Introduction to Machine Learning

Brown University CSCI 1950-F, Spring 2011

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Lecture 4: Bayesian Estimation

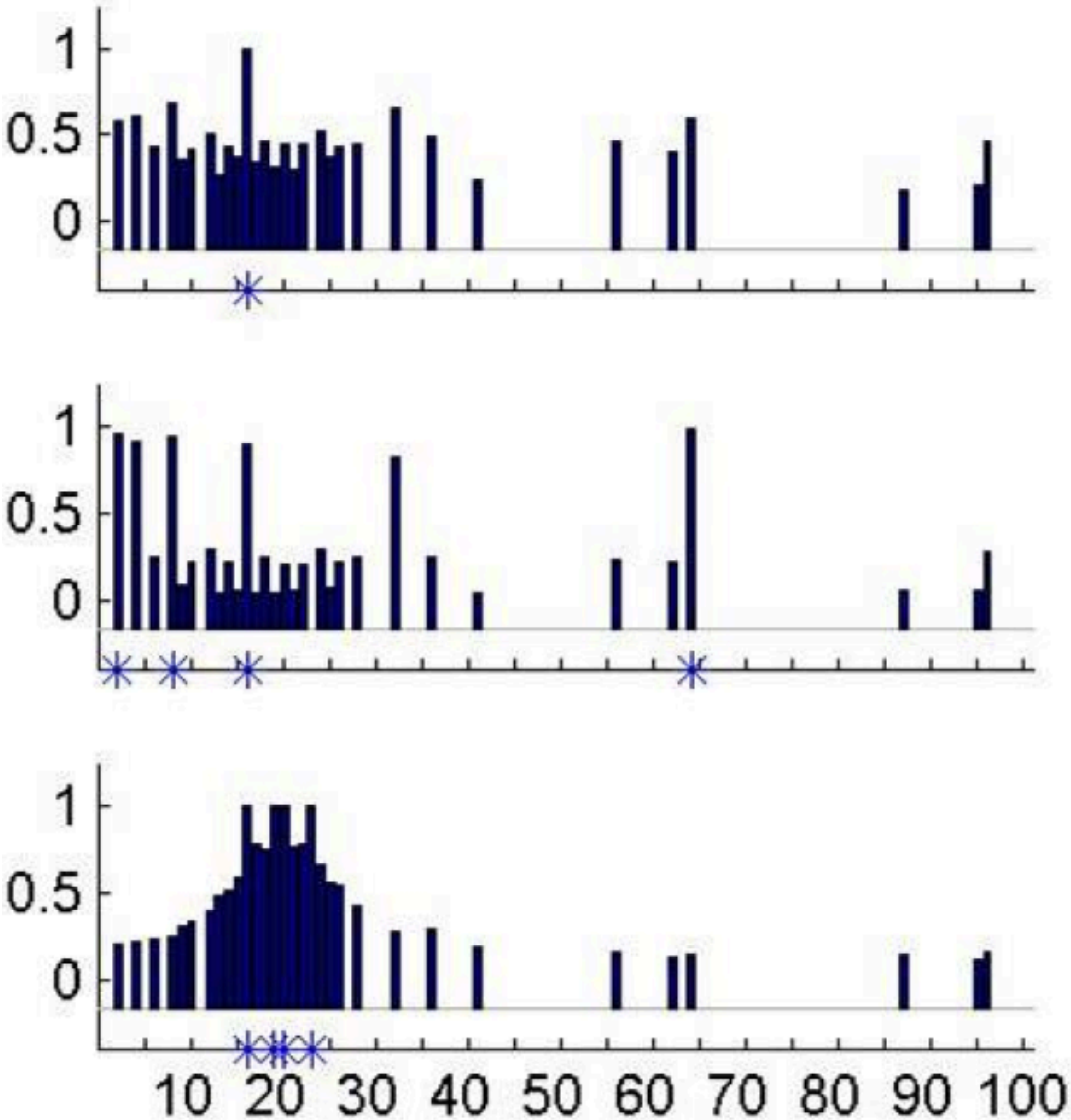
Many figures courtesy Kevin Murphy's textbook, *Machine Learning: A Probabilistic Perspective*

The Number Game

- I am thinking of some arithmetical concept, such as:
  - Prime numbers
  - Numbers between 1 and 10
  - Even numbers
  - …
- I give you a series of randomly chosen *positive* examples from the chosen class
- Question: Are other test digits also in the class?

