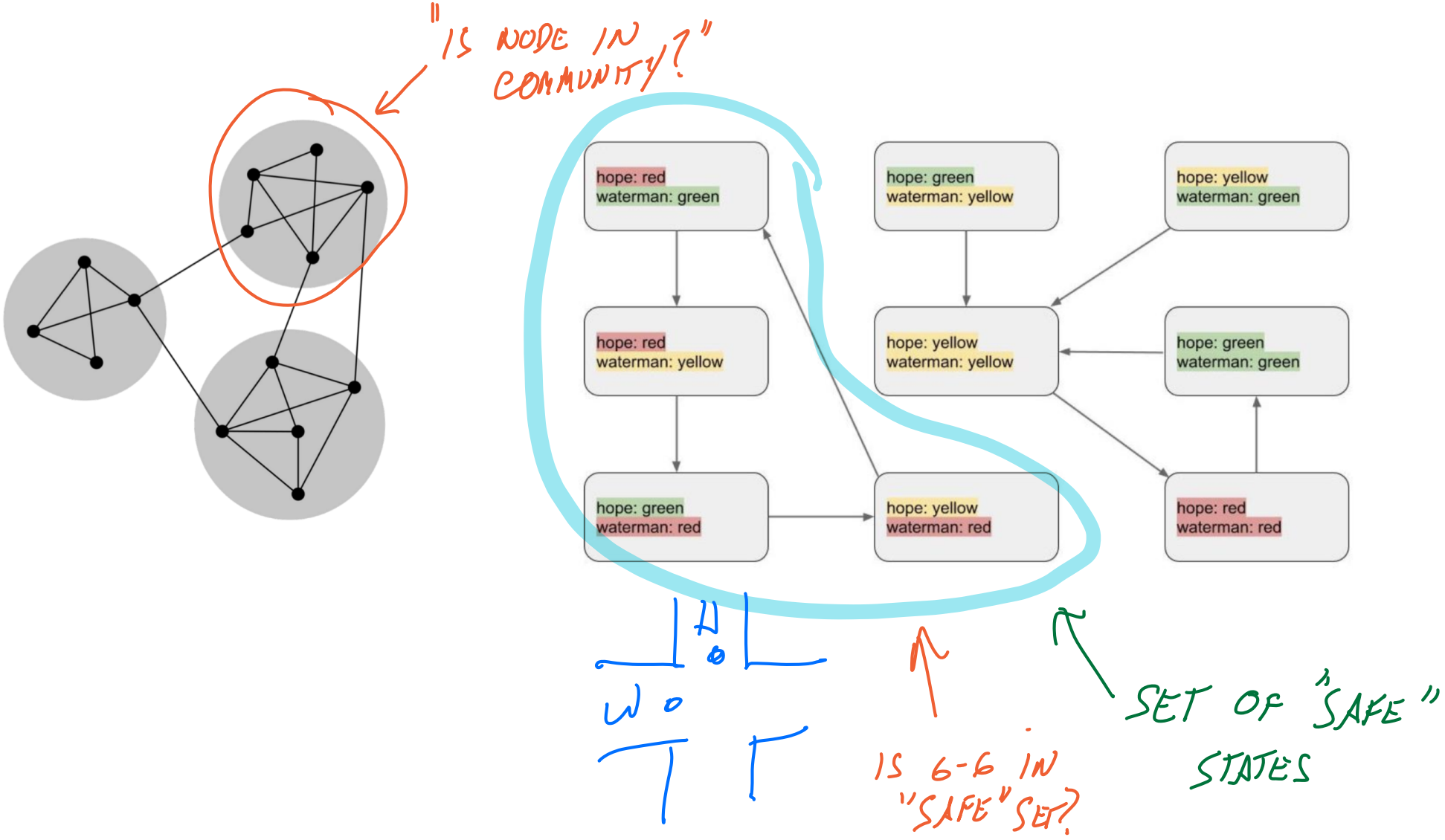
How would we represent a collection of nodes within, eg., a community?

=> Problem: what if we have a LOT of nodes?



How would represent membership in a collection?

Good initial choice is a hashset...

