EPIC Ascending Auctions

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We note that just as welfare-maximizing direct mechanisms must charge Groves payments to be DSIC, welfare-maximizing indirect mechanisms must charge Groves payments to be EPIC. We then investigate the converse: are welfare-maximizing indirect mechanisms that charge Groves payments necessarily EPIC? The answer to this question gives rise to a recipe for designing EPIC indirect mechanisms.

1 *Groves Payments are Necessary*

The "Groves uniqueness" theorem states that among all direct, welfare-maximizing mechanisms, the Groves mechanism is the unique DSIC mechanism.[1,2] In other words, no direct mechanism can be both welfare maximizing and DSIC, unless it yields a Groves outcome. In this section, we set out to answer the analogous question, as it pertains to indirect mechanisms. That is, is it also the case that no indirect mechanism can be both welfare maximizing and DSIC, and not yield a Groves outcome?

[1] up to an additive constant

[2] assuming connected type spaces

Proposition 1.1. *If an indirect mechanism is DSIC and welfare maximizing, then the outcome must be a Groves outcome:*[3,4] *i.e., a welfare-maximizing allocation and Groves payments.*[5]

[3] up to an additive constant

[4] assuming connected type spaces

[5] Hereafter, we drop the qualifiers "up to an additive constant" and "assuming connected type spaces," but these qualifiers apply throughout.

Proof Sketch. Consider an indirect mechanism that is DSIC and welfare maximizing. Apply the revelation principle. The result is a DSIC outcome-preserving (i.e., welfare-maximizing) direct mechanism. But the unique direct DSIC and welfare-maximizing mechanism is the Groves mechanism. Therefore, the outcome of the indirect mechanism is likewise a Groves outcome: i.e., welfare-maximizing together with Groves payments. □

This proposition yields a strategy for designing ascending auctions that yield a Groves outcome: Simply check that the auction is DSIC and welfare-maximizing, and then *boom!*—Groves payments, and hence a Groves outcome, come for free.

But DSIC is a very strong condition. SAAs, assuming additive valuations, are not DSIC (even up to ϵ). So does this mean their outcome is not necessarily a Groves outcome? Not so fast! SAAs are EPIC and welfare maximizing up to ϵ. And since the notions of EPIC and DSIC coincide in direct mechanisms,[6] we can relax the antecedent in the previous proposition from DSIC to EPIC.

[6] See the course lecture notes on The Revelation Principle.

Proposition 1.2. *If an indirect mechanism is EPIC and welfare maximizing, then the outcome must be a Groves outcome: i.e., a welfare-maximizing allocation and Groves payments.*

Proof Sketch. Apply the revelation principle. The result is an EPIC outcome-preserving (i.e., welfare-maximizing) direct mechanism. But EPIC and DSIC coincide in direct mechanisms, and the unique direct DSIC and welfare-maximizing mechanism is the Groves mechanism. Therefore, the outcome of the indirect mechanism is likewise a Groves outcome: i.e., welfare-maximizing together with Groves payments. □

In sum, a strategy for proving an ascending auction yields a Groves outcome is to show that it is EPIC and welfare-maximizing and then *boom!*—Groves payments, and hence a Groves outcome, come for free.

2 *Groves Payments are Sufficient*

While it is nice to know that EPIC, welfare-maximizing ascending auctions recover a Groves outcome, what we would really like to know is *how to design ascending auctions with incentive guarantees*. Hence, we are driven to investigate the converse: if sincere bidding in a welfare-maximizing indirect mechanism yields Groves payments, is it necessarily EPIC?

If only life were so simple . . . the answer to this question is no. Consider the usual English auction with the following funky (or perhaps ridiculous) modification:

> If when the price is q, a bidder does not bid, but then when the price is $q + \epsilon$, the bidder does bid, the auction ends, and that bidder wins the good at the price q.

In this funky English auction, sincere bidding yields the VCG outcome. However, if all other bidders are sincere, it is better for a bidder to sit out the first round, and then win the good in the second round at price 0. Therefore, for an auction to be EPIC, it is not enough for sincere bidding to yield the VCG outcome.

We will resolve this issue by supplying an additional condition, beyond a mere Groves outcome, which guarantees EPIC. Before doing so, however, we restrict the space of bidding strategies to restore equivalence between DSIC and EPIC in ascending auctions. (Recall that these two properties are equivalent in direct mechanisms.)

