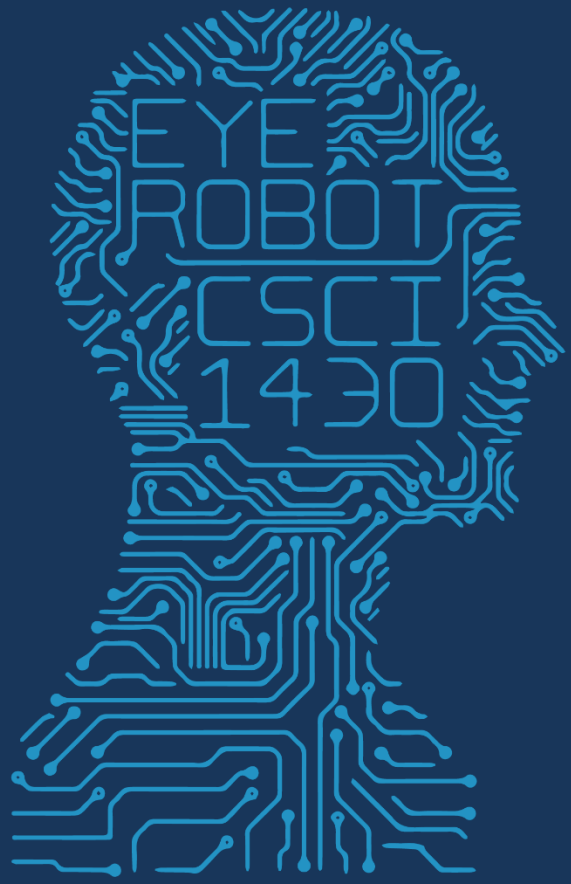
1950

Future Vision

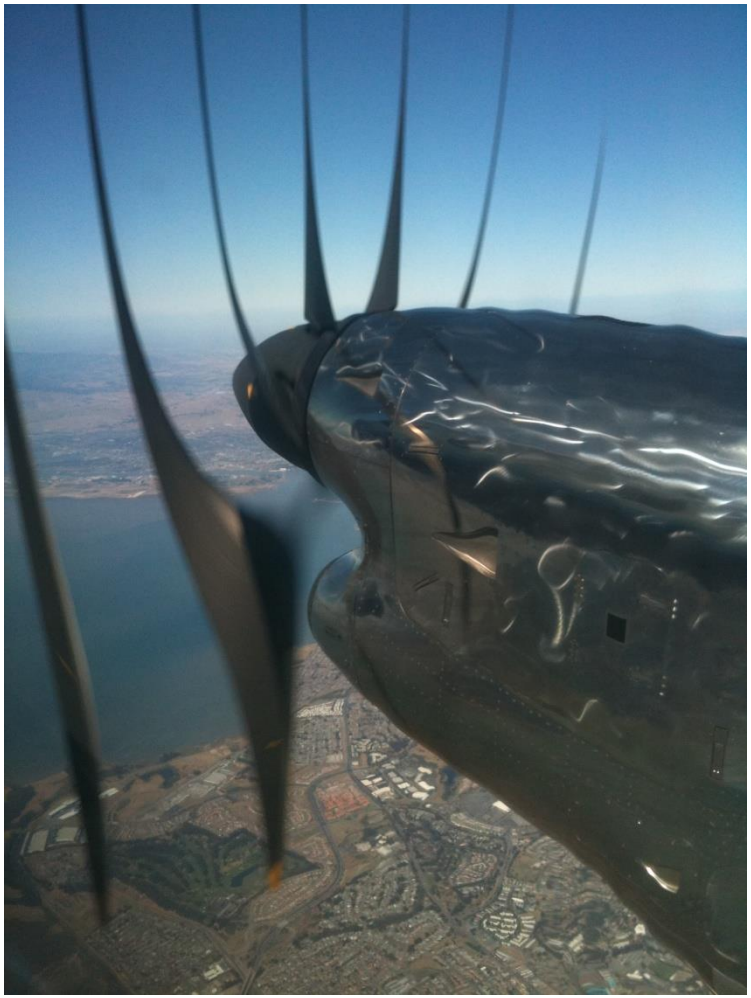


2017 MWF 1pm

Computer Vision

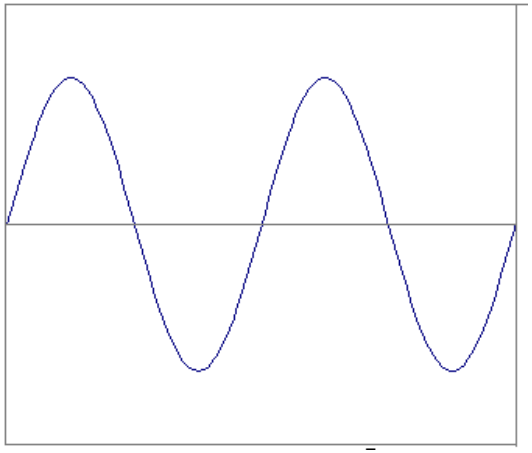

DO NOT STEP

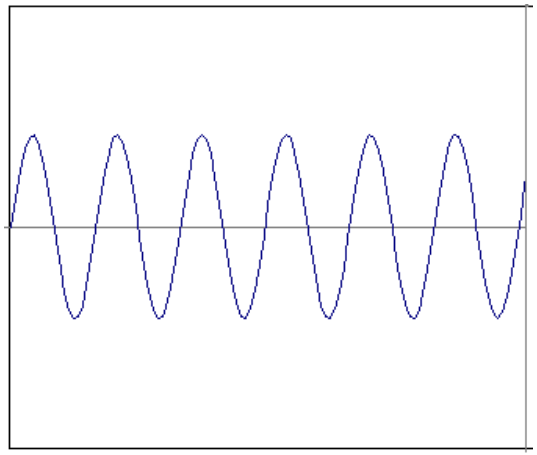
Capture Frequency - Rolling `Shutter’



James Hays

I understand frequency as in waves…





…but how does this relate to the complex signals we see in natural images?

…to image frequency?

Another way of thinking about frequency

FOURIER SERIES & FOURIER TRANSFORMS

Fourier series

A bold idea (1807):

Any *univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.*

Jean Baptiste Joseph Fourier (1768-1830)

Fourier series

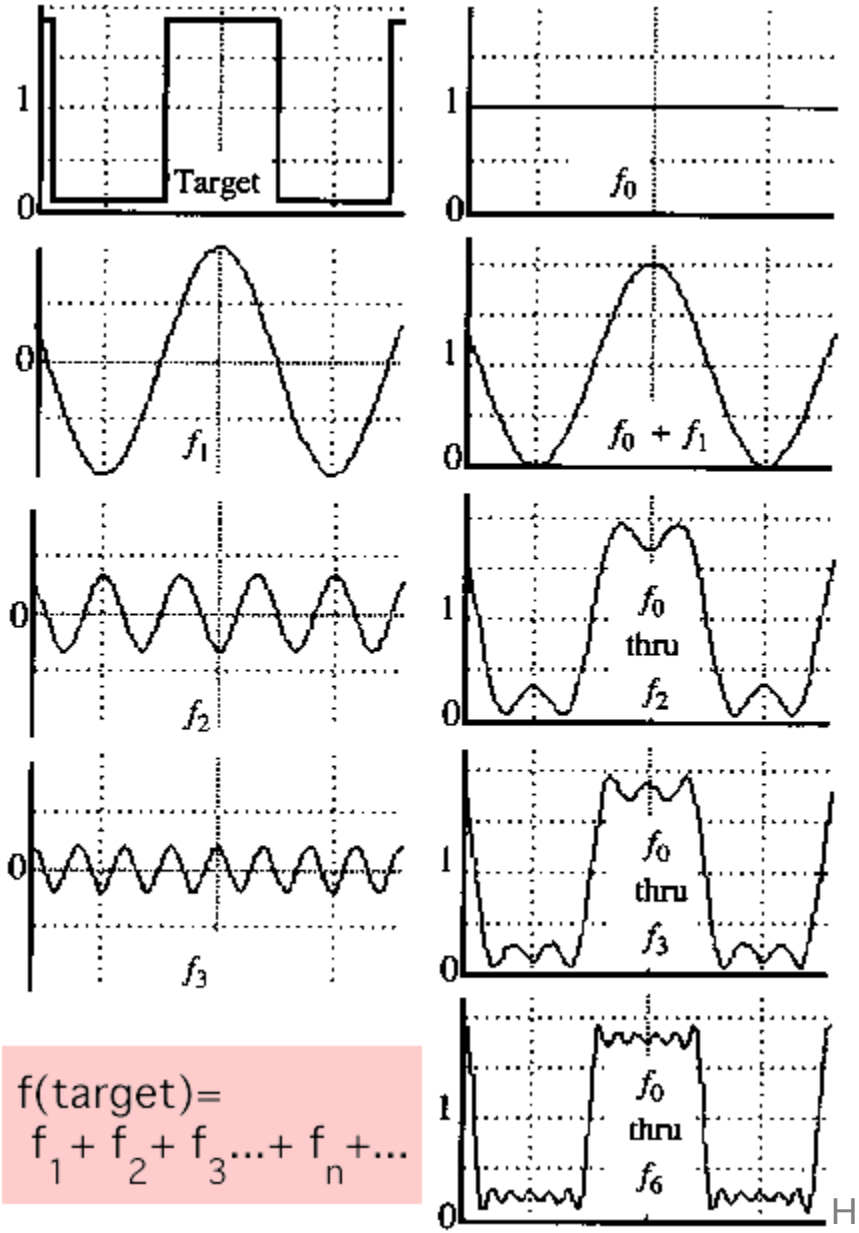
A bold idea (1807):

Any *univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.*

Our building block:

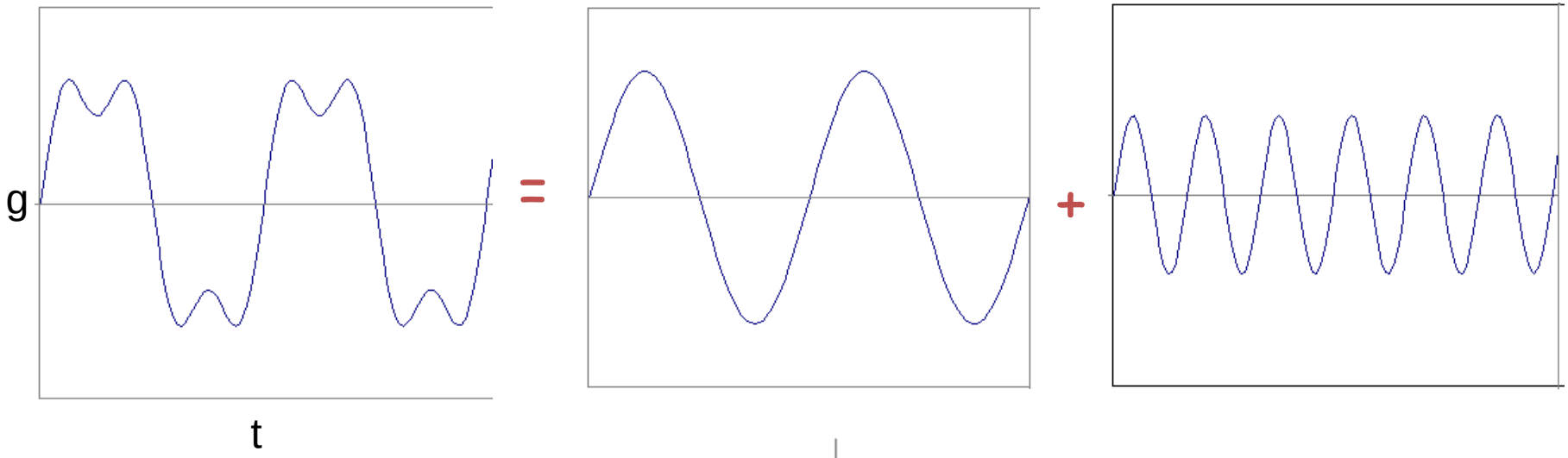
$$A\sin(\omega t) + B\cos(\omega t)$$

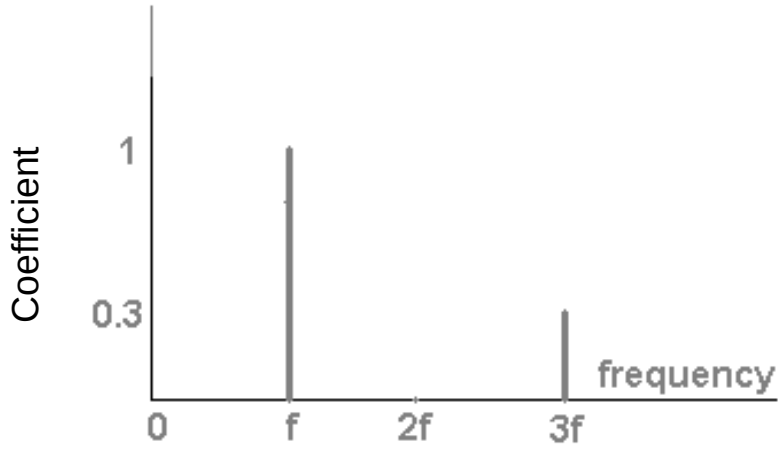
Add enough of them to get any signal *g(t)* you want!



$t = [0,2]$, $f = 1$

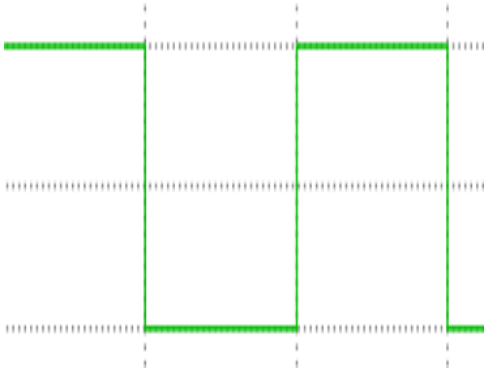
$$g(t) = \sin(2\pi f\, t) + (1/3)\sin(2\pi(3f)\, t)$$



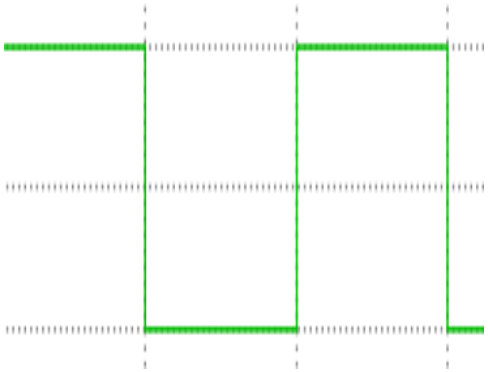


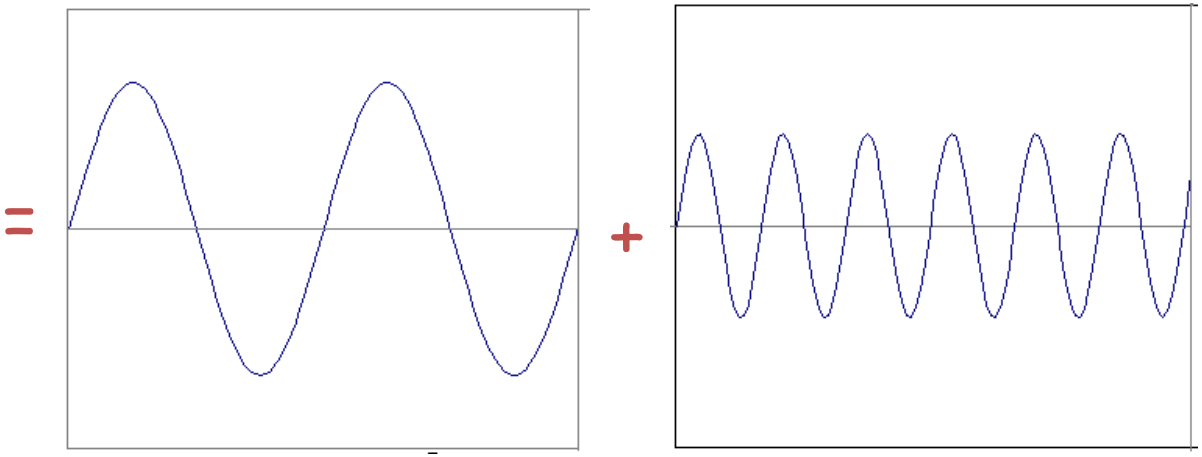
Slides: Efros

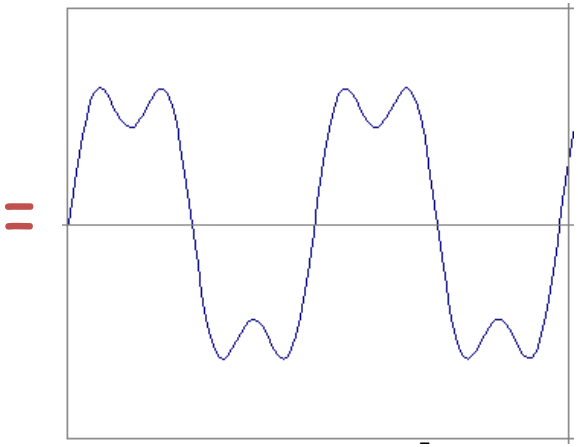
Square wave spectra



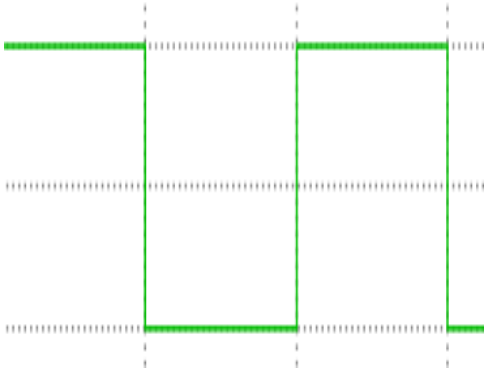
Square wave spectra







Square wave spectra



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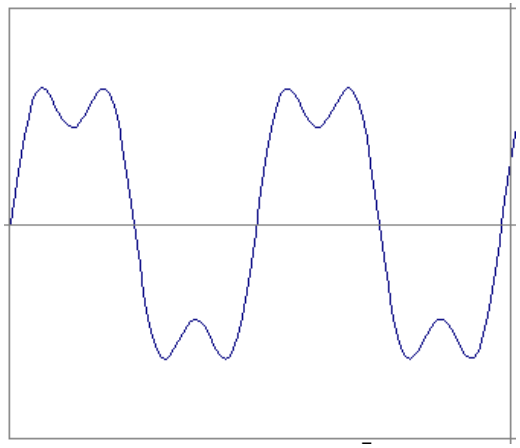

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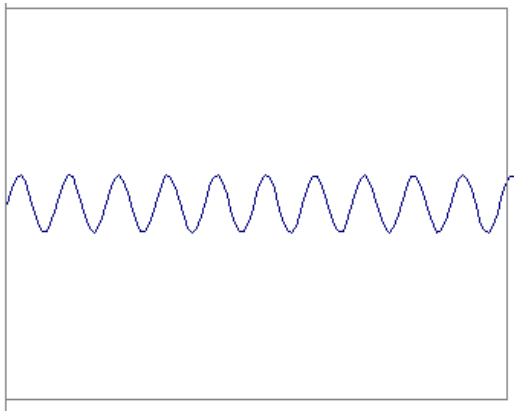

