

# Introduction to Machine Learning

Brown University CSCI 1950-F, Spring 2011  
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Lecture 20: Principal Components Analysis  
& Factor Analysis

Many figures courtesy Kevin Murphy's textbook,  
*Machine Learning: A Probabilistic Perspective*

# Dimensionality Reduction

*Supervised Learning*

*Unsupervised Learning*

*Discrete*

*Continuous*

classification or  
categorization

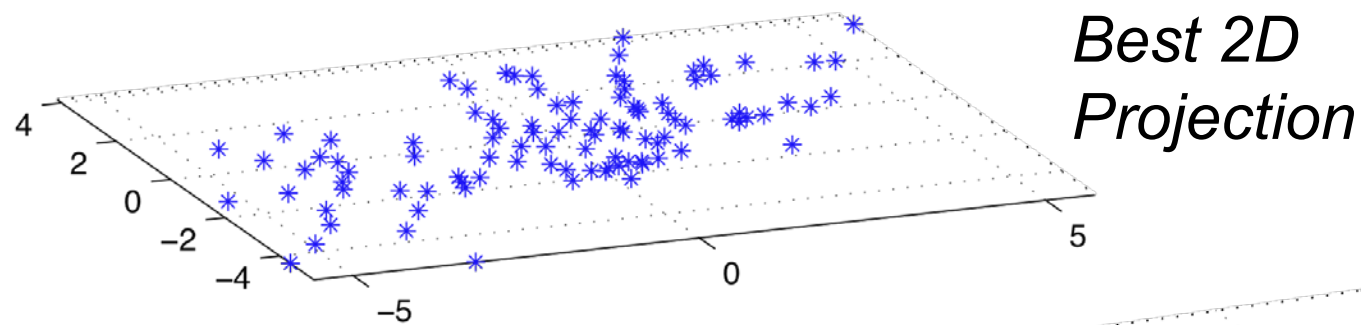
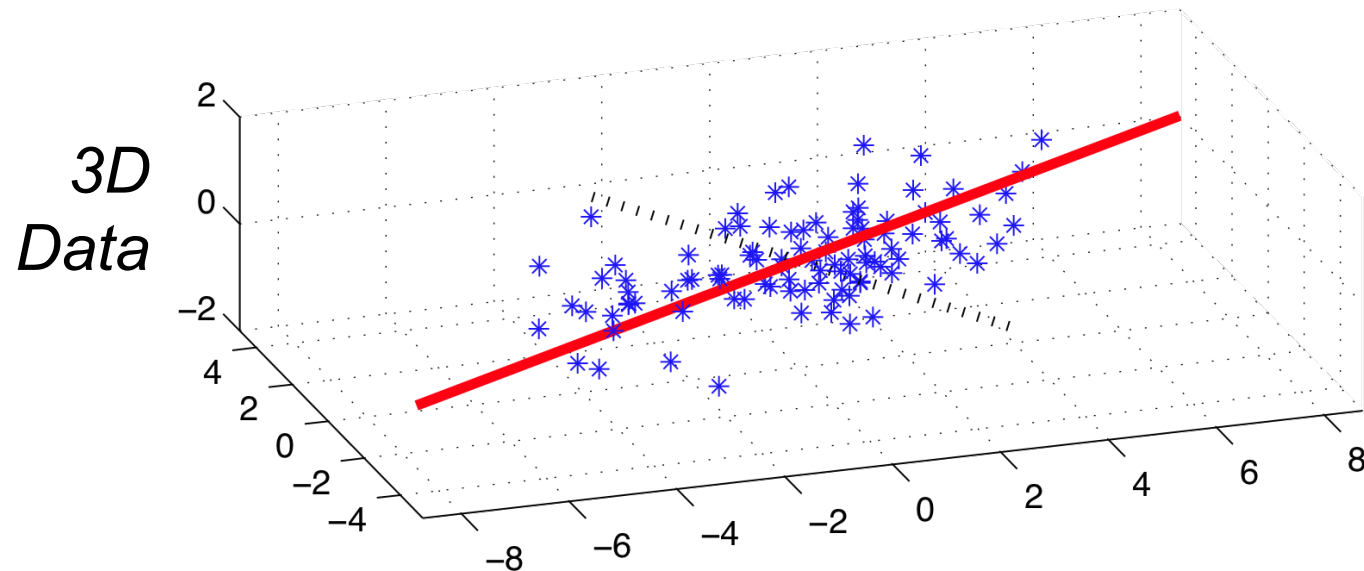
clustering

