

Problem Set 7

*Instructor: Anna Lysyanskaya***PRF Candidates**

Suppose that $\{F_S : \{0, 1\}^k \mapsto \{0, 1\}^k \mid s \in \{0, 1\}^k\}$ is a pseudo-random family of functions from k -bit input to k -bit output, indexed by a k -bit key (“seed”). We would like to construct a new PRF family. Consider the following constructions, and for each show whether it is good or bad (namely whether the specified family is pseudo-random or not, for the appropriate domain and range). If your answer is “yes,” give an outline of a reduction. If your answer is “no,” give an example F , some convincing evidence that F is a PRF, and an attack on the construction of F^i when it is based on your F .

Below, “ \circ ” denotes concatenation, and \bar{x} denotes the bitwise negation of x .

- (a) $F_S^1(x) = F_S(x) \circ F_S(\bar{x})$.
- (b) $F_S^2(x) = F_{0^k}(x) \circ F_S(x)$.
- (c) $F_S^3(x) = F_{S_1}(x) \circ F_{S_2}(x)$, where $S_1 = F_S(0^k)$ and $S_2 = F_S(1^k)$.
- (d) $F_S^4(x) = F_S(x) \oplus x$.
- (e) $F_S^5(x) = F_S(x) \oplus S$.