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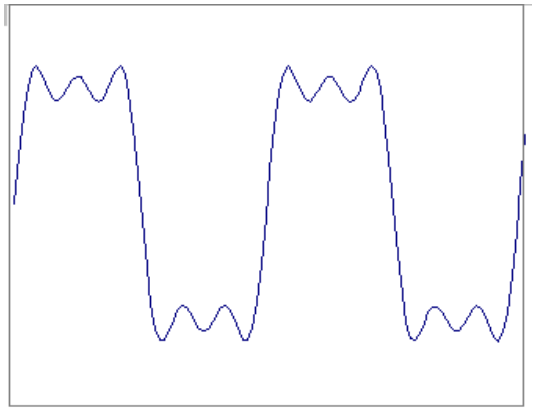
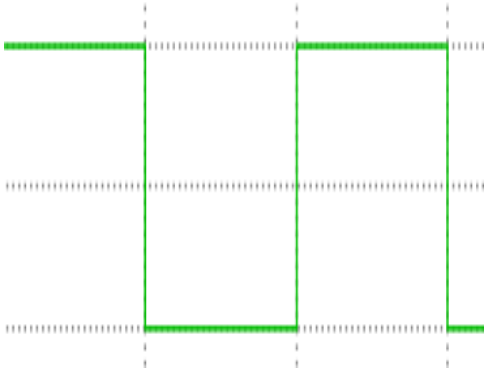
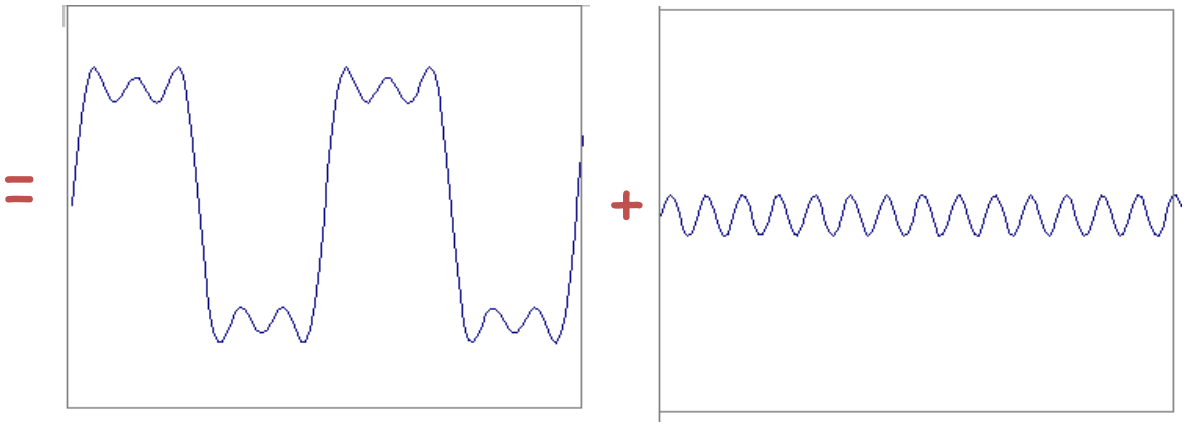
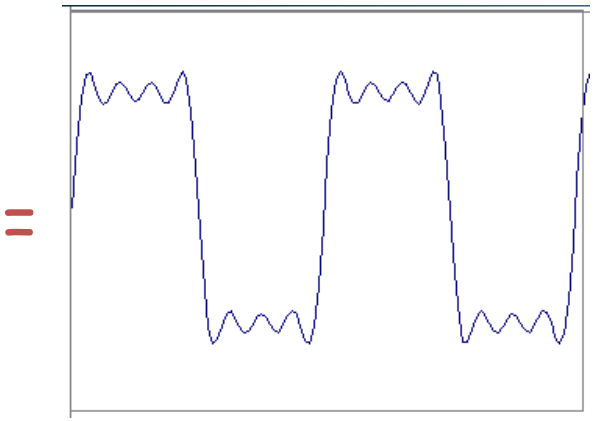


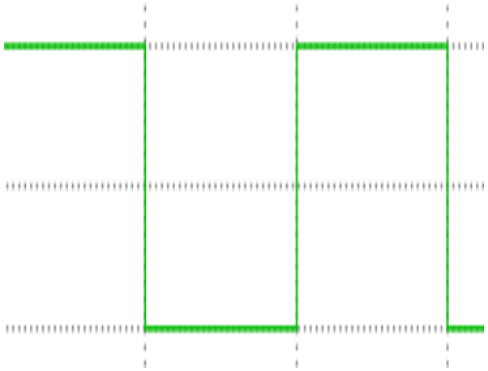
Square wave spectra







Square wave spectra



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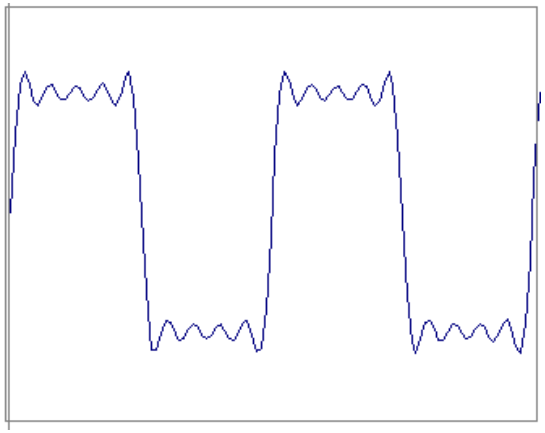

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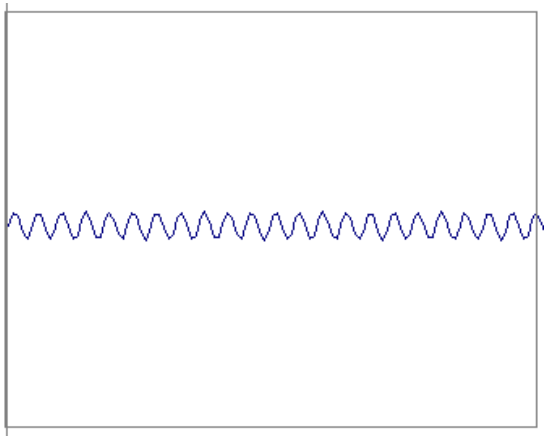

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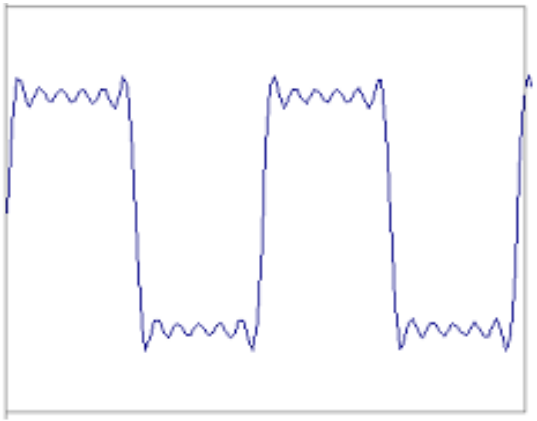
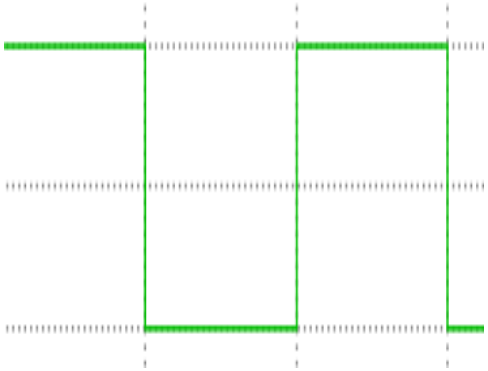
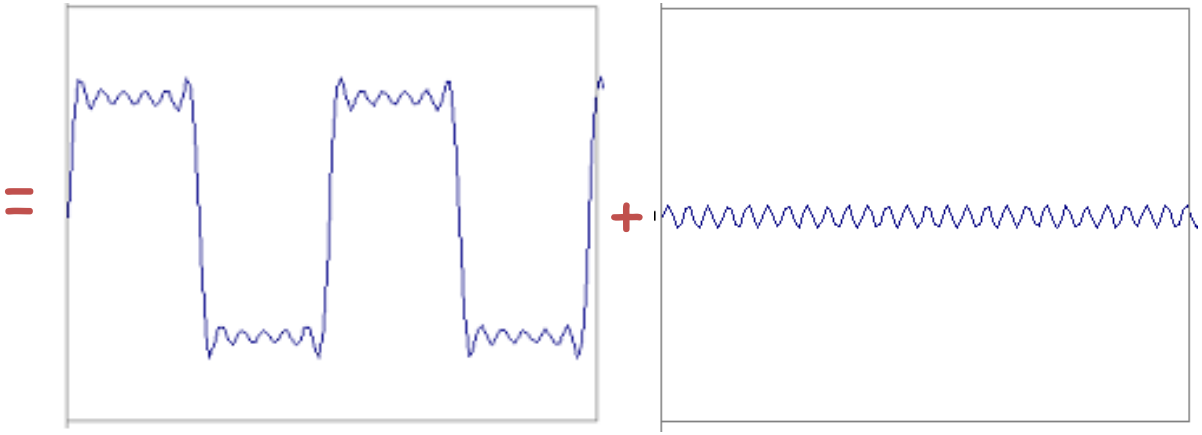
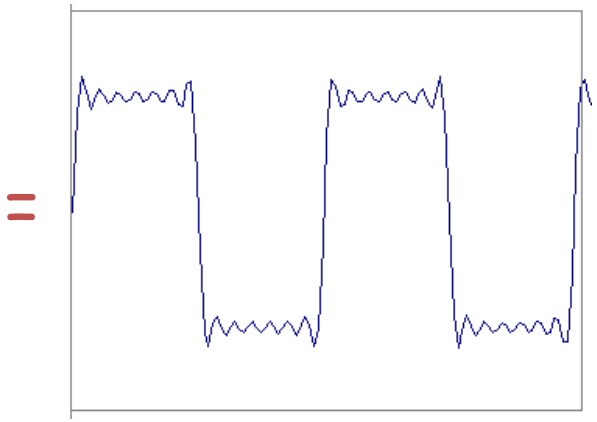


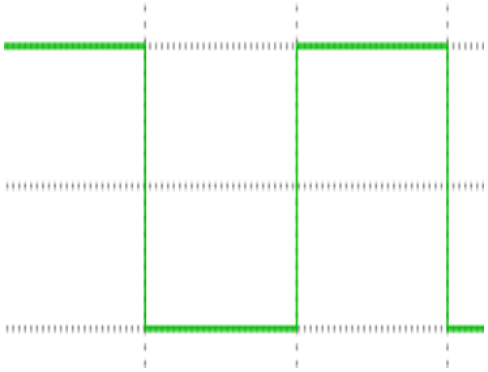
Square wave spectra



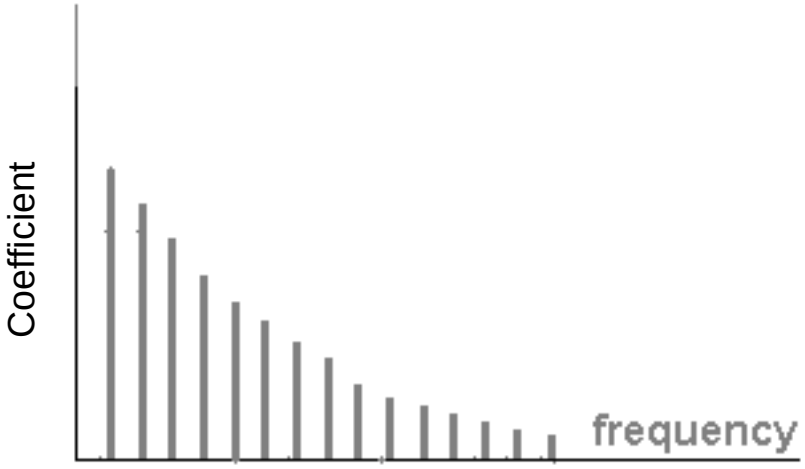




Square wave spectra



$$= A \sum_{f=1}^{\infty} \frac{1}{f} \sin(2\pi f t)$$



Jean Baptiste Joseph Fourier (1768-1830)

A bold idea (1807):

