Introduction to Machine Learning

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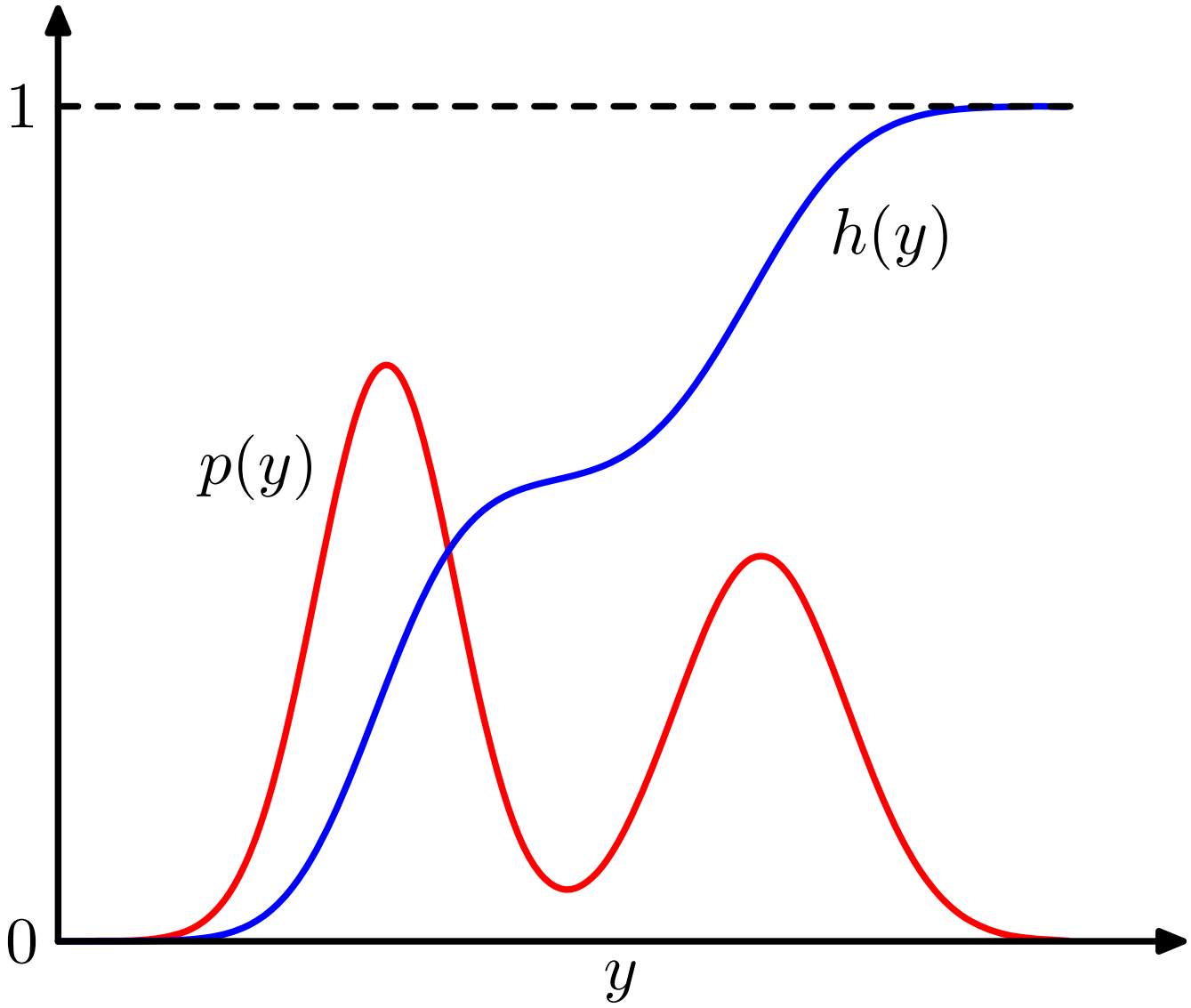
Lecture 23: Monte Carlo Methods

Many figures courtesy Kevin Murphy's textbook,
Machine Learning: A Probabilistic Perspective

Uses of Monte Carlo Methods

- Given estimated parameters for some statistical model, quantitatively or qualitatively assess accuracy of fit
- Parameter estimation when closed forms unavailable
- Parameter estimation for models with hidden “nuisance” variables (alternative to the EM algorithm)
- General approach to applying computational resources to solve statistical learning problems…

Random Number Generation



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Monte Carlo Estimators

$$\mathbb{E}_p[f(x)] = \int_{\mathcal{X}} f(x)p(x)\,dx$$

$$\approx \frac{1}{L}\sum_{\ell=1}^{L} f(x^{(\ell)}) = \mathbb{E}_{\tilde{p}}[f(x)]$$

