Useful formulae

$$\int_a^b x^n \, dx = \frac{b^{n+1} - a^{n+1}}{n+1}$$

$$\begin{aligned}
\text{Beta}(\theta \mid a, b) &= \frac{1}{B(a,b)} \, \theta^{a-1} \, (1-\theta)^{b-1} \qquad \text{for } 0 \le \theta \le 1 \\
B(a,b) &= \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} \\
\text{E}[\,\theta \mid a, b\,] &= \int_0^1 \theta \; \text{Beta}(\theta \mid a, b)\, d\theta = \frac{a}{a+b} \\
\text{Var}[\,\theta \mid a, b\,] &= \text{E}[\,\theta^2 \mid a, b\,] - \text{E}[\,\theta \mid a, b\,]^2 = \frac{ab}{(a+b)^2(a+b+1)}
\end{aligned}$$

$$\begin{aligned}
\text{Unif}(\theta \mid a, b) &= \begin{cases} 1/(b-a) & \text{if } a \le \theta \le b \\ 0 & \text{otherwise} \end{cases} \\
\text{E}[\,\theta \mid a, b\,] &= \int_{-\infty}^{\infty} \theta \; \text{Unif}(\theta \mid a, b)\, d\theta = \frac{a+b}{2} \\
\text{Var}[\,\theta \mid a, b\,] &= \text{E}[\,\theta^2 \mid a, b\,] - \text{E}[\,\theta \mid a, b\,]^2 = \frac{(b-a)^2}{12}
\end{aligned}$$

$$\begin{aligned}
\text{Normal}(\theta \mid \mu, \lambda) &= \frac{1}{\sqrt{2\pi\lambda}} \; \exp\left\{ -\frac{(\theta-\mu)^2}{2\lambda} \right\} \\
\text{E}[\,\theta \mid \mu, \lambda\,] &= \int_{-\infty}^{\infty} \theta \; \text{Normal}(\theta \mid \mu, \lambda)\, d\theta = \mu \\
\text{Var}[\,\theta \mid \mu, \lambda\,] &= \text{E}[\,\theta^2 \mid \mu, \lambda\,] - \text{E}[\,\theta \mid \mu, \lambda\,]^2 = \lambda
\end{aligned}$$