> ***Any*** *univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.*

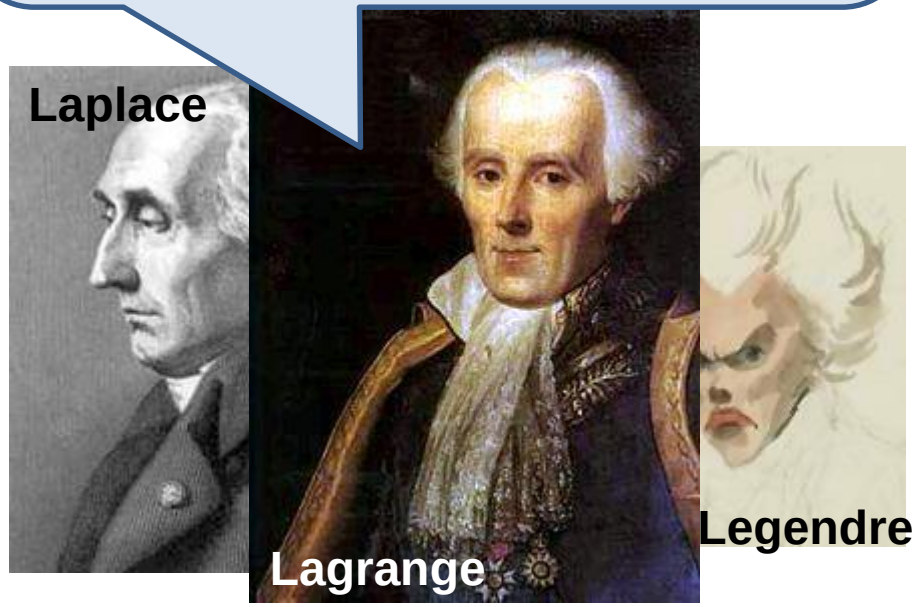
Don't believe it?

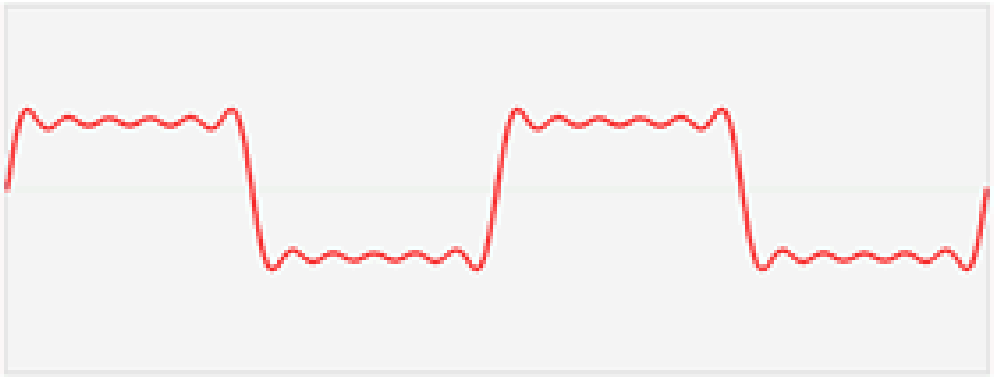
- Neither did Lagrange, Laplace, Poisson and other big wigs
- Not translated into English until 1878!

But it's (mostly) true!

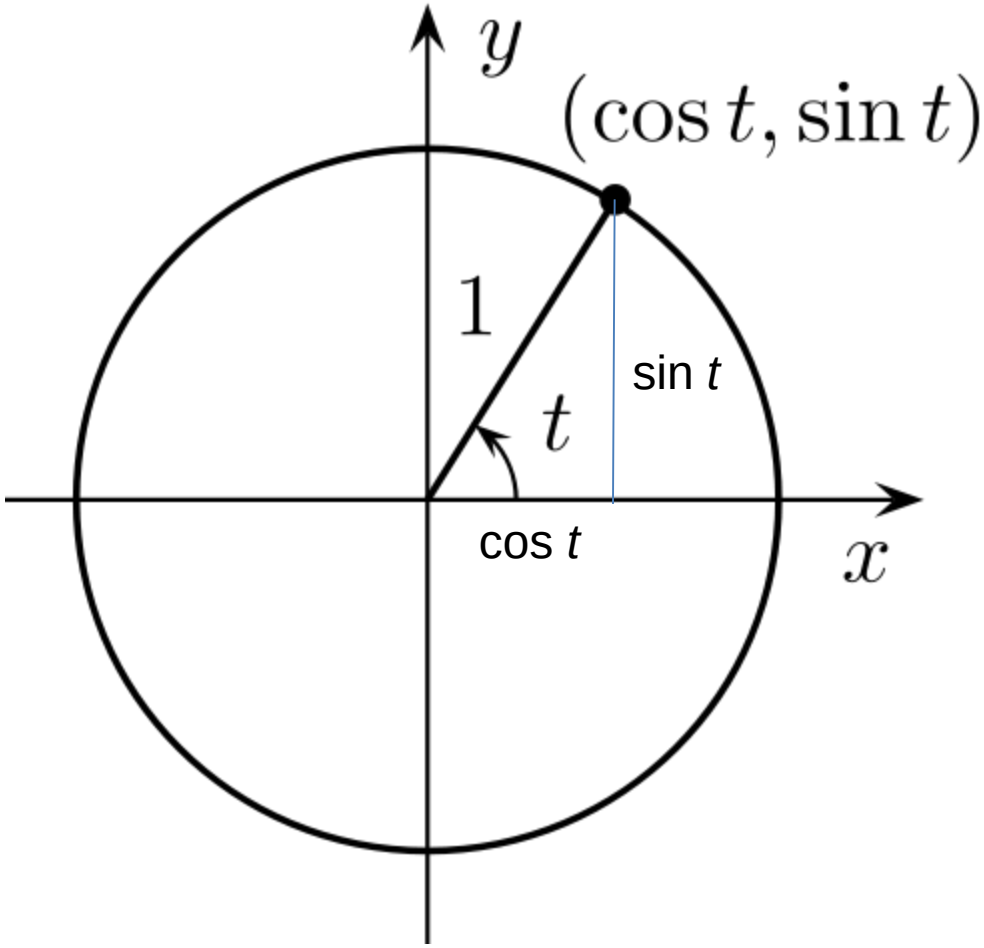
- Called Fourier Series
- There are some subtle restrictions