regression

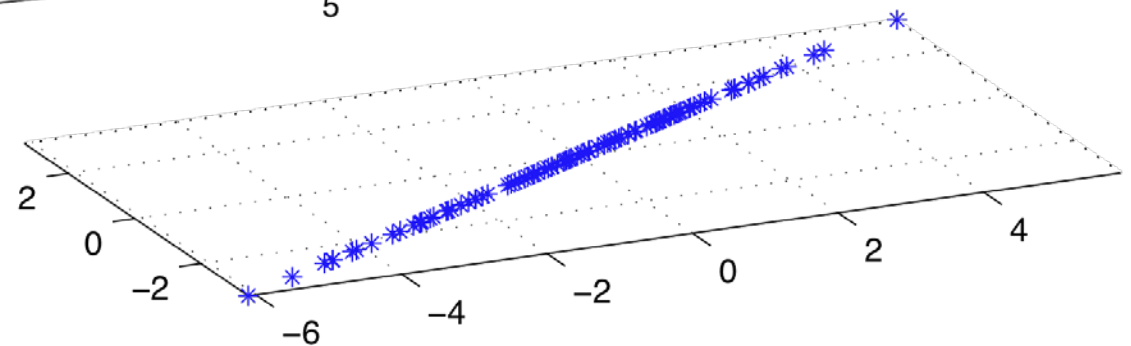
dimensionality  
reduction

- **Goal:** Infer label/response  $y$  given only features  $x$
- **Classical:** Find latent variables  $y$  good for *compression* of  $x$
- **Probabilistic learning:** Estimate parameters of joint distribution  $p(x,y)$  which *maximize marginal probability*  $p(x)$

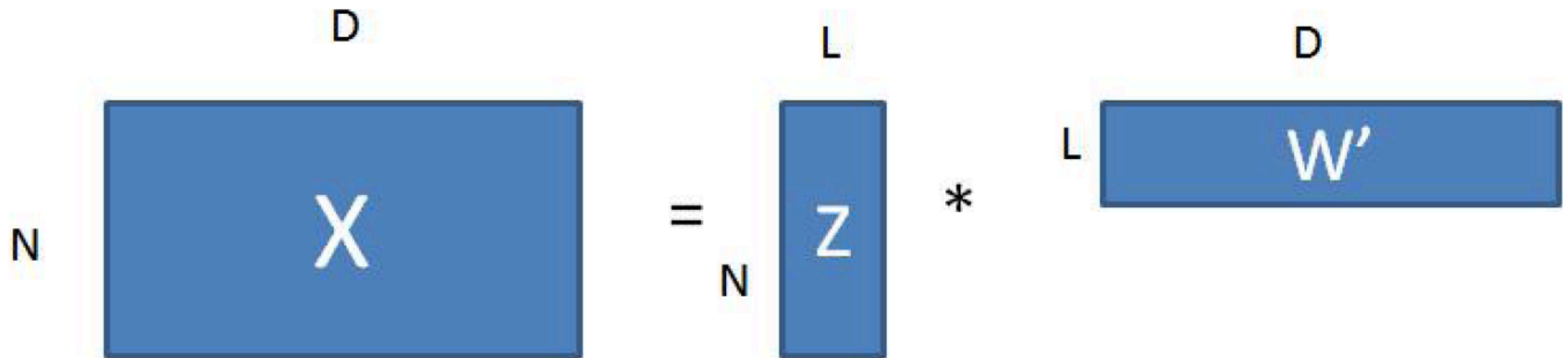
# Principal Components Analysis (PCA)