Problem: how do we name the nodes in a set?
=> Can be especially problematic when there are a lot of nodes (eg. really lage graph)

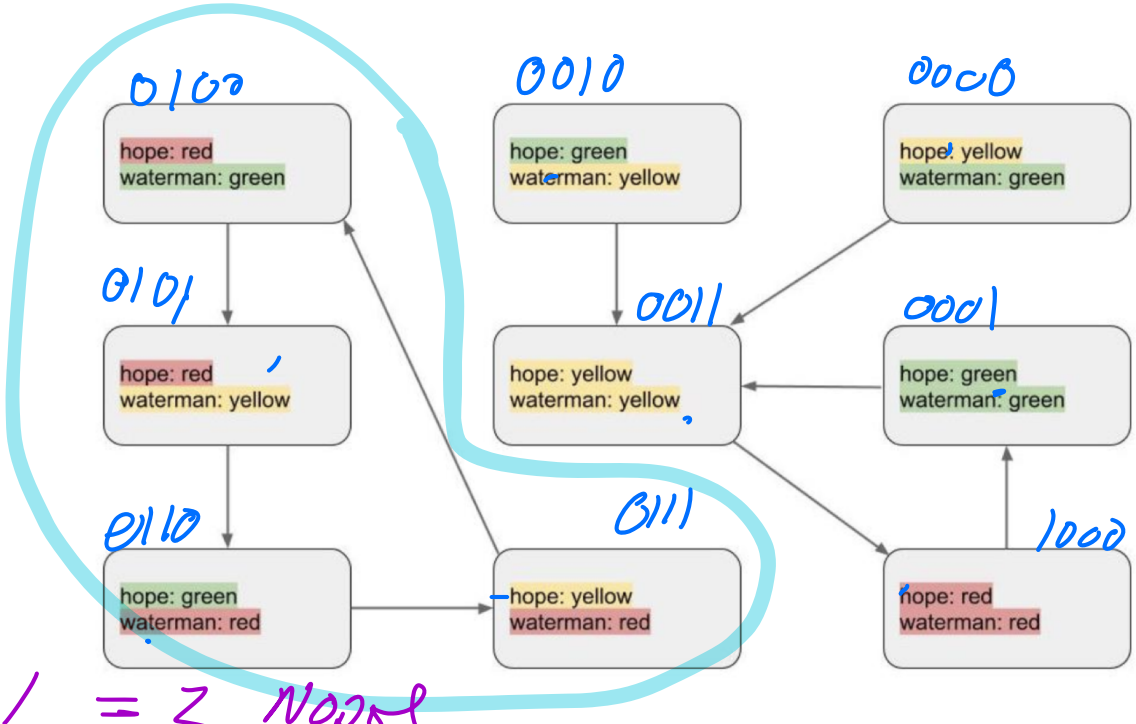
- Idea: Could assign a name to each Node object: {"node1", "node2", ...}

- Another idea: Could just use the memory addresses: {@1127, @4452, ...}
  => Storing a set of numbers (addresses) would use less space than strings)

What else could we do?

Claim: if we get creative about how we name the objects, can represent membership in a space-efficient way

Idea: smallest piece of information in a computer is a bit (0, 1)
- Assign a name as a sequence of bits (bit vector, or a bit string)



1 BIT => 0 1 = 2 NODES

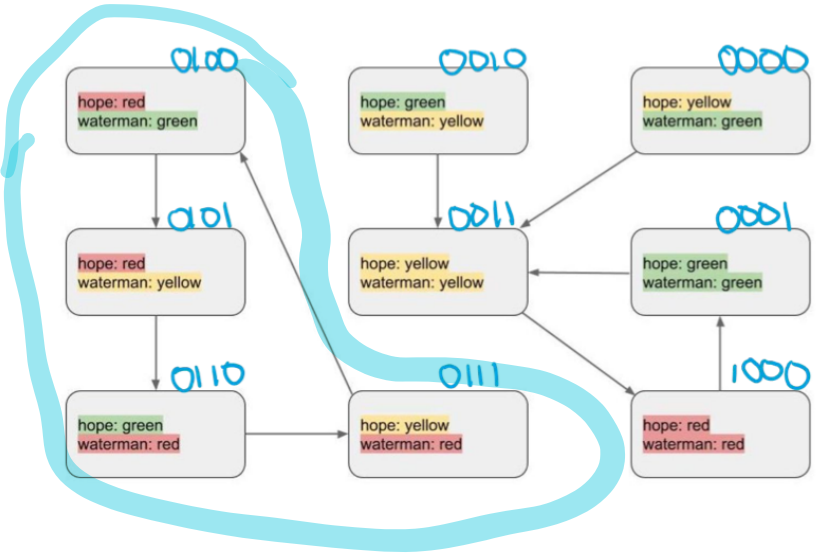
2 BITS => 00 01 10 11 => 4 NODES.