> *...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.*



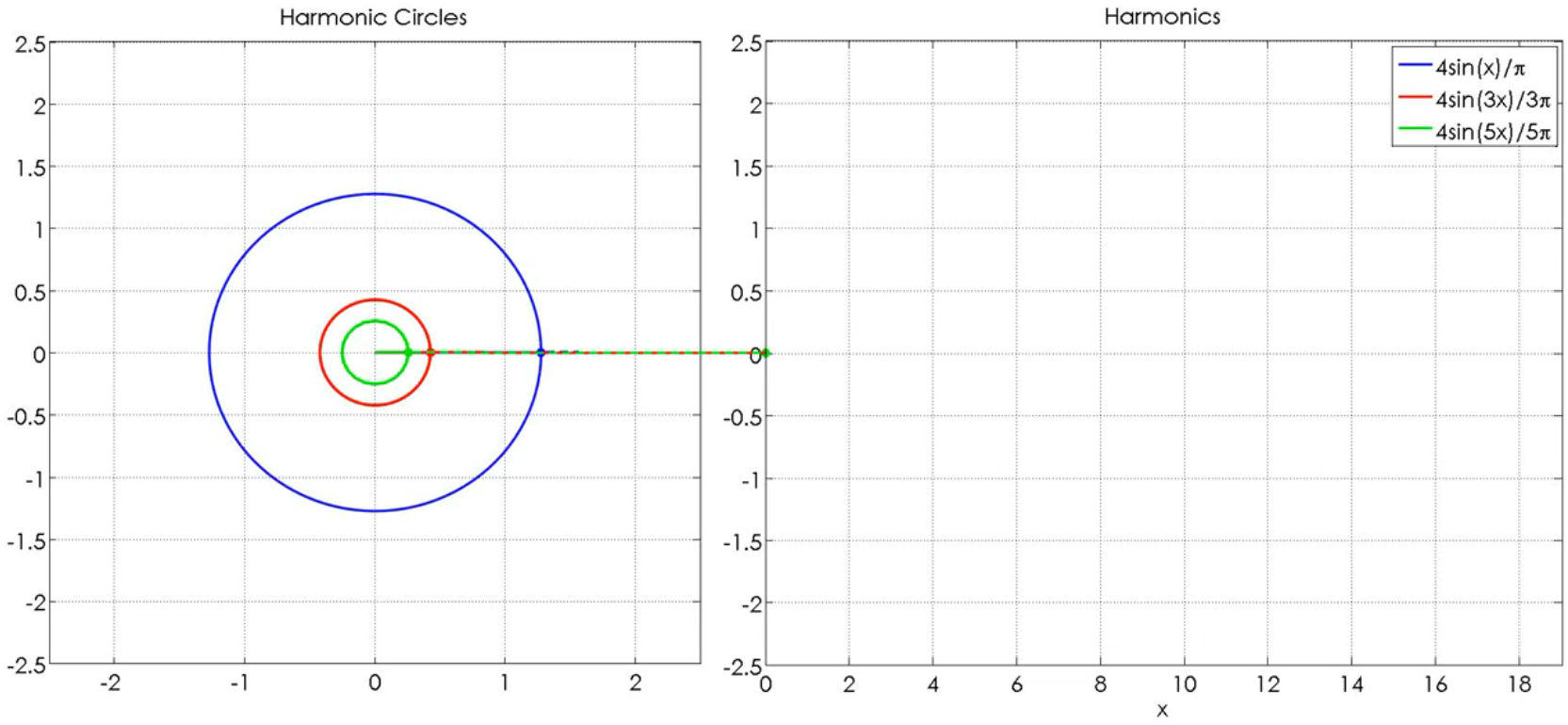


Sine/cosine and circle

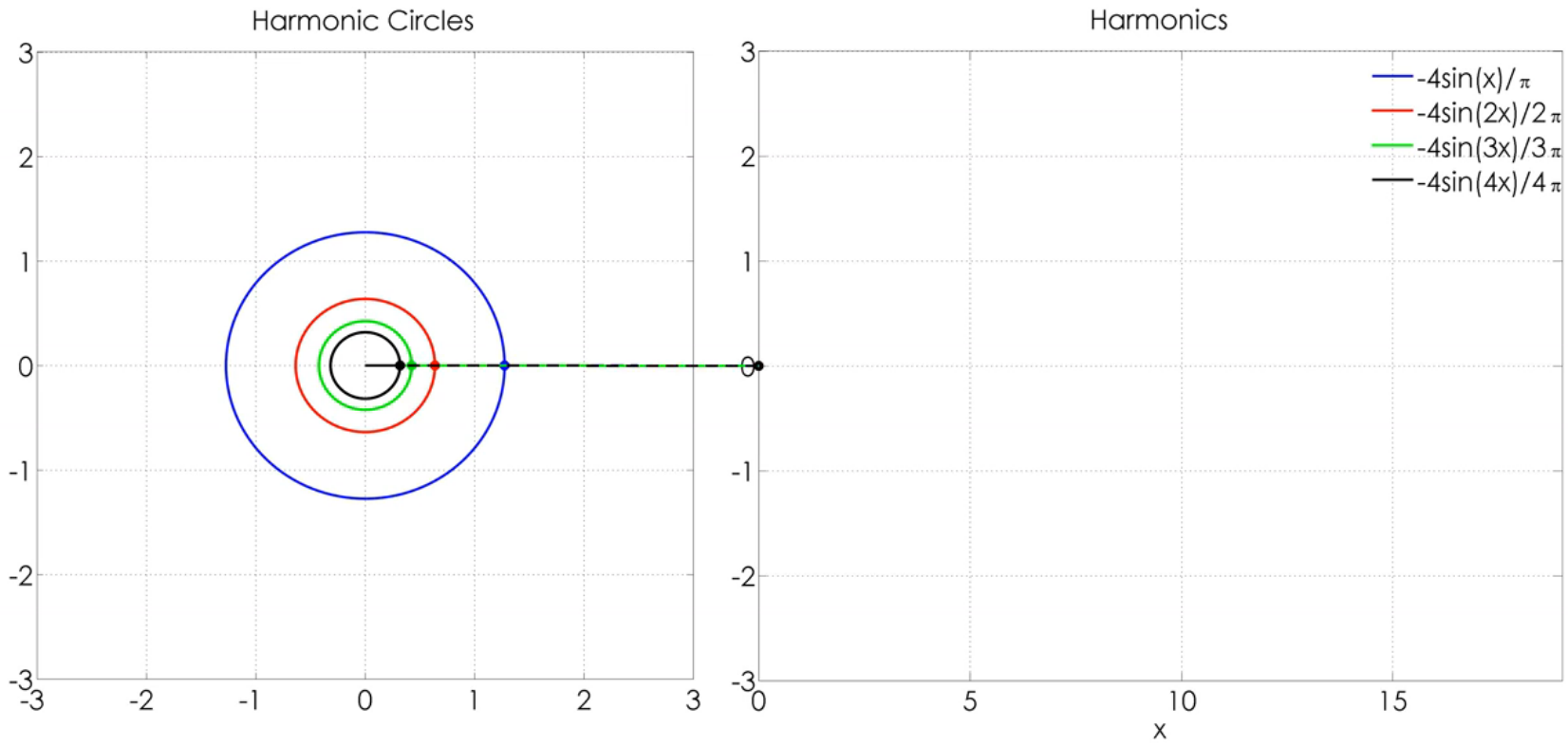


Wikipedia – Unit Circle

Square wave (approx.)

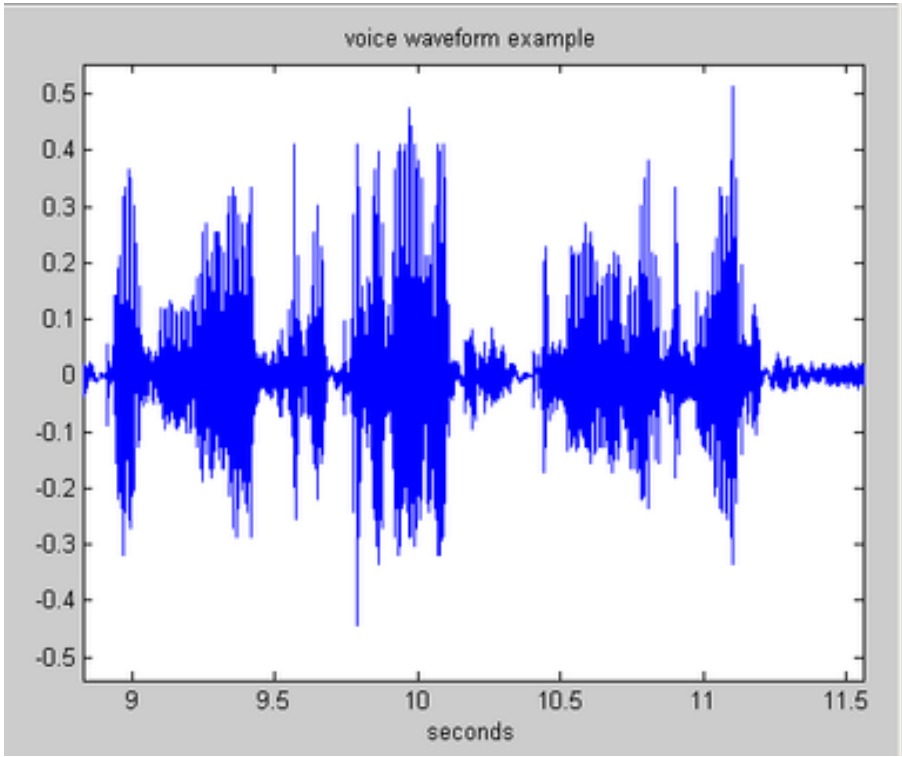


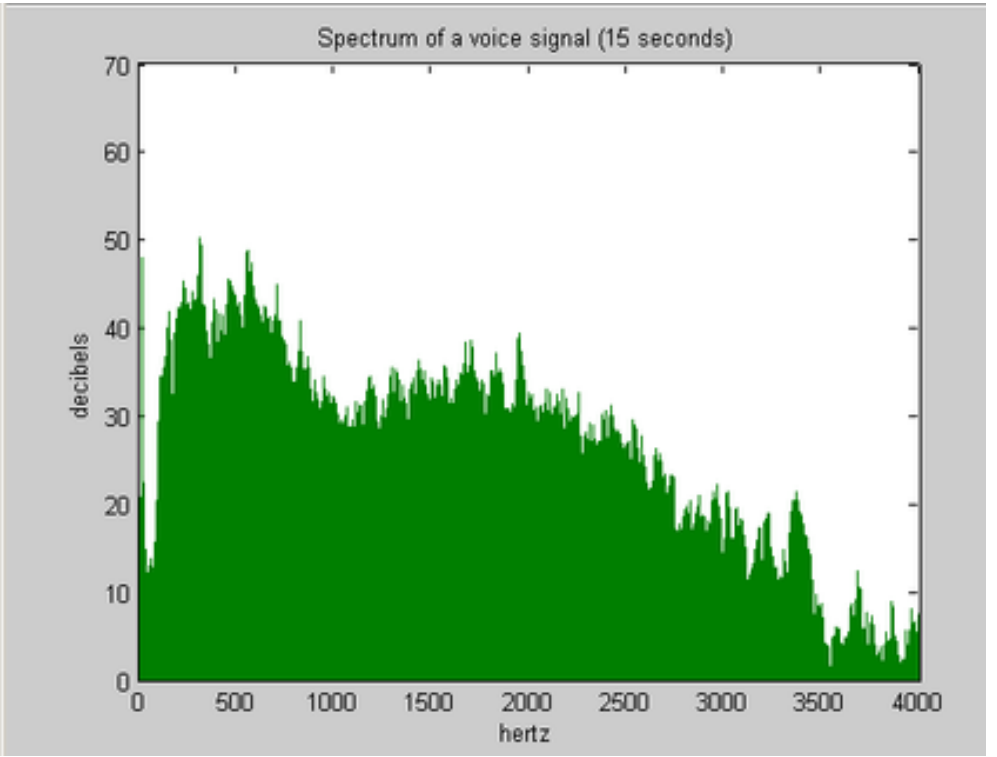
Sawtooth wave (approx.)



