CS195Y - Spring 2014 **Spencer Gordon(slg1)**

Notes on Linear Temporal Logic

1 Syntax

For a given finite set A of propositional (or Boolean) variables $p, q, \ldots$, the set of formulas of LTL (denoted $\mathrm{LTL}(A)$) is inductively defined as the smallest set satisfying the following:

1. For every variable $p \in A$, $p \in \mathrm{LTL}(A)$.
2. If $\alpha \in \mathrm{LTL}(A)$ and $\beta \in \mathrm{LTL}(A)$, then $\alpha \vee \beta \in \mathrm{LTL}(A)$.
3. If $\alpha \in \mathrm{LTL}(A)$ and $\beta \in \mathrm{LTL}(A)$, then $\alpha \wedge \beta \in \mathrm{LTL}(A)$.
4. If $\alpha \in \mathrm{LTL}(A)$, then $\neg\alpha \in \mathrm{LTL}(A)$.
5. If $\alpha \in \mathrm{LTL}(A)$, then $\mathbf{X}\alpha \in \mathrm{LTL}(A)$.
6. If $\alpha \in \mathrm{LTL}(A)$, then $\mathbf{G}\alpha \in \mathrm{LTL}(A)$.
7. If $\alpha \in \mathrm{LTL}(A)$, then $\mathbf{F}\alpha \in \mathrm{LTL}(A)$.
8. If $\alpha, \beta \in \mathrm{LTL}(A)$, then $\alpha \,\mathbf{U}\, \beta \in \mathrm{LTL}(A)$.

2 Semantics

Instances in LTL consist of infinite sequences of the form $w = I_0, I_1, \ldots$ where each of the I_k is a total function that maps each of the propositional variables in A to a truth value:

For an instance w, we define two pieces of notation:

$w(i) \stackrel{\text{def}}{=} I_i$	The i-th entry in the sequence.
$w^i \stackrel{\text{def}}{=} I_i, I_{i+1}, \ldots$	The sequence starting at the i-th entry.

We now need to define the satisfaction relation for LTL. For any $\alpha \in \mathrm{LTL}(A)$, we'll write $w \vDash \alpha$ when w satisfies α, where satisfaction is defined as follows:

1. If α is a propositional variable p, then $w \vDash \alpha$ iff $w(0)$ maps p to true.
2. If α is of the form $\beta \vee \gamma$, then $w \vDash \alpha$ iff $w \vDash \beta$ or $w \vDash \gamma$.
3. If α is of the form $\beta \wedge \gamma$, then $w \vDash \alpha$ iff $w \vDash \beta$ and $w \vDash \gamma$.
4. If α is of the form $\neg\beta$, then $w \vDash \alpha$ iff $w \nvDash \beta$.
5. If α is of the form $\mathbf{X}\beta$, then $w \vDash \alpha$ iff $w^1 \vDash \beta$.
6. If α is of the form $\mathbf{G}\beta$, then $w \vDash \alpha$ iff for all $i \in \mathbb{N}$, $w^i \vDash \beta$.

7. If α is of the form $\mathbf{F}\beta$, then $w \vDash \alpha$ iff there exists an $i \in \mathbb{N}$ such that $w^i \vDash \beta$.
8. If α is of the form $\beta \,\mathbf{U}\, \gamma$, then $w \vDash \alpha$ iff there exists an $i \in \mathbb{N}$ such that $w^i \vDash \gamma$ and for all $0 \leq j < i$, $w^j \vDash \beta$.