Homework 3: The Vickrey-Clarke-Groves Mechanism

CS 1951k/2951z

2020-02-07

Due Date: Tuesday, February 11, 2020. 9:00 PM.

We encourage you to work in groups of size two. Each group need only submit one solution. Your submission must be typeset using LaTeX. Please submit via Gradescope with you and your partner's Banner ID's and which course (CS1951k/CS2951z) you are taking.

For 1000-level credit, you need to solve the first four problems. For 2000-level credit, you must solve all five.

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1 Payment Bounds

The VCG mechanism collects reports from all bidders and then computes a welfare-maximizing allocation ω^* together with corresponding payments that ensure that the outcome is DSIC.

1. Prove that VCG payments are lower-bounded by 0.
2. Prove that VCG payments are upper-bounded by $b_i(\omega^*)$, for all bidders $i \in N$.

2 Revenue

This problem asks you to construct examples of multi-parameter auctions, by which we mean you should specify 1. a set of goods, and 2. each bidder's valuation. Let $R(N)$ be the revenue generated by the VCG mechanism, given a set of bidders N.

1. Show, by example, that the following is possible: $R(N \setminus i) > R(N)$, for some $i \in N$.
2. Construct a non-trivial example of a multi-parameter auction in which VCG $R(N) = 0$. (By non-trivial, we mean that each bidder's valuation should be non-negative for every bundle and strictly positive valuation for at least one bundle.)

3 Strategy

This problem highlights two shortcomings of the VCG mechanism.

1. Show that, when the VCG mechanism is used, it is possible for two bidders not to be allocated anything when bidding truthfully, but to obtain strictly positive utility if they tacitly collude, by both submitting untruthful bids.

2. **False-name bids**. Suppose that a single bidder submits multiple false bids to the VCG mechanism. That is, the mechanism thinks there are n bidders, but in reality, at least two of the n bids are submitted by a single bidder i. Show that bidder i can obtain strictly higher utility by submitting multiple false bids than by submitting one truthful bid.

4 *Ties*

The revenue earned in a VCG auction does not depend on the choice of welfare-maximizing allocation. In other words, ties do not impact the auctioneer. Prove this claim in the case of 2 goods and 2 bidders.

5 *Unit Demand*

A **unit-demand** valuation is one in which bidders value a bundle based on only the value of their favorite good in the bundle. More formally, let S denote the set of goods, and let v_{ij} represent bidder i's value for good $j \in S$. If bidder i has unit demand, then

$$v_i(S) = \max_{j \in S} v_{ij}, \quad \forall S \subseteq G. \tag{1}$$

Prove that the VCG mechanism can be run in time polynomial in the number of bidders and goods, assuming unit-demand valuations.