3 BITS => 000 001 010 011 => 8 NODES.
100 101 110 111

4 BITS 16 NODES

Set[0100, 0101, 0110, 0111]

Claim: if we get creative about how we name the objects, can represent membership in a space-efficient way



① Idea: give each state a name
- Smallest piece of information in a computer is a bit (0 or 1)
- Assign name to each state "bit string" (sequence of bits)'

For n states, how many bits? Log2(N)

REPRESENT AS BITS:

2 STATES => 0, 1 => 1 BIT

4 POSSIBLE BITSTRINGS

4 STATES => 00 01 10 11 => 2 BITS

8 POSSIBLE BITSTRINGS

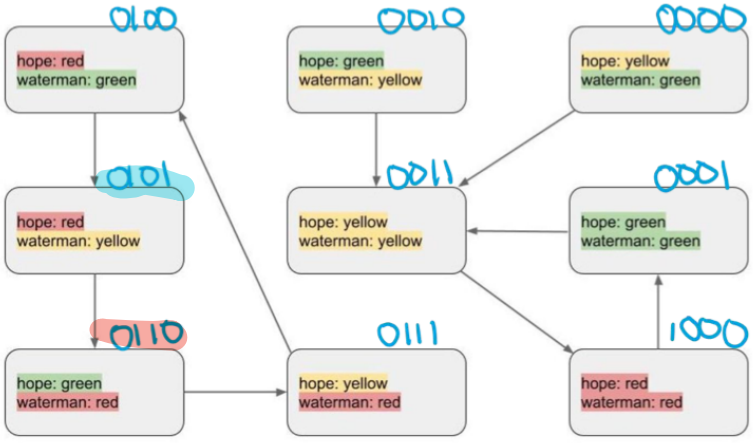
8 => 000 001 010 011 100 101 110 111 => 3 BITS

In general: N bits => 2^N possible "things" (in this case states)
OR
M states => log2(M) bits
log2(9) = 3.1 => 4 (need all names to be same length)

Using bitstrings would let us make the set:
Set[0100, 0101, 0110, 0111]

=> This is much smaller than using objects or addresses, but still grows linearly!

Can we do better?



=> Using this idea, we can represent each state as a bitstring of 0's and 1's

0 1 0 0

b_1 b_2 b_3 b_4

> Notation: we write state of individual bit as b0, b1, …, where b0 could be 0 or 1 (like a Boolean variable)

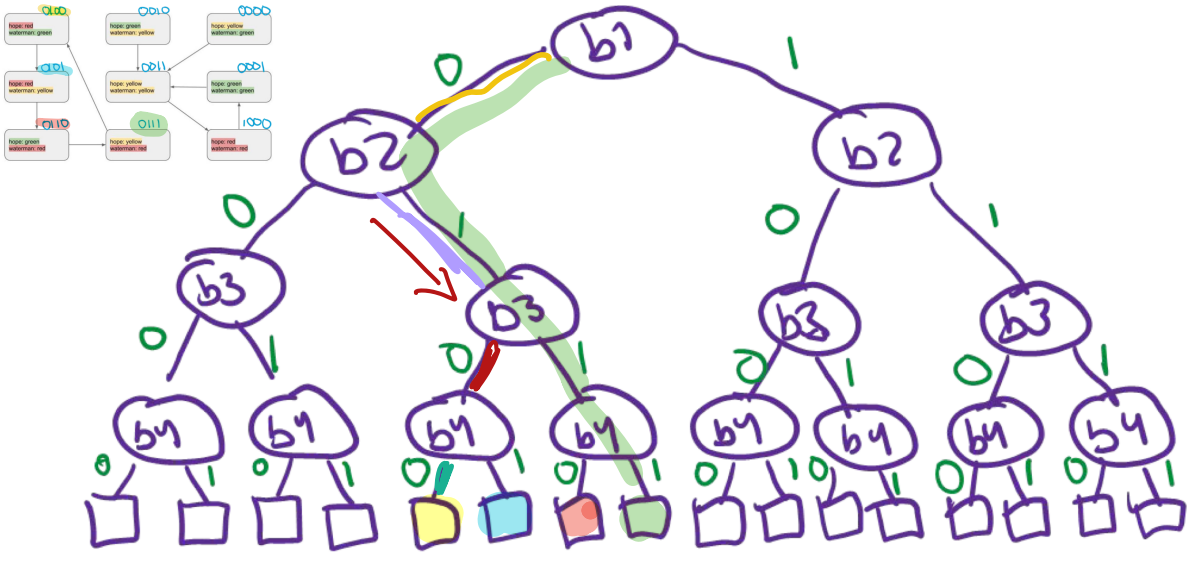
What can we do with this?