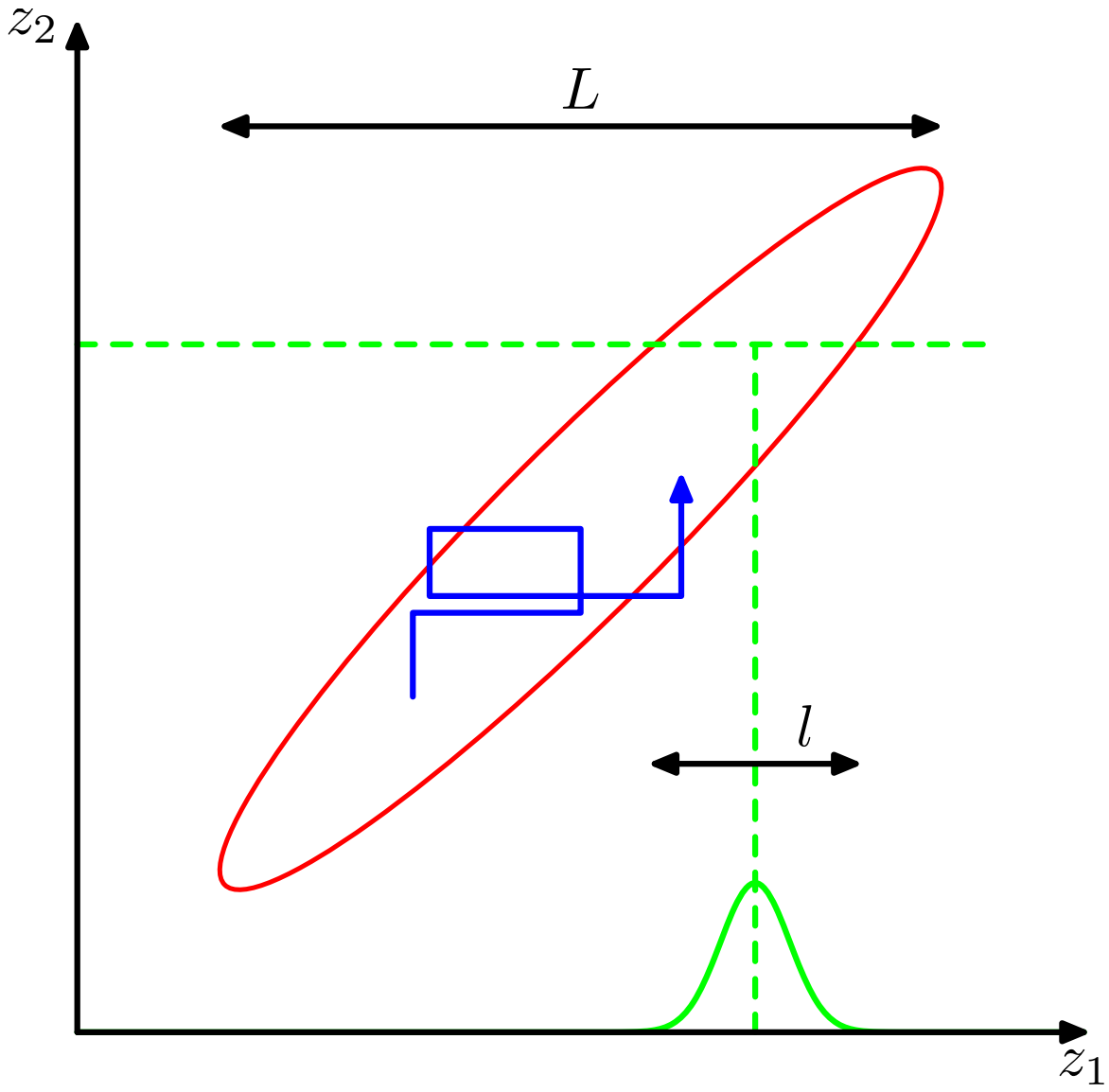
$\{x^{(\ell)}\}_{\ell=1}^{L}$ *independent samples*

$$\tilde{p}(x) = \frac{1}{L}\sum_{\ell=1}^{L} \delta(x, x^{(\ell)})$$

Good properties if *L* sufficiently large:

- Bias and variance
- Weak law of large numbers
- Strong law of large numbers

Gibbs Sampler for a 2D Gaussian



For 3 variables:

$$x_1^{s+1} \sim p(x_1 | x_2^s, x_3^s)$$

$$x_2^{s+1} \sim p(x_2 | x_1^{s+1}, x_3^s)$$

$$x_3^{s+1} \sim p(x_3 | x_1^{s+1}, x_2^{s+1})$$

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