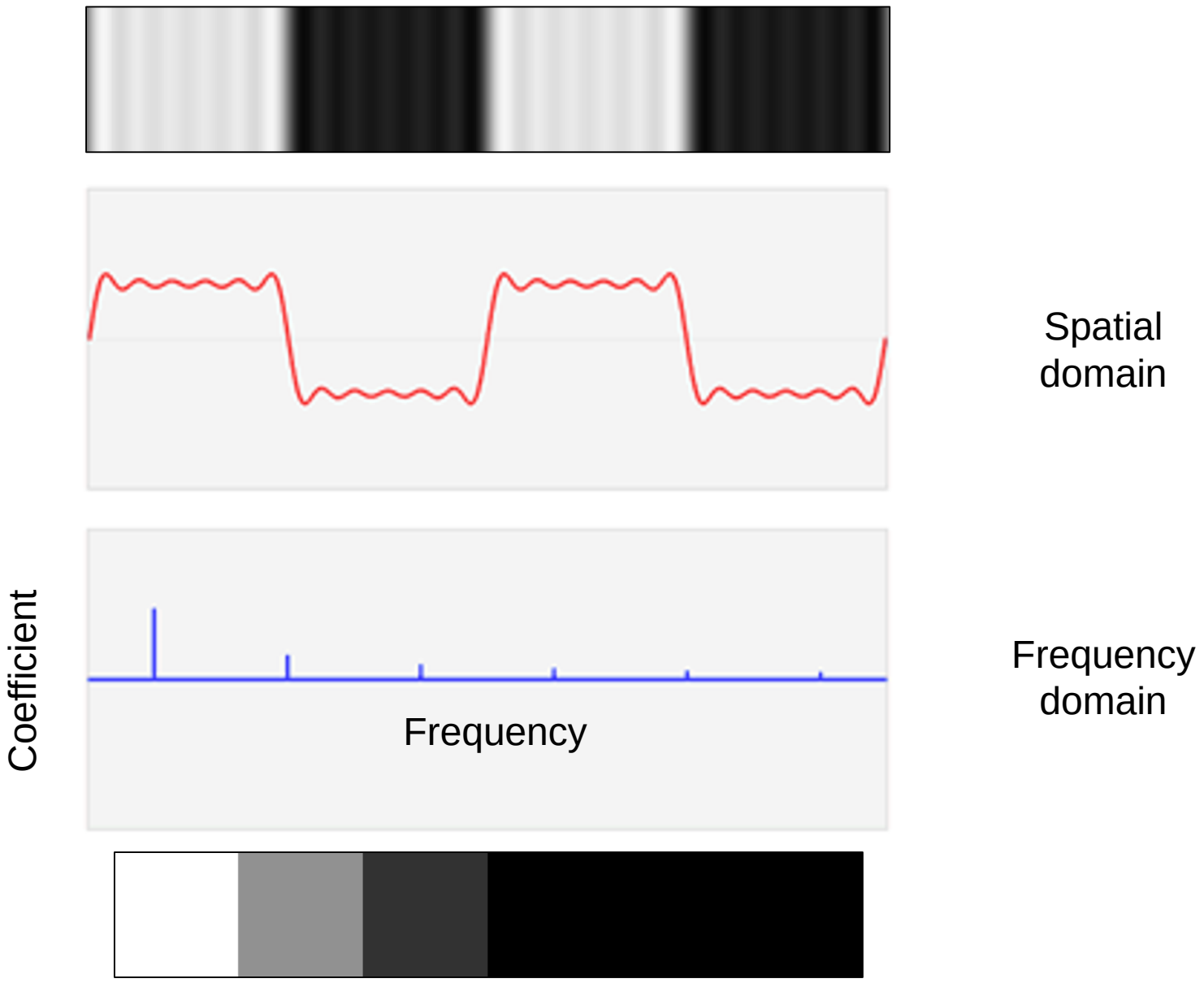
Example: Music

- We think of music in terms of frequencies at different magnitudes





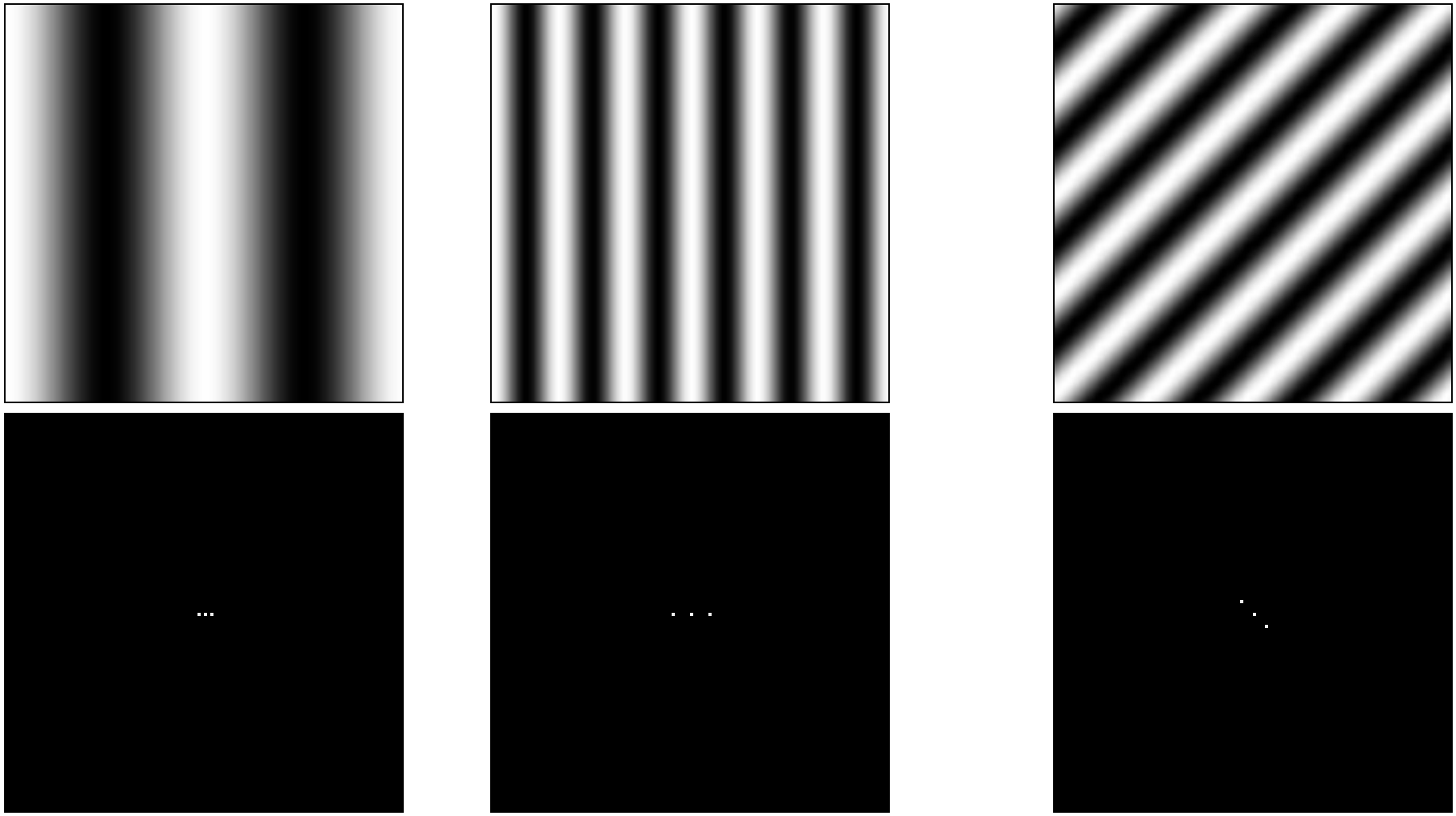
Slide: Hoiem



Wikipedia – Fourier transform

Fourier analysis in images

Spatial domain images

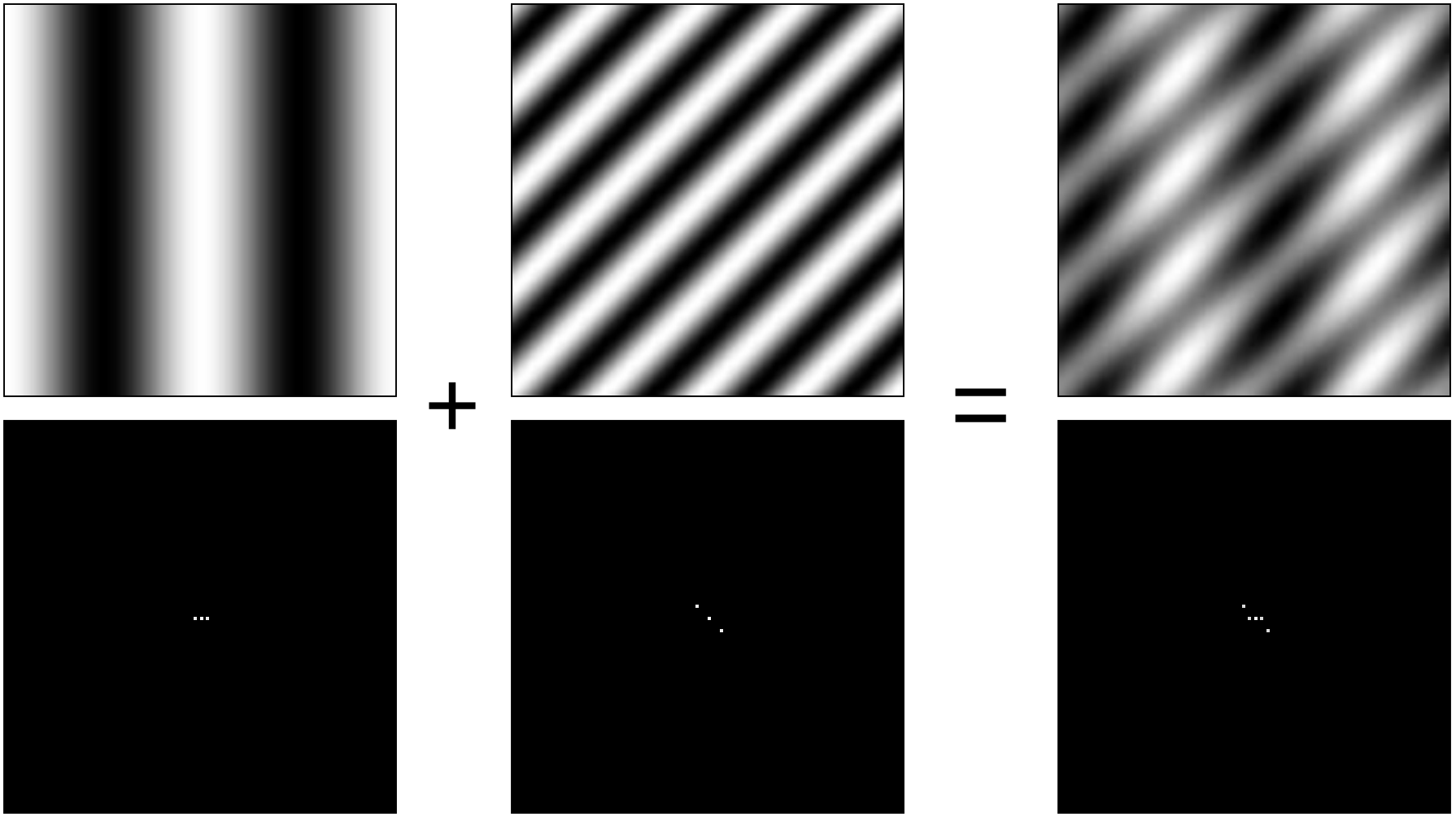


Fourier decomposition frequency amplitude images

http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering More: http://www.cs.unm.edu/~brayer/vision/fourier.html

Signals can be composed

Spatial domain images



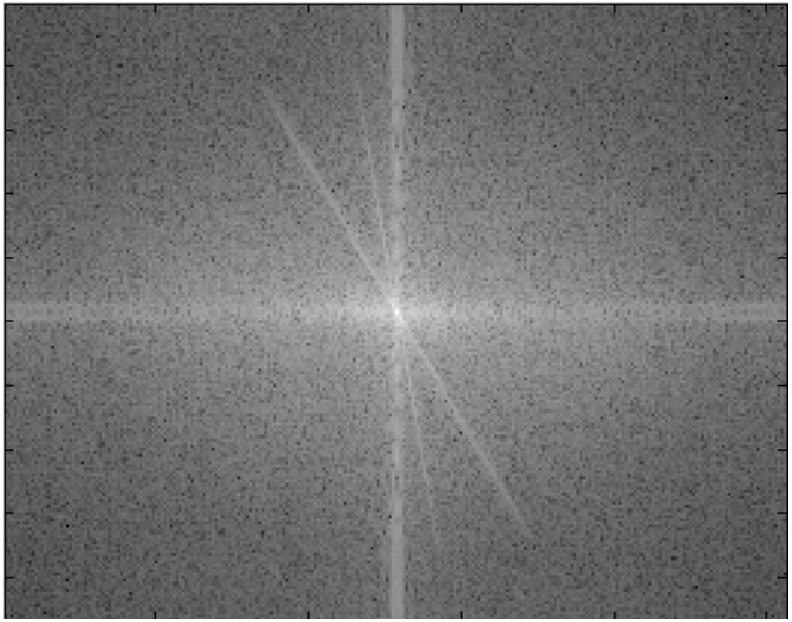
Fourier decomposition frequency amplitude images

http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering More: http://www.cs.unm.edu/~brayer/vision/fourier.html

Natural image

Natural image

Fourier decomposition
Frequency coefficients (amplitude)



What does it mean to be at pixel x,y?
What does it mean to be more or less bright in the Fourier decomposition image?

