

## Prediction Market 3

CSCI 1951k/2951z

2020-04-27

---

In the next few labs, you will be competing in a few different prediction markets. Please try to solve the assigned problems—analytically and/or with Monte Carlo simulations—before coming to lab. Your grade will reflect your effort, more than your accuracy.

---

### 1 Prediction Markets

A **prediction market** is a market in which securities are traded based on the predicted outcomes of future events. Prediction markets can be based on the future outcomes of sporting events, elections, or even the weather. They are interesting because they effectively aggregate the beliefs of all participants, so that the market price conveys the collective belief/probability of an outcome.

### 2 Market with One Decoy

Recall that in the first market, we flipped two coins:  $C$ , the true coin; and  $D$ , the decoy coin. Both were fair coins. We then independently and uniformly at random picked one of the two coin flips to reveal to each student.

In the second market, we again flipped the true coin,  $C$ , but we also flipped a *unique* decoy coin,  $D$ , for each student in the class. Again, both were fair coins. We then independently and uniformly at random chose whether to reveal to each student the true coin or their decoy.

In this third market, we will again flip a (fair) true coin,  $C$ . In addition, we will assign each student  $s$  an integer  $t_s$  indicating their number of decoys. These numbers will be common knowledge among all the students in lab.

We will then flip  $t_s$  (fair) decoy coins for each student  $s$ , and independently and uniformly at random choose whether to reveal to each student the true coin flip or one of their decoys. For example, if  $t_s = 10$  for a student  $s$ , we will choose what we reveal to student  $s$  from among 11 coin flips (the true coin and the 10 decoys).

You, the students, will then begin trading contracts that are worth \$100 if the true coin is heads, and \$0, if it is tails. Once trading subsides, we will reveal the value of  $C$  and  $D$ , and settle all contracts.

### 3 Questions

As usual, assume you are told “heads”.

1. Unlike the two previous markets we studied, this market is **asymmetric**, because different students have different beliefs about the value of the true coin. Would you prefer to be assigned a higher or a lower  $t_s$  value? Why?
2. If your value of  $t_s$  is 1, what is your belief that C is heads?
3. If your value of  $t_s$  is 1000000, what is your belief that C is heads?
4. If your value of  $t_s$  is 10, what is your belief that C is heads? (Hint: Use Monte Carlo simulations.)
5. Assume your value of  $t_s$  is 1, and some other student’s value is 10, and that that student was also told “heads”. Your belief is then the answer to question 2, and their belief, the answer to question 4.

Assume you two share your information with each other. You should now agree on a belief—you have the same information after all! Should your new, shared belief be closer to your original belief or to their original belief?