- Represent states VERY succinctly (lower space complexity)
- Can represent set of reachable states as a decision tree => Binary Decision Diagram (BDD)

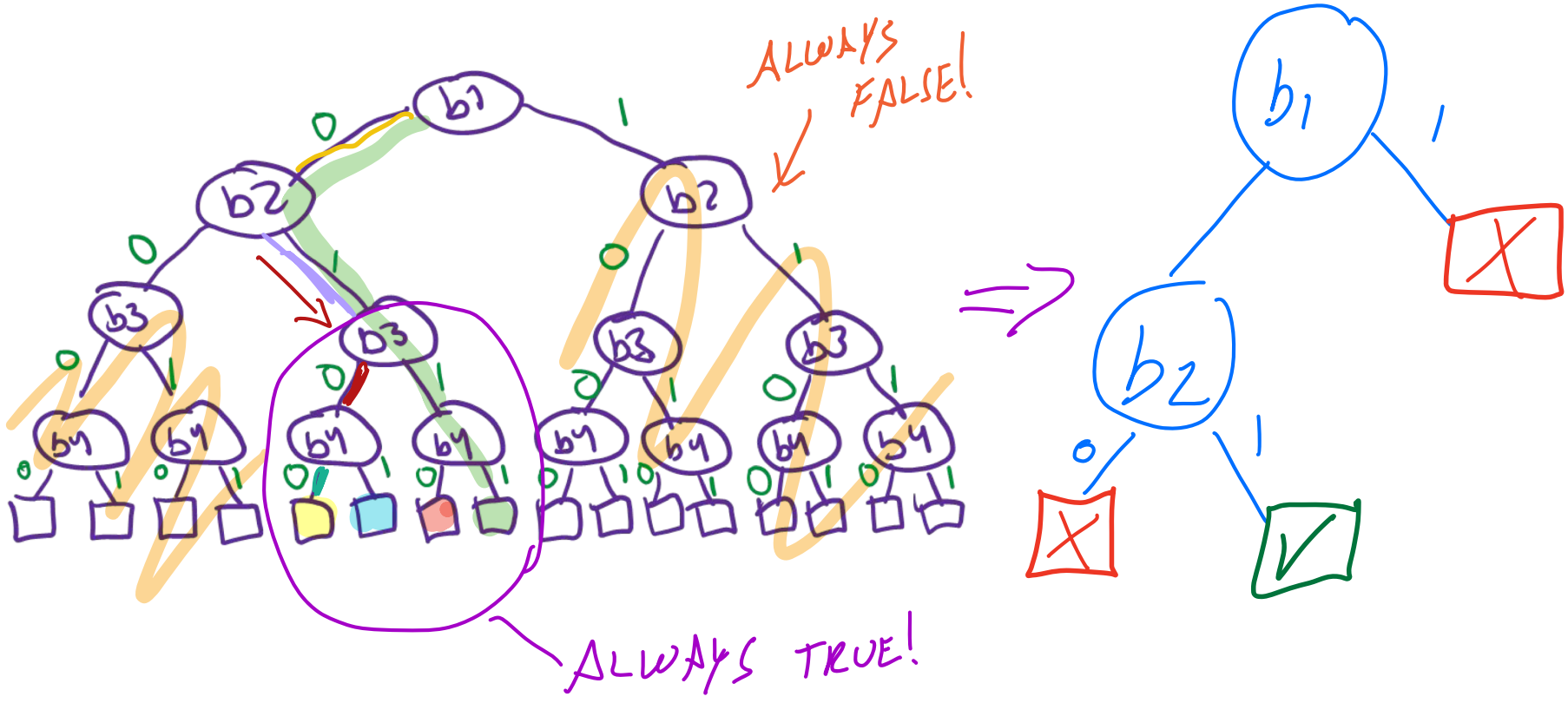
BDD: How it works (high level)

- Each bit is like a step in a decision tree: value is either 0 or 1. After that, consider the next bit
- Each "leaf" is a decision on whether or not the node is in the set

- Can make this very compact...