Tenenbaum 1999

Predictions of 20 Humans



D = {16}

D = {16,8,2,64}

D = {16,23,19,20}

A Bayesian Model

Likelihood:

$$p(\mathcal{D}|h) = \left[\frac{1}{\text{size}(h)}\right]^n = \left[\frac{1}{|h|}\right]^n$$

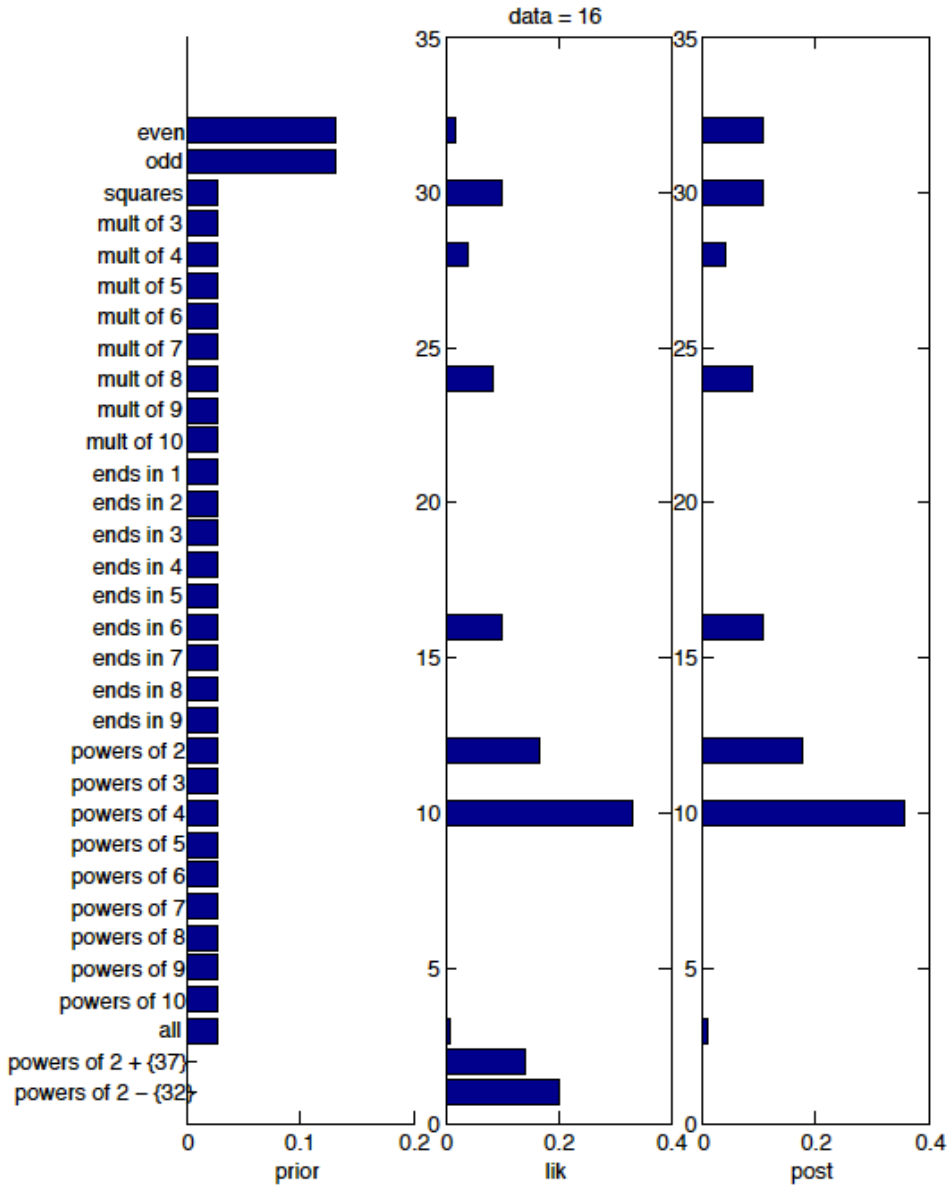
- Assume examples are sampled uniformly at random from all numbers that are consistent with the hypothesis
- Size principle: Favors smallest consistent hypotheses

