## 4/8 - Soundness & Completeness

See the Coq demo for the commutativity of and in the recording (first 15 min)

- Soundness no false positives
- *Completeness* no false negatives

These are broad terms for a search

- In other words, if we get something, it really does satisfy our constraints (sound)
- And we can get all possible things satisfying our constraints (complete)
- We can also apply these to the search for proofs

We can apply this to satisfiability and provability:

- Satisfiability notation: I | = a (I satisfies a), where I is an instance
- Provability notation:  $\Gamma \mid -a$  ( $\Gamma$  proves a), where  $\Gamma$  is our premises

So what does it mean for ND to be sound & complete?

- Why can't we prove A  $|-A \land B$ 
  - It's because there's a counterexample (i.e. an instance where the above is false)
- ND is sound exactly when if  $\Gamma \mid -a$ , then for all instances I, I  $\mid = \Gamma \rightarrow I \mid = a$ .
- ND is *complete* exactly when for all instances I, if  $I = \Gamma \rightarrow I = a$ , then  $\Gamma = a$ .