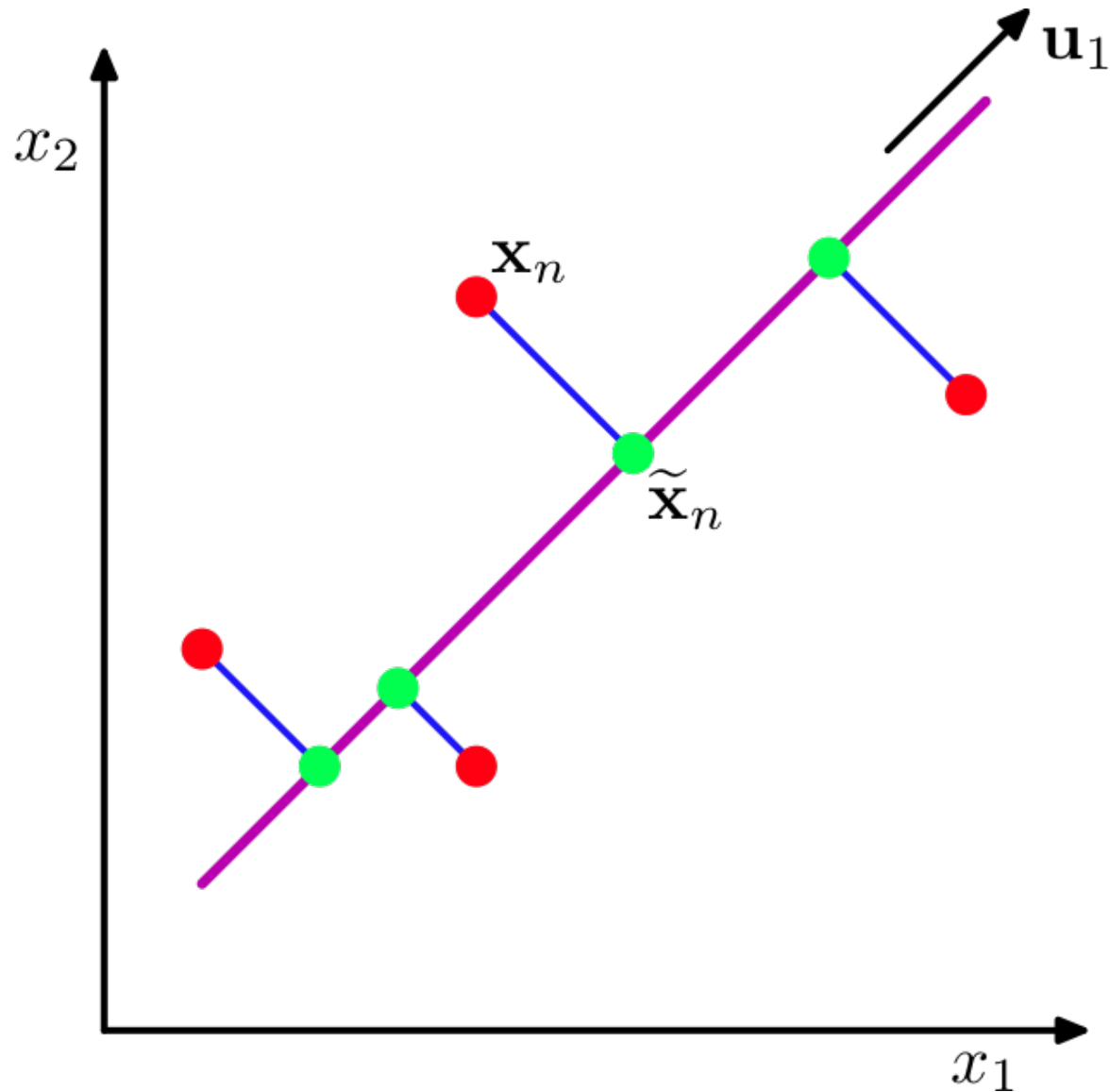
*Best 1D Projection*



# PCA as Low Rank Approximation

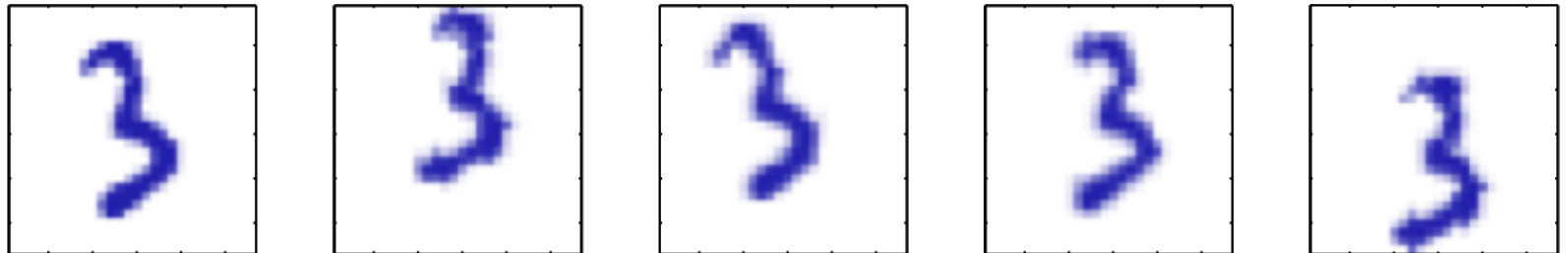


# Maximizes Variance & Minimizes Error



# PCA for Handwritten Digits

**Data:**  
*Randomly  
translated &  
rotated 3's*



Mean

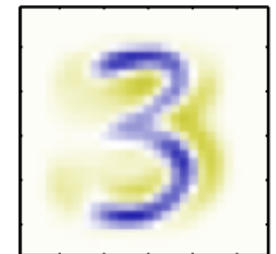
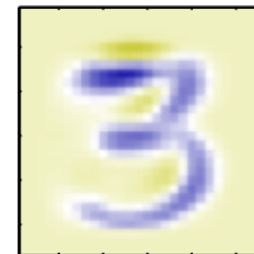
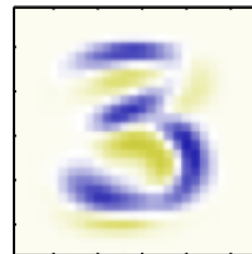
