Homework for CS 2510: Approximation Algorithms

Due February 16, 2018 at the beginning of class

1. Problem 1.1 (Partial cover)
2. Problem 1.3 (Asymmetric TSP)
3. Problem 1.6, part b (Node-weighted Steiner tree)
4. The following problem:

   One way of formulating the *maximum st-flow* problem is as follows.

   Let G be a directed graph, and let s and t be two vertices of G. Fix an assignment $e \mapsto c_e$ of nonnegative numbers to edges of G. Let $\mathcal{P}_{st}$ be the set of s-to-t directed paths in G.

   Consider the linear program

$$\max \sum \{x_P \ : \ P \in \mathcal{P}_{st}\} \text{such that}$$
$$\begin{aligned} \sum x_P \ : \ P \in \mathcal{P}_{st}, e \in P &\leq c_e \forall e \\ x &\geq 0 \end{aligned}$$

   This linear program has a variable x_P for each s-to-t path P. It has a constraint for each edge e. A solution x assigns a weight to each s-to-t path such that, for each edge e, the sum of weights of all paths containing e is at most c_e. The goal is to maximize the sum of weights.

   (a) Find the dual linear program.

   (b) Show that the dual linear program (and therefore also the primal) can be solved by giving a polynomial-time separation oracle for the dual linear program.

   (c) Suppose you have an optimal integral solution to the dual linear program (i.e. the values assigned to the dual variables are integers). Explain how to interpret such a solution as a solution to a familiar optimization problem.