Definition 2.1. A strategy c_i for bidder i is called **consistent** iff it is "consistent" with some type $v_i \in T_i$. In other words, there must exist a type v_i such that sincere bidding yields precisely the behavior of $c_i(t_i)$. We denote the space of bidder i's consistent strategies by C_i.

Definition 2.2. A strategy s_i for bidder $i \in N$ is called **dominant *up to consistency*** if it is (weakly) optimal relative to all other consistent strategies, regardless of all the other bidders' strategies and types, but again assuming only consistent strategies, even on the part of other bidders: i.e., for all bidders $i \in N$,

$$u_i(s_i(v_i), \mathbf{c}_{-i}(\mathbf{t}_{-i})) \geq u_i(c_i(v_i), \mathbf{c}_{-i}(\mathbf{t}_{-i})), \quad \forall c_i \in S_i, \forall \mathbf{c}_{-i} \in C_{-i}, \forall \mathbf{t} \in T.$$

In words, a strategy is DSIC up to consistency if bidding sincerely is an *ex-post* best response for each bidder relative to other consistent strategies, regardless of how the other agents bid, assuming they, too, bid consistently.

Definition 2.3. A strategy profile comprises a **dominant strategy equilibrium *up to consistency*** if all bidders' strategies are dominant up to consistency. An auction is called **dominant-strategy incentive compatible *up to consistency*** if sincere bidding is a dominant strategy up to consistency.

Strategy profiles can likewise be defined to be EPNE up to consistency, and EPIC up to consistency, if sincere bidding is an EPNE up to consistency. The next lemma establishes the relationship between EPIC and DSIC in indirect mechanisms.

Proposition 2.4. *If an indirect mechanism is EPIC up to consistency, then it is also DSIC up to consistency.*

Proof. Since M is EPIC up to consistency, for all bidders $i \in N$ and for all (true) value profiles $\mathbf{t} \in T$, the sincere bidding profile $\mathbf{s} \in S$ satisfies

$$u_i(s_i(v_i), s_{-i}(\mathbf{t}_{-i})) \geq u_i(c_i(v_i), s_{-i}(\mathbf{t}_{-i})), \quad \forall c_i \in S_i.$$

Our goal is to show that M is also DSIC up to consistency: i.e., for all bidders $i \in N$ and for all (true) values $v_i \in T_i$,

$$u_i(s_i(v_i), \mathbf{c}_{-i}(\mathbf{t}_{-i})) \geq u_i(c_i(v_i), \mathbf{c}_{-i}(\mathbf{t}_{-i})), \quad \forall c_i \in S_i, \forall \mathbf{c}_{-i} \in C_{-i}, \forall \mathbf{t}_{-i} \in T_{-i}.$$

Fix a bidder i and their true value v_i. For two arbitrary value profiles $\mathbf{t}'_{-i}, \mathbf{t}''_{-i} \in T$, since M is EPIC up to consistency,

$$u_i(s_i(v_i), s_{-i}(\mathbf{t}'_{-i})) \geq u_i(c_i(v_i), s_{-i}(\mathbf{t}'_{-i})), \quad \forall c_i \in S_i.$$

$$u_i(s_i(v_i), s_{-i}(\mathbf{t}''_{-i})) \geq u_i(c_i(v_i), s_{-i}(\mathbf{t}''_{-i})), \quad \forall c_i \in S_i.$$

Hence, sincere bidding is a best response for bidder i, relative to other consistent strategies, assuming others are also bidding sincerely, regardless their value profiles. But if all other bidders are bidding sincerely relative to some value profile or another, then they are bidding consistently! In other words, sincere bidding is a dominant-strategy *up to consistency*: i.e., for all bidders $i \in N$,

$$u_i(s_i(v_i), \mathbf{c}_{-i}(\mathbf{t}_{-i})) \geq u_i(c_i(v_i), \mathbf{c}_{-i}(\mathbf{t}_{-i})), \quad \forall c_i \in S_i, \forall \mathbf{c}_{-i} \in C_{-i}, \forall \mathbf{t}_{-i} \in T_{-i}.$$

Since bidder i was arbitrary, sincere bidding is a dominant-strategy equilibrium up to consistency (i.e., it is a best response for all bidders up to consistency). □

Therefore, just as the notions of DSIC and EPIC coincide in direct mechanisms, DSIC and EPIC, up to consistency, coincide in indirect mechanisms. We use both these facts to prove the next lemma:

Lemma 2.5. *If sincere bidding in a welfare-maximizing indirect mechanism yields Groves payments, then it is DSIC/EPIC up to consistency.*

Proof Sketch. Apply the revelation principle. The result is a direct outcome-preserving (i.e., welfare-maximizing) mechanism in which truthful bidding yields a Groves payments: i.e., the ensuing direct mechanism *is* a Groves mechanism. But the Groves mechanism is DSIC and EPIC, meaning truthful bidding is an equilibrium (regardless of types). So likewise, in the indirect mechanism, sincere bidding must have been an equilibrium (regardless of types). In particular, sincere bidding must have been a best response relative to all strategies *that depend only on agents' types:* i.e., relative to all consistent strategies.[7] In other words, the indirect mechanism must have been DSIC and EPIC, up to consistency. □

[7] We cannot say anything about how effective a strategy may have been relative to inconsistent strategies.

The following theorem follows immediately from this lemma:

Theorem 2.6. *If sincere bidding in a welfare-maximizing indirect mechanism yields Groves payments, then it is EPIC, as long as no inconsistent strategy yields greater utility than consistent bidding for any one bidder, assuming all the others bid sincerely.*

Remark 2.7. The following is also true, however it does not have much bite, since relatively few indirect mechanisms are in fact DSIC: If sincere bidding in a welfare-maximizing indirect mechanism yields Groves payments, then it is DSIC, as long as no inconsistent strategy yields greater utility than consistent bidding for any bidder, assuming *arbitrary* bidding on the part of others.

Theorem 2.6 yields a recipe for designing EPIC indirect auctions:

1. Design an allocation rule that is welfare maximizing, assuming sincere bidding.

2. Show that sincere bidding yields Groves payments.

3. Argue that no inconsistent strategy is a profitable deviation: i.e., none yields greater utility than sincere bidding for any bidder.

We apply this recipe in upcoming lectures to design EPIC ascending auctions for different valuation classes (specifically, diminishing marginal values and unit demand). But first, we use it to establish that k parallel English auctions are indeed EPIC up to $k\epsilon$ assuming additive valuations and, in general, heterogeneous goods.

1. Assuming sincere bidding, a single English auction is welfare maximizing up to ϵ, because allocating to the highest bidder implies allocating to the bidder with the highest valuation, up to ϵ.

   - If, when the auction ends, exactly one bidder remains, that bidder's valuation is the highest, so the good is allocated to the highest bidder.
   - If, however, the auctions ends without any remaining bidders, then the highest two or more valuations are all within ϵ of one another, so the good might not be allocated to a bidder with the highest valuation, but it is still allocated to a bidder whose valuation is within ϵ of the highest valuation.

   Therefore, assuming additive valuations and sincere bidding, it follows that k parallel English auctions are therefore welfare maximizing up to $k\epsilon$.

2. Assuming sincere bidding, the final price p^* in a single English auction is the second-highest valuation up to ϵ, because the bidder with the second-highest valuation is the last to drop out, meaning their valuation lies somewhere in the range of $[p^* - \epsilon, p^*]$. As the second-highest valuation is the VCG payment, the final price in a single English auction is the VCG payment up to ϵ.

   While VCG payments are generally defined per bidder, in the case of additive valuations, those per-bidder payments reduce to prices per good, as each one is the second-highest valuation for that good. Moreover, the VCG payment for a bundle of goods is the sum of the second-highest valuations of all the goods in that bundle. Therefore, assuming additive valuations and sincere bidding, the price of a bundle of goods of size $m \leq k$ in k parallel English auctions is within $m\epsilon$ of the VCG payment.

3. As sincere bidding in k parallel English auctions is welfare maximizing up to $k\epsilon$, and terminates at VCG payments up to $m\epsilon$, it follows that sincere bidding is an EPNE up to $k\epsilon = \max\{k\epsilon, m\epsilon\}$ among consistent strategies. It remains to show that no inconsistent strategy is a profitable deviation from sincere bidding up to

some additive error: i.e., no inconsistent strategy yields substantially greater utility than sincere bidding for any bidder.[8]

[8] Problem 4

The proof proceeds by showing that any deviation via an inconsistent strategy can be replicated by a sincere one up to $2k\epsilon$. Then, since sincere bidding is an EPNE up to $k\epsilon$ among (only) consistent strategies, sincere bidding is an EPNE up to $2k\epsilon = \max\{k\epsilon, 2k\epsilon\}$ among both consistent and inconsistent strategies.