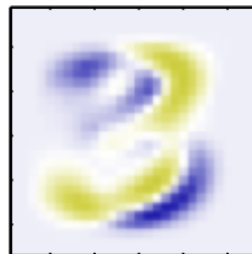
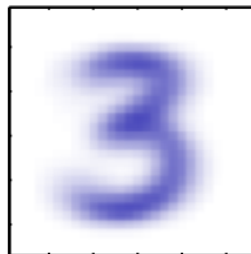
$$\lambda_1 = 3.4 \cdot 10^5$$

$$\lambda_2 = 2.8 \cdot 10^5$$

$$\lambda_3 = 2.4 \cdot 10^5$$

$$\lambda_4 = 1.6 \cdot 10^5$$

**PCA Model:**  
*Mean &  
top four  
eigenvectors*



Original

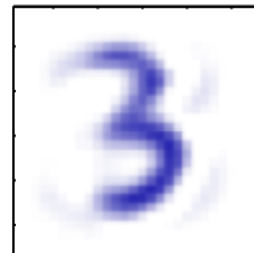
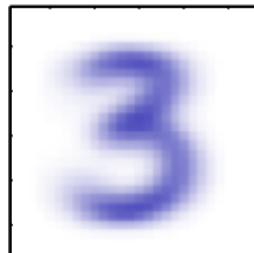
$M = 1$