If we are clever about how to select the names, can reduce a large decision tree into a small one (similar to the decision tree project!). For example, if b1==1 => always false



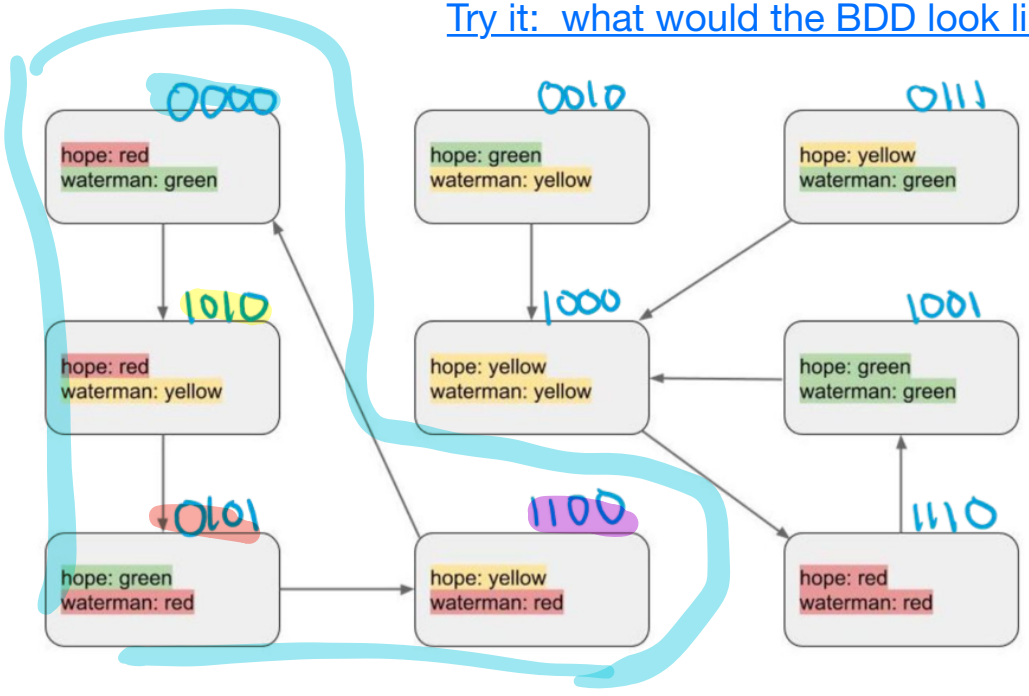
Can also write this as a formula:

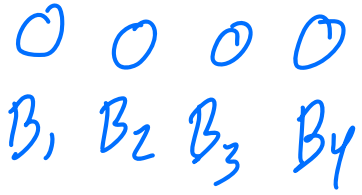
Reachable: (b1 == 0) AND (b2 == 1)

Could also write as: (not b1) and (b2)

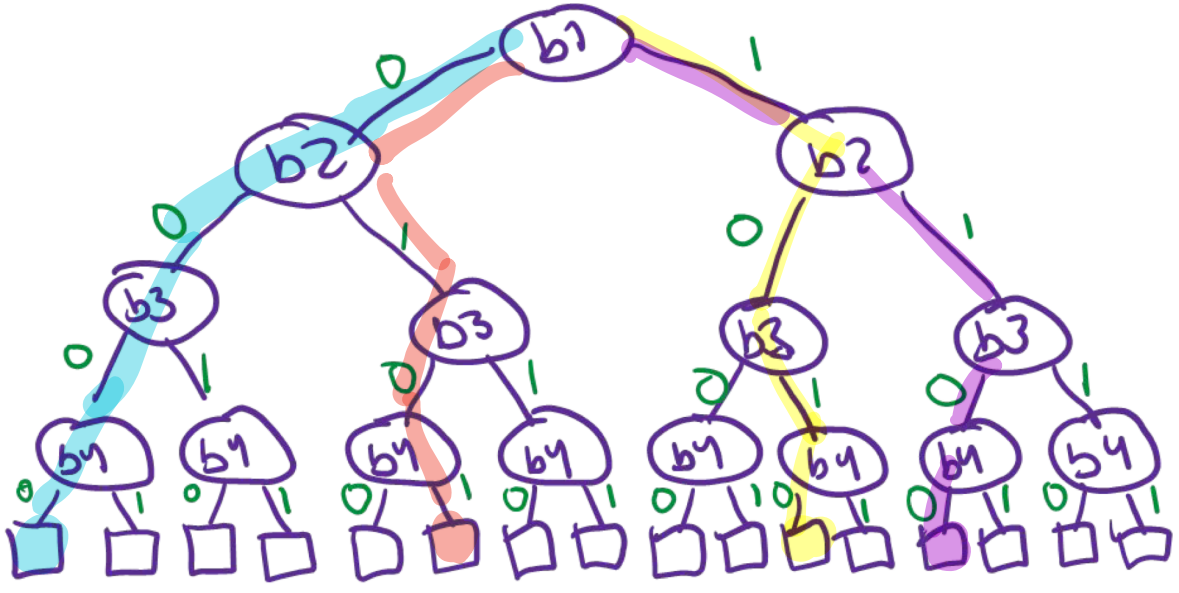
==> To check if the state is safe, Don't even need to store a list of reachable states! Just need to check if the bitstring matches the formula!