Prior:

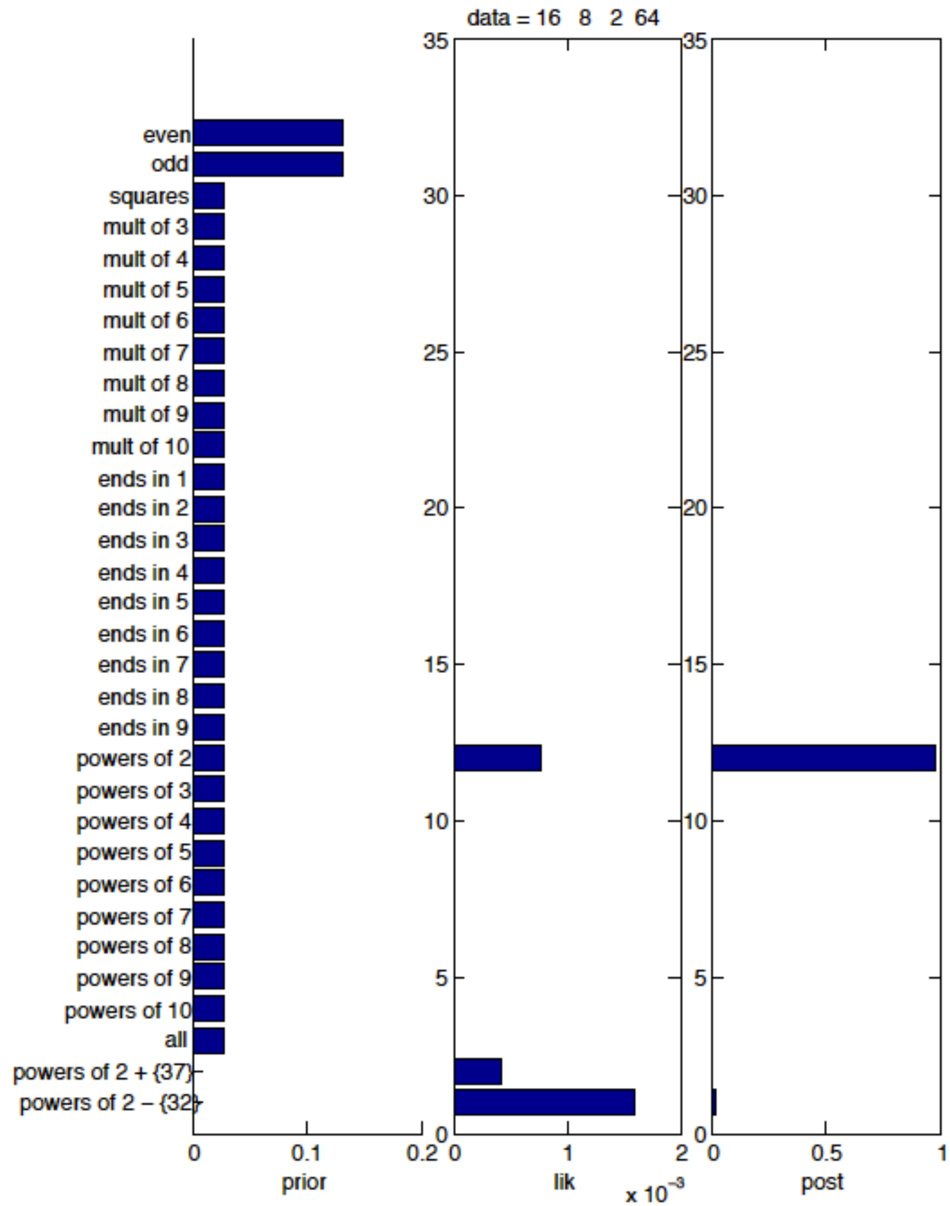
- Based on prior experience, some hypotheses are more probable ("natural") than others
  - Powers of 2
  - Powers of 2 except 32, plus 37
- Subjectivity: May depend on observer's experience

D = {5, 34, 2, 89, 1, 13}?