Brian Pauw demo

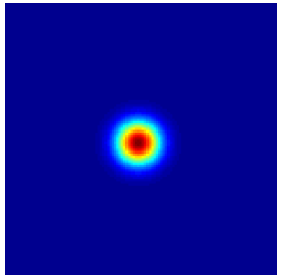
- Live Fourier decomposition images
  - Using FFT2 function

- I hacked it a bit for MATLAB
- http://www.lookingatnothing.com/index.php/archives/991

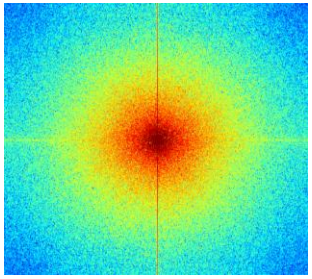
Think-Pair-Share

Match the spatial domain image to the Fourier magnitude image

1



2



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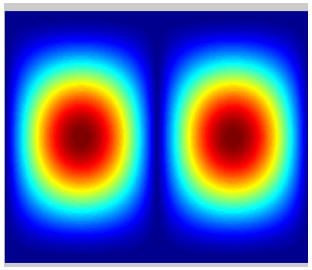

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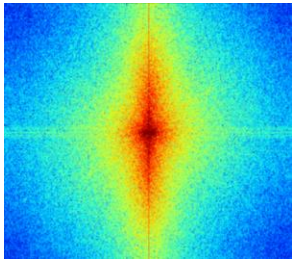

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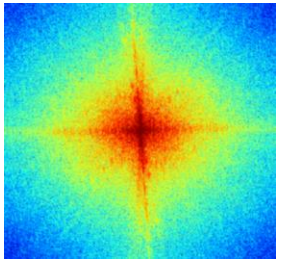
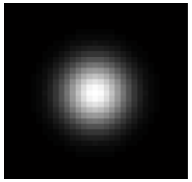


A

B

C

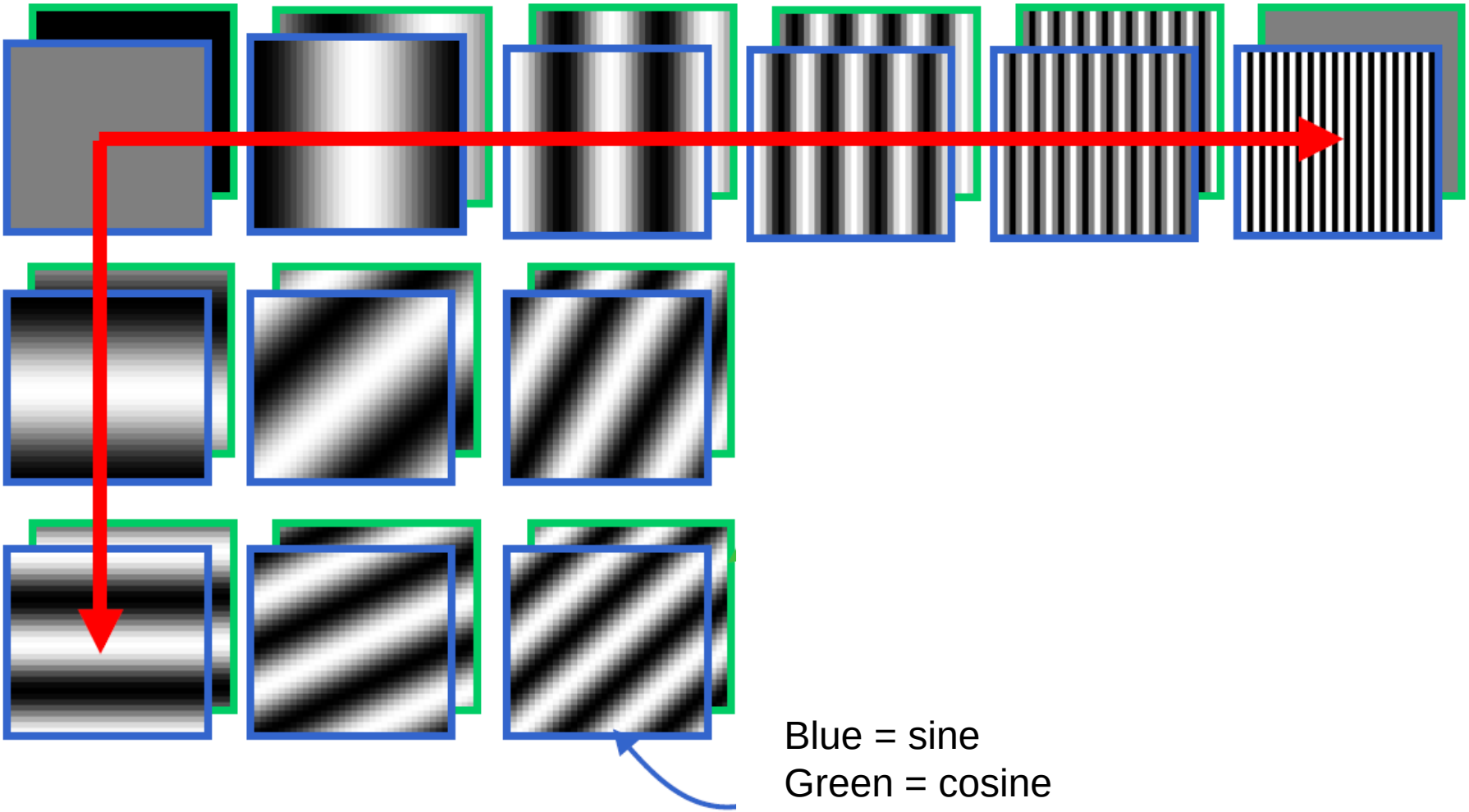
D



E

Fourier Bases

Teases away ‘fast vs. slow’ changes in the image.

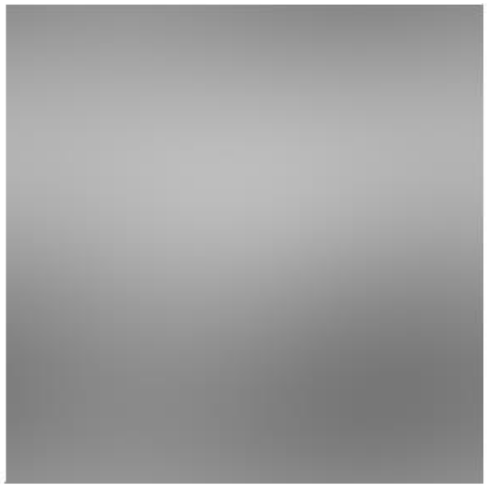


This change of basis is the Fourier Transform

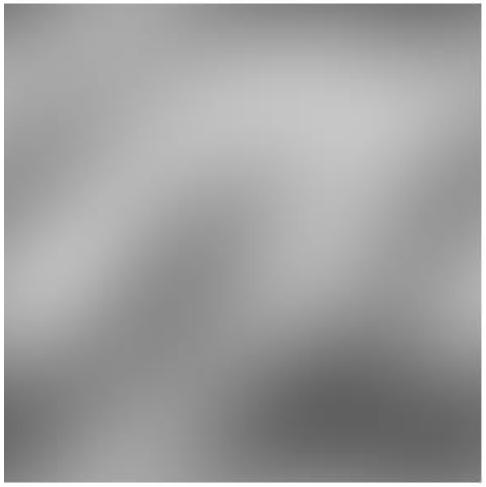
Basis reconstruction

Full image

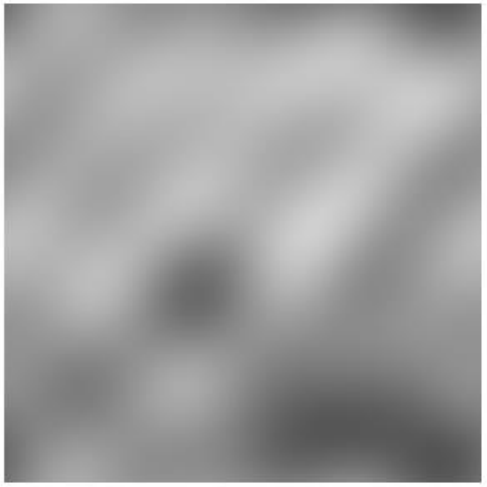
First 1 basis fn



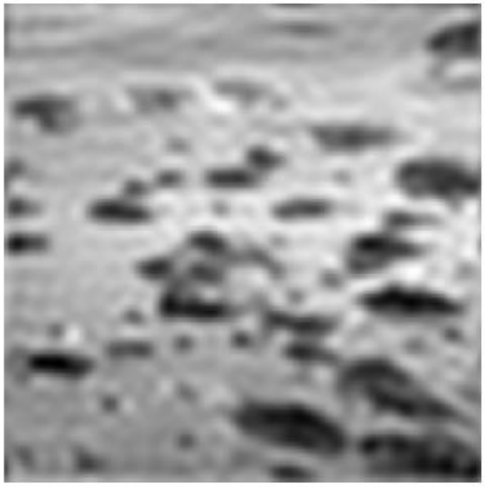
First 4 basis fns



First 9 basis fns



First 16 basis fns



First 400 basis fns

Fourier Transform

- Stores the amplitude and phase at each frequency:
  - For mathematical convenience, this is often notated in terms of real and complex numbers
  - Related by Euler's formula