$M = 10$

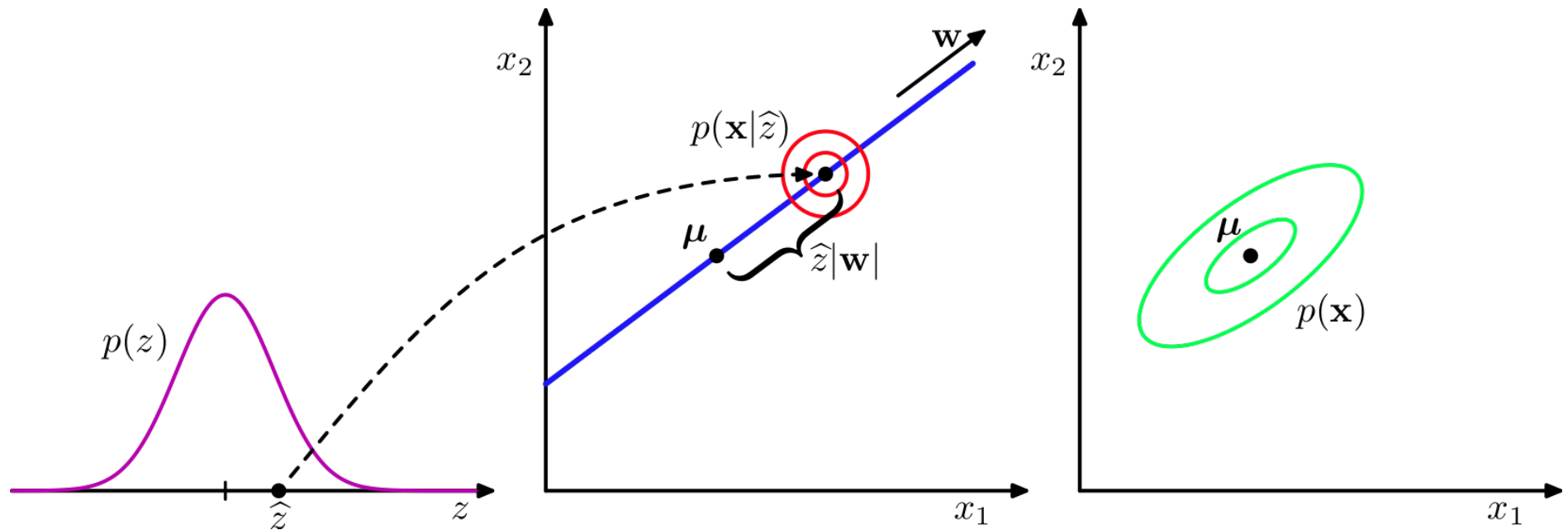
$M = 50$

$M = 250$

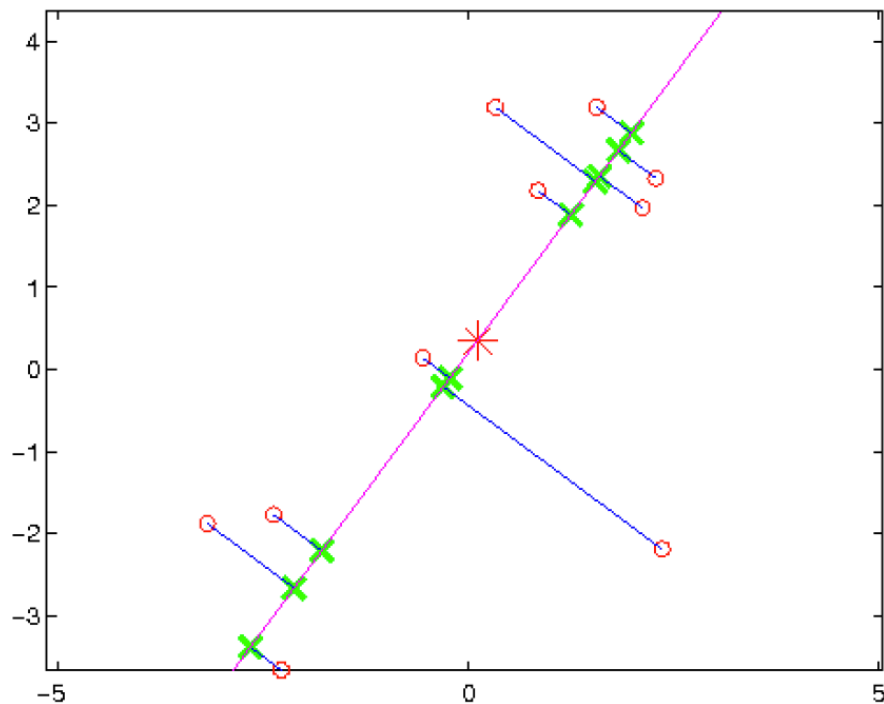
**Compression:**  
*Quality vs.  
latent  
dimension*