Posterior Distributions

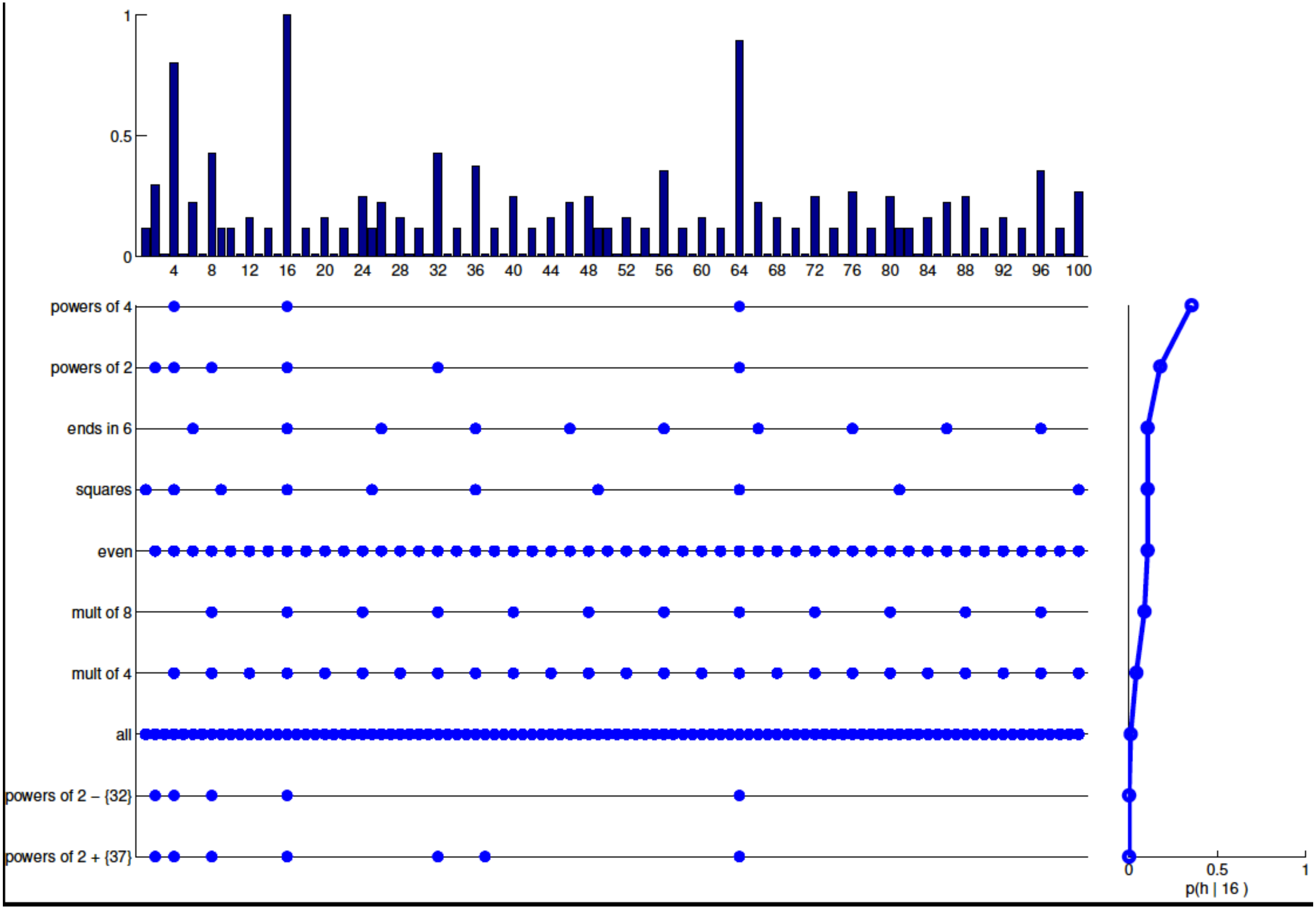


D = {16}

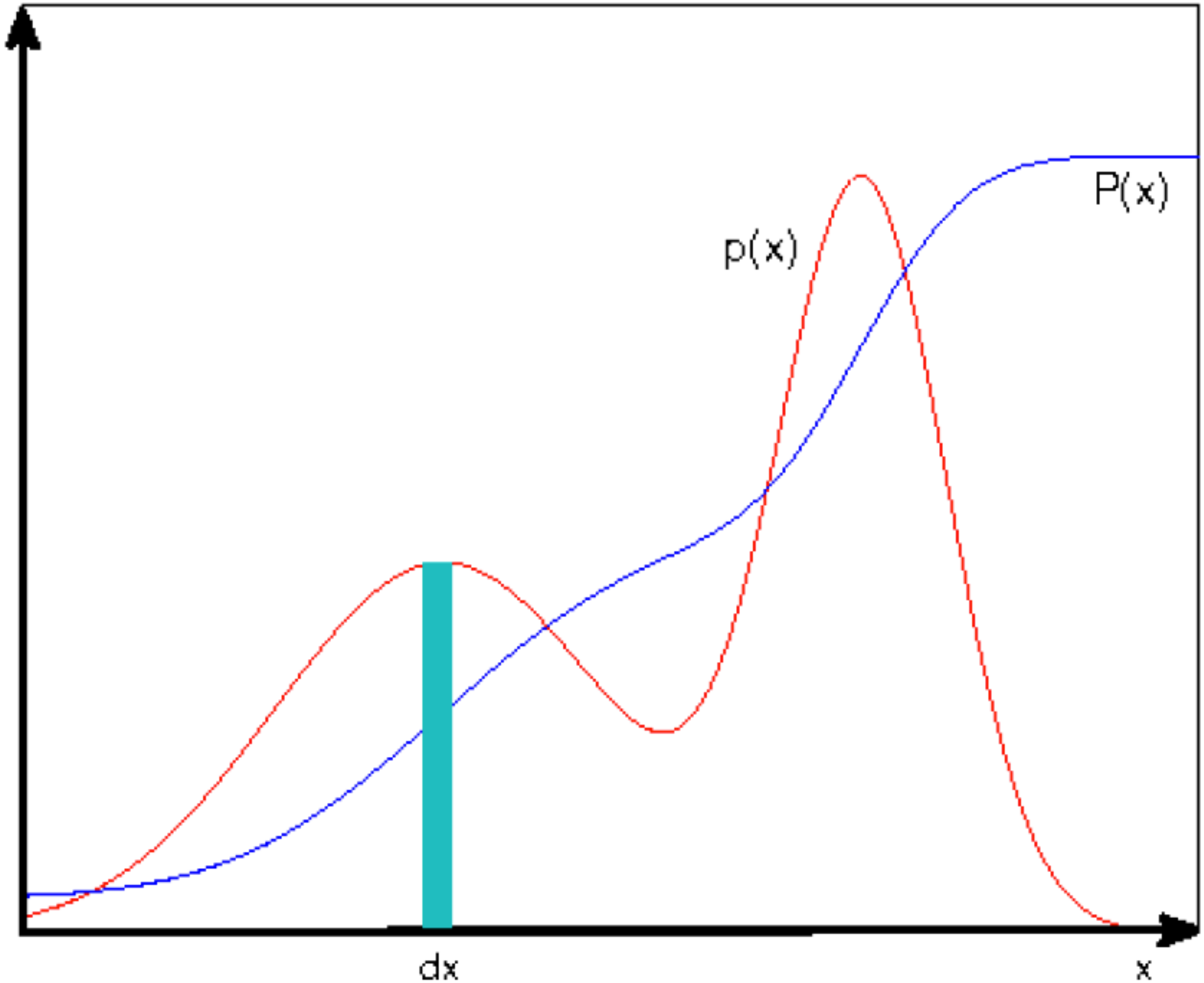


D = {16,8,2,64}

Posterior Predictive Distribution



Continuous Random Variables



Gaussian (Normal) Distributions

$$\mathcal{N}(x|\mu,\sigma^2) \quad := \quad \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \qquad\qquad \Phi(x;\mu,\sigma^2) := \int_{-\infty}^{x} \mathcal{N}(z|\mu,\sigma^2)dz$$