Fourier Transform

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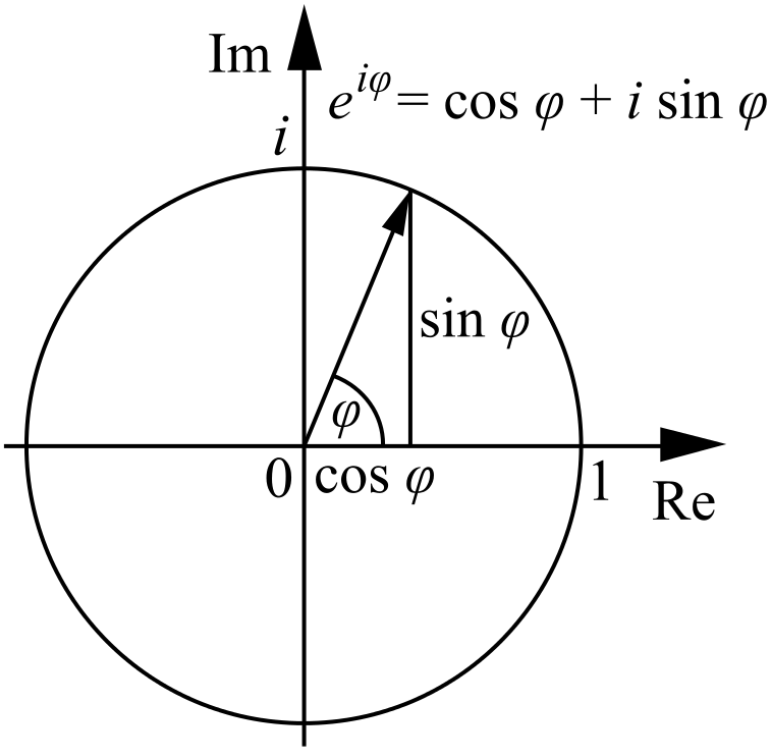

Fourier Transform

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  - For mathematical convenience, this is often notated in terms of real and complex numbers
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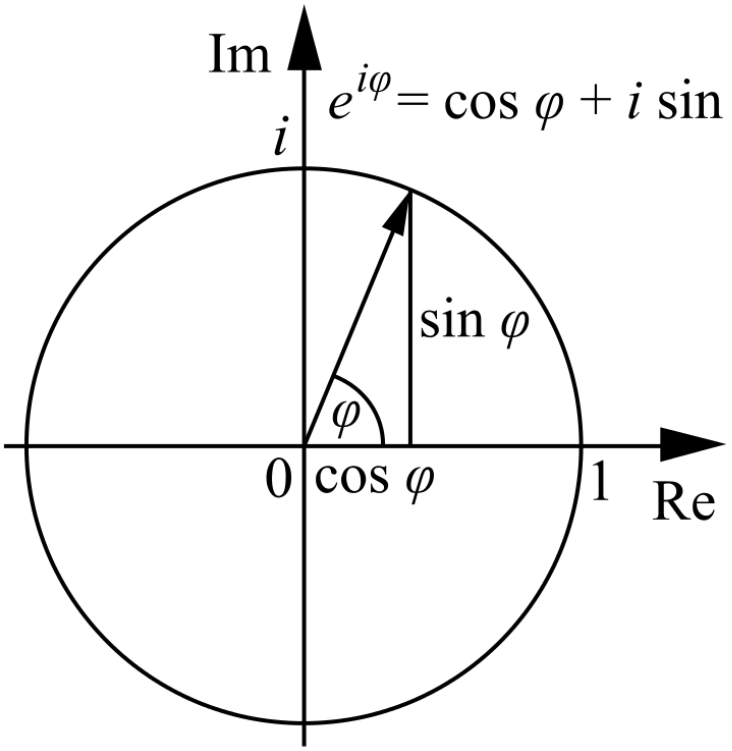


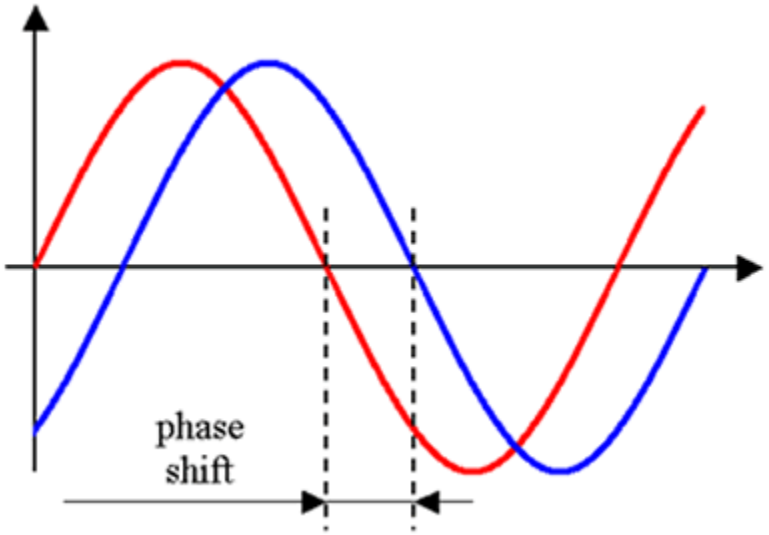
Amplitude encodes how much signal there is at a particular frequency:

$$A = \pm\sqrt{\mathrm{Re}(\varphi)^2 + \mathrm{Im}(\varphi)^2}$$

Phase encodes spatial information (indirectly):

$$\phi = \tan^{-1}\frac{\mathrm{Im}(\varphi)}{\mathrm{Re}(\varphi)}$$

Amplitude / Phase



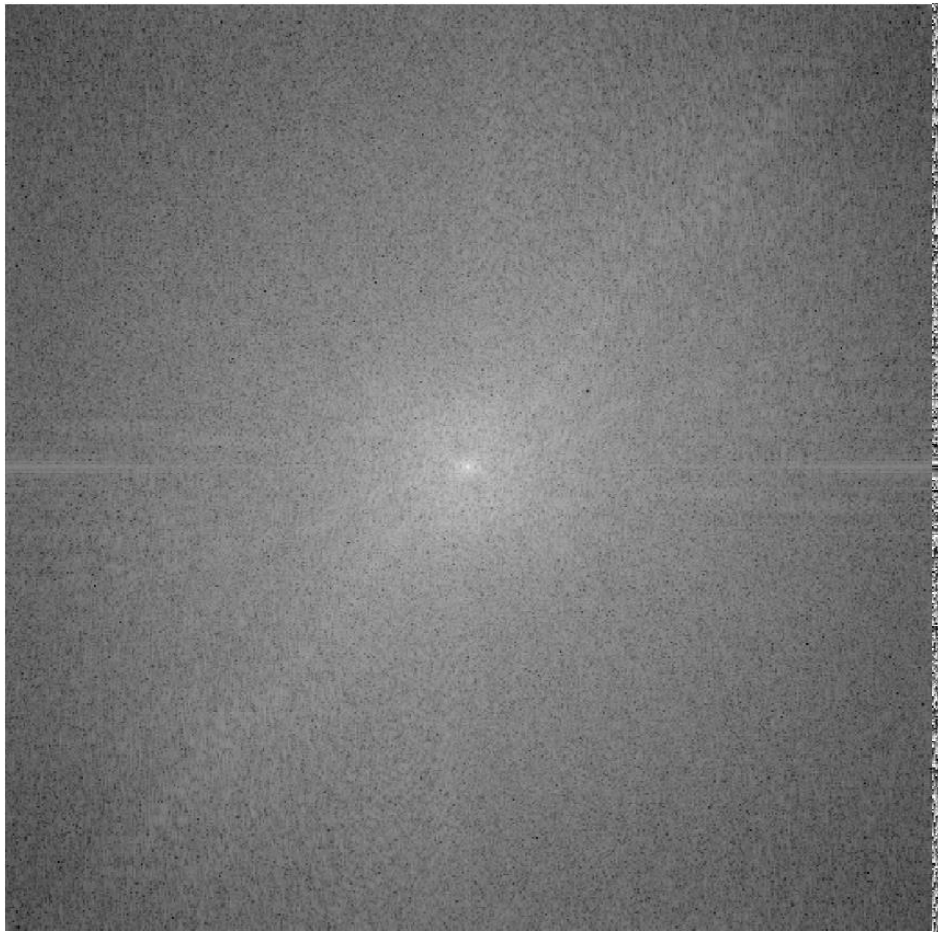
- Amplitude tells you “how much”
- Phase tells you “where”

- Translate the image?
  - Amplitude unchanged
  - Adds a constant to the phase.

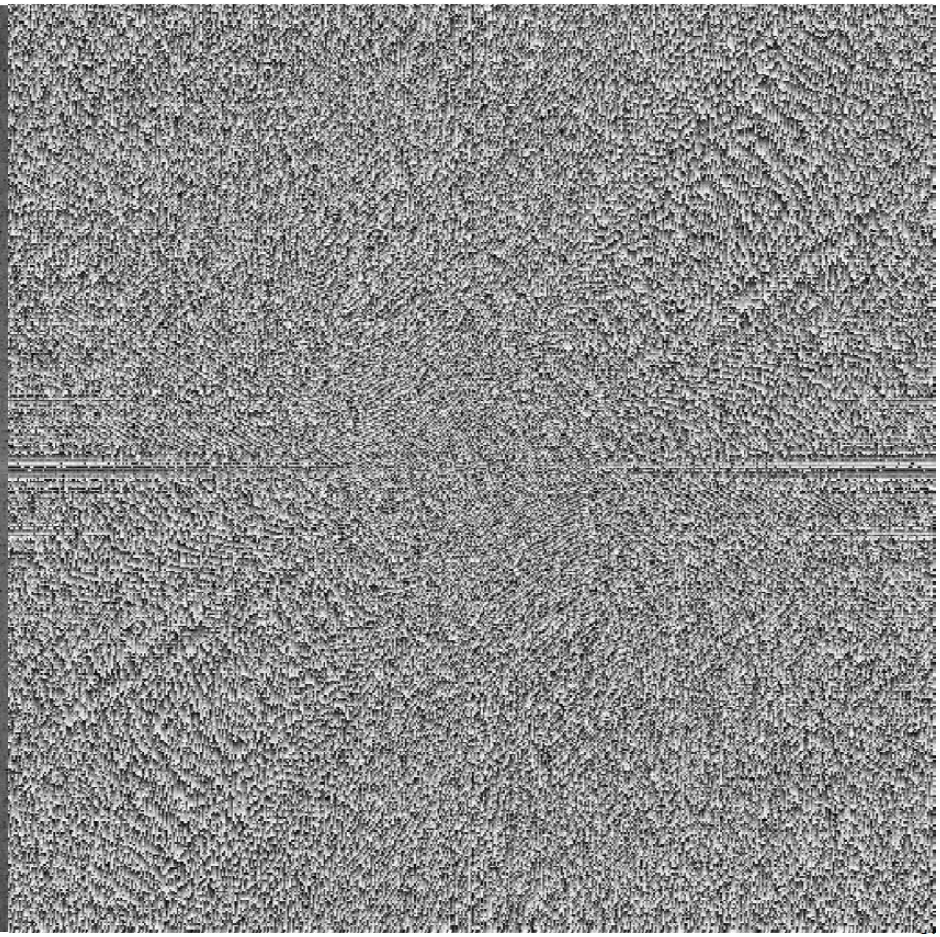
What about phase?

What about phase?

Amplitude



Phase



What about phase?

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