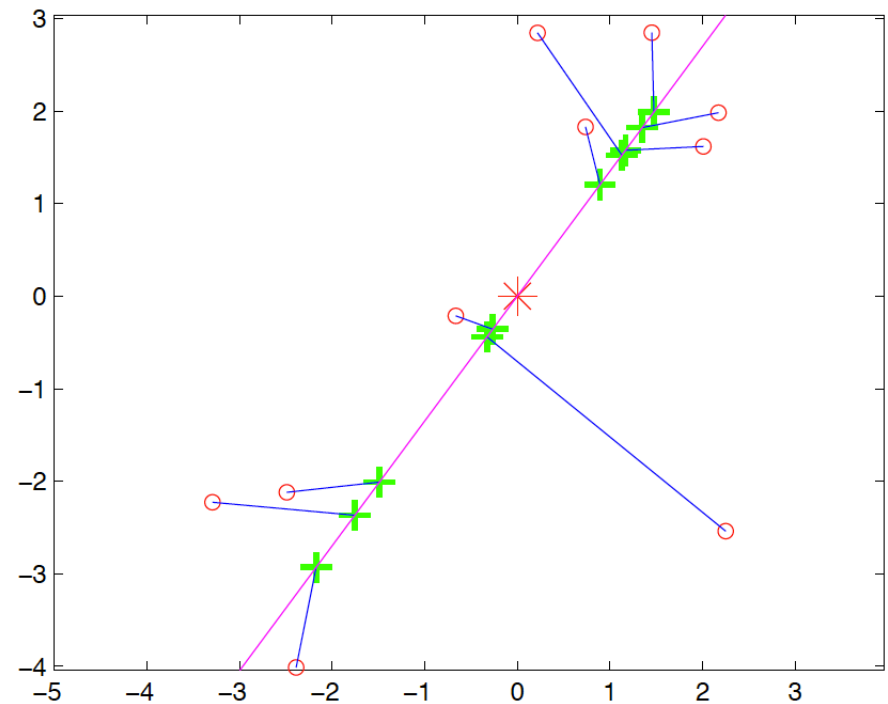
# Probabilistic PCA



# PCA versus Probabilistic PCA



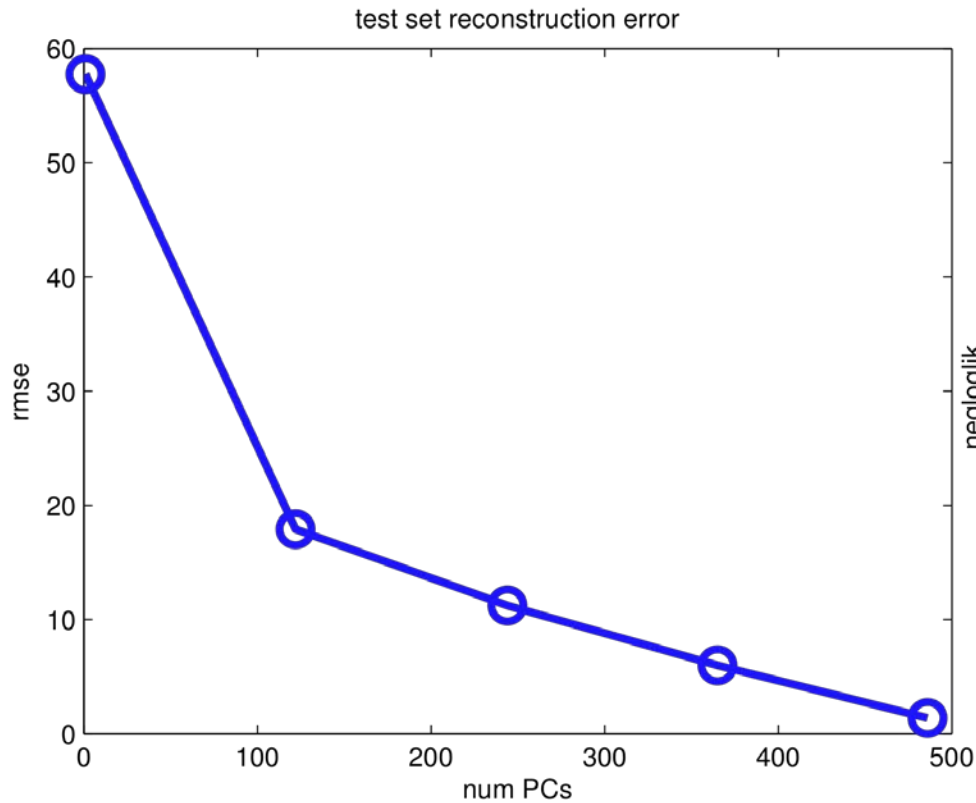
*Standard PCA*  
(orthogonal projection)



