Homework 8: EPIC Auctions

CS 1951k/2951z

2020-04-02

Due Date: Tuesday, April 7, 2020. 9:00 PM.

We encourage you to work in groups of size two. Each group need only submit one solution. Your submission must be typeset using LaTeX. Please submit via Gradescope with you and your partner's Banner ID's and which course (CS1951k/CS2951z) you are taking.

For 1000-level credit, you need only solve the first three problems. For 2000-level credit, you should solve all four problems.

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1 *Vanilla English Auctions*

1. We say that an ascending auction is **DSIC up to** ϵ iff no bidder who deviates from sincere bidding can improve upon sincere bidding by more than ϵ. Likewise, we can define the notion of any equilibrium "up to ϵ," where by deviating from the equilibrium strategy, no bidder can improve their expected utility by more than ϵ, assuming the other players are conforming. Show by counterexample that the English auction is not DSIC, even up to ϵ.

   Keep in mind that the other bidders need not bid sincerely, and that strategies in the English auction can be outright bizarre: i.e., behavior from one round to the next need not be consistent.

2. Prove that sincere bidding in the English auction is an ex-post Nash equilibrium (EPNE), up to ϵ.

3. Apply the Revelation Principle to transform the English auction, in which sincere bidding is an ϵ-EPNE, into a direct mechanism, which is DSIC up to ϵ. Describe in a sentence (or two) why it is reasonable that applying the Revelation Principle would close the loopholes in the English auction.

2 *A Modified English Auction*

This problem concerns a variant of the k-good English auction that does not allow bidders to re-enter after exiting (i.e., skipping even one round of bidding in) the auction. The k goods are assumed to be homogeneous: i.e., identical copies of one another.

This modified k-good English auction can be formally specified as follows: Given a fixed price increment ϵ,

- Initialize the set of bidders $S_0 = N$ and the price $p_0 = 0$.
- At each round $i = 1, 2, \ldots,$
  - Let price $p_i = i\epsilon$.
  - Let S_i be the bidders in S_{i-1} who remains interested at price p_i. **N.B.** No bidder who expressed disinterest earlier can re-express their interest at some later round.
- Increment i until $|S_i| \leq k$. Call the final round t.
  - Give $|S_t|$ of the k goods to the bidders in S_t at price $t\epsilon$.
  - Give the remaining $k - |S_t|$ of the goods (if any) to random bidders in $S_{t-1} \setminus S_t$ at price $(t-1)\epsilon$.

Bidders with unit demand valuations do not demand more than one good. More specifically, their value for any bundle they might be allocated is simply their maximum value across all individual goods: i.e., a bidder i's valuation is **unit demand** if their value for a bundle of goods $X \subseteq G$ is given by

$$v_i(X) = \max_{j \in X} v_i(j),$$

where $v_i(j)$ denotes i's value for good j.

Assuming unit demand valuations, show the following:

1. The modified k-good English auction is still not DSIC. **Hint:** It suffices to exhibit a counterexample in the single-good case: i.e., when $k = 1$.
2. This modified k-good English auction is DSIC up to $k\epsilon$.

3 *Approximate Welfare-Maximization*

1. Prove that sincere bidding in a k-good English auction yields total welfare within $k\epsilon$ of the optimal. Assume unit demand valuations.
2. What is the additive loss in welfare when running k parallel *single-good* English auctions, assuming sincere bidding and additive valuations on the part of the bidders?
3. In the above setups, does it matter whether the auction allows re-entry or not?

4 *k Parallel English Auctions are EPIC*

Complete the proof that k parallel English auctions are EPIC, assuming additive valuations, by following step 3 of the EPIC auction design recipe. More specifically, show that for every inconsistent bidding strategy, there exists a consistent bidding strategy that generates at least as much utility. **Hint:** You may assume *integer* valuations and $\epsilon = 1$, so that there are no small additive discretization errors.