Homework No. 1

Please solve the first 8 problems, and state whether or not you are able to solve the next 3. Homeworks are due at the beginning of class. You do not have to type it up but you will be penalized if we can't read it.

Don't forget your name and cs login. Let us know if you need an account

Problem 1

Find a nonzero vector in the nullspace of the following matrix:

$$M = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Find a basis for the range of M.

Problem 2

Suppose that a function $f(x)$ can be written as a sum of sines and cosines:

$$f(x) = a_1 \sin(1x) + a_2 \sin(2x) + ... + a_k \sin(kx) + b_1 \cos(1x) + b_2 \cos(2x) + ... + b_k \cos(kx)$$

Suppose also that $f(t) = f(-t)$ for every x. Explain why all the a_i's must be zero.

Problem 3

What is the integral of x^3 from 1 to 4?

Problem 4

Solve $f'(x) = 3f(x)$ given that $f(0) = 7$.

Problem 5

Explain why $\log((x + y)/2)$ is no less than the average of $\log(x)$ and $\log(y)$.

Problem 6

Find a vector whose dot product with $(1, 3, 7)$ is zero.

Problem 7

a)

Take N samples: $x_1, x_2 \cdots, x_N$ of the uniform probability distribution over $(-1, 1)$. Form the random variable $Z_1 = \sum_{i=1}^{N} x_i$, compute the mean of Z_1 and the variance.

b)

Let Z_2 be defined similarly, but all the x_i's are drawn from a standard normal (Gaussian) distribution (mean 0 and variance 1). What are μ and σ^2 for Z_2?

c)

What's the ratio $\frac{\sigma_{Z_1}}{\sigma_{Z_2}}$ as $N \to \infty$?

Problem 8

a)

$\int\limits_{\mathfrak{R}^+} zx^2 dA$ where $\mathfrak{R}^+$ is defined as the upper hemisphere of the unit sphere. $(z \geq 0)$

b)

$F(x,y,z) = (zx, z, -y)$; find $\int\limits_{\mathfrak{R}^+} F \cdot n dA$

Please state whether or not you are able to solve the following problems:

Problem 9

Find a basis for the nullspace (kernel) of the transformation:

$$T : R^3 \to R : (x,y,z) \to (3x + 2y + 6)$$

Problem 10

a)

Explain why the set of continuous real-valued functions on the interval $[0,1]$ is a vector space over the reals.

b)

If we define

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx$$

explain why it's an inner product on that space.

c)

Suppose that $f(x) = 2x - 1$. Find a nonzero function g whose inner product with f is zero.

Problem 11

Suppose that A is the matrix

$$\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$$

If you multiply A by every unit vector in the plane, you get a collection of vectors in R^2 that happens to be an ellipse.

- a)What's the area of the ellipse
- b) How far is the farthest point on the ellipse from the origin? (You should answer this without ever multiplying the matrix by any unit vectors, or taking any derivatives.)

Section

Section times are as follows

- Monday 4:30-6:30pm
- Friday 2:00pm - 4:00pm
- Saturday 3:00pm -5:00pm

Write which sections you can make in order of preference below. We will do our best to accommodate them (indicate if you absolutely can not make a time).