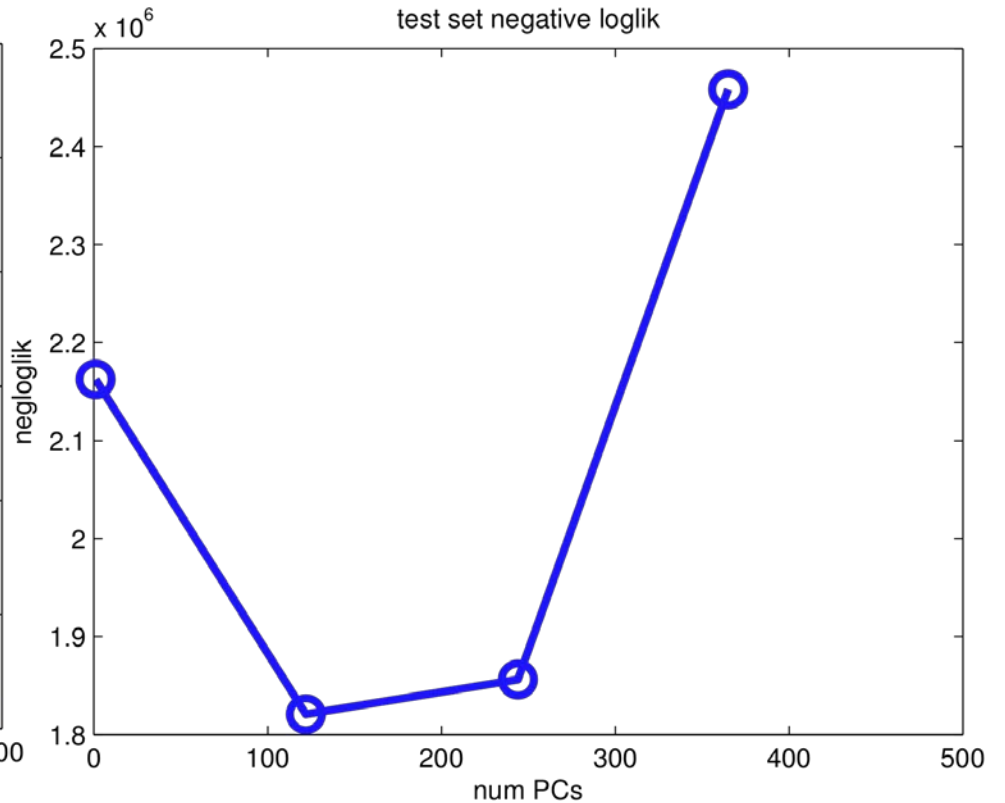
*Probabilistic PCA*  
(shrunk towards mean)



# Prediction of Validation Data



*Standard PCA*  
(reconstruction error)



*Probabilistic PCA*  
(negative log likelihood)

# Factor Analysis Example

## Features of Cars in 2004

Suggested retail price in USD

Price to dealer in USD

Engine size in liters

Num. cylinders

Horsepower

City MPG

Highway MPG

Weight in pounds

Wheelbase in inches

Length in inches

Width in inches