Try it: what would the BDD look like if we picked these names instead?





Formula would be a lot more complicated!
((not b1) and (not b2) and (not b3) and (not b4)) OR
((b1) and (not b2) and (b3) and (not b4) OR
…



Takeaways:
- Coming up with a good naming of states is nontrivial
  => Problems are provably computationally hard, people work on this for specific settings

- Depending on what the BDD looks like, logical formula may get complex

Want to learn more?
Consider taking: Logic for Systems
=> Learn about algorithms people have made to represent these in more concise ways, make tools to help, software to use

Compression: how to store text efficiently?

"NATS"
↳ 8 BITS/CHAR (ASCII)
32 BITS/CHAR (UNICODE)

ASCII: Use 8 bit bitstring for each character
=> 4 * 8 bits = 32 bits total

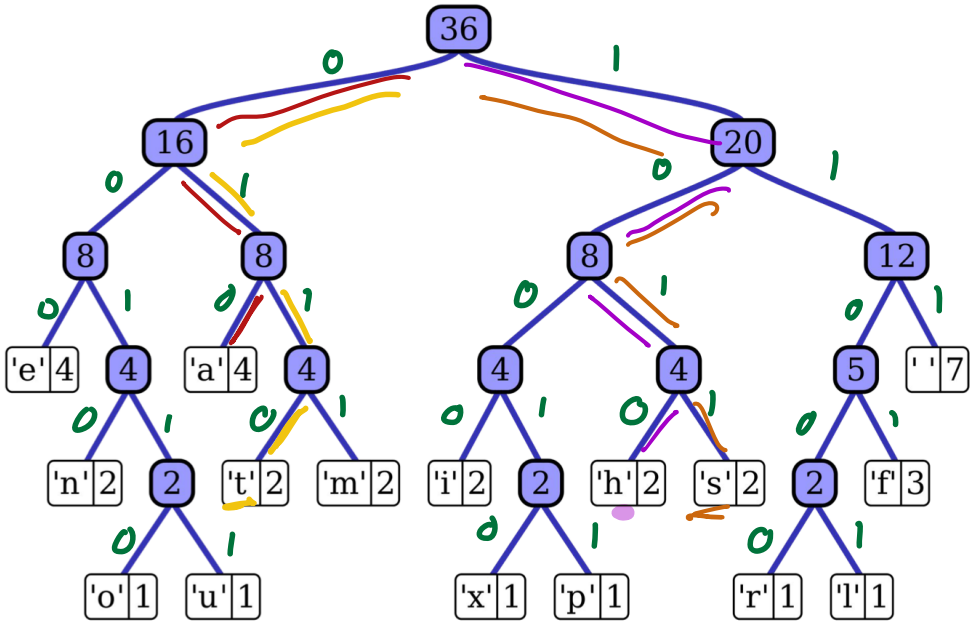
Storage: increases linearly with number of characters
BigWiki: 141 million characters * 8 bits ~= 136MB

Idea: language (and most data) isn't random
Compression: use fewer bits to represent data that appears more frequently
"e" appears more often than "x"—so maybe we can use fewer bits?

Huffman coding:
- for some input data, find unique bitstring to represent characters, use fewer bits for more common letters

Takeaways:
=> Leveraging patterns in data for efficiency
=> Encoding pattern in BDD structure (use to convert)

"this is an example of a huffman tree"



https://en.wikipedia.org/wiki/Huffman_coding

de		
a	4	010
e	4	000
f	3	1101
h	2	1010
i	2	1000
m	2	0111
n	2	0010
s	2	1011
t	2	0110
l	1	11001
o	1	00110
p	1	10011
r	1	11000
u	1	00111
x	1	10010

"NATS"
h: 1010 4
a: 010 3
t: 0110 4
S: 1011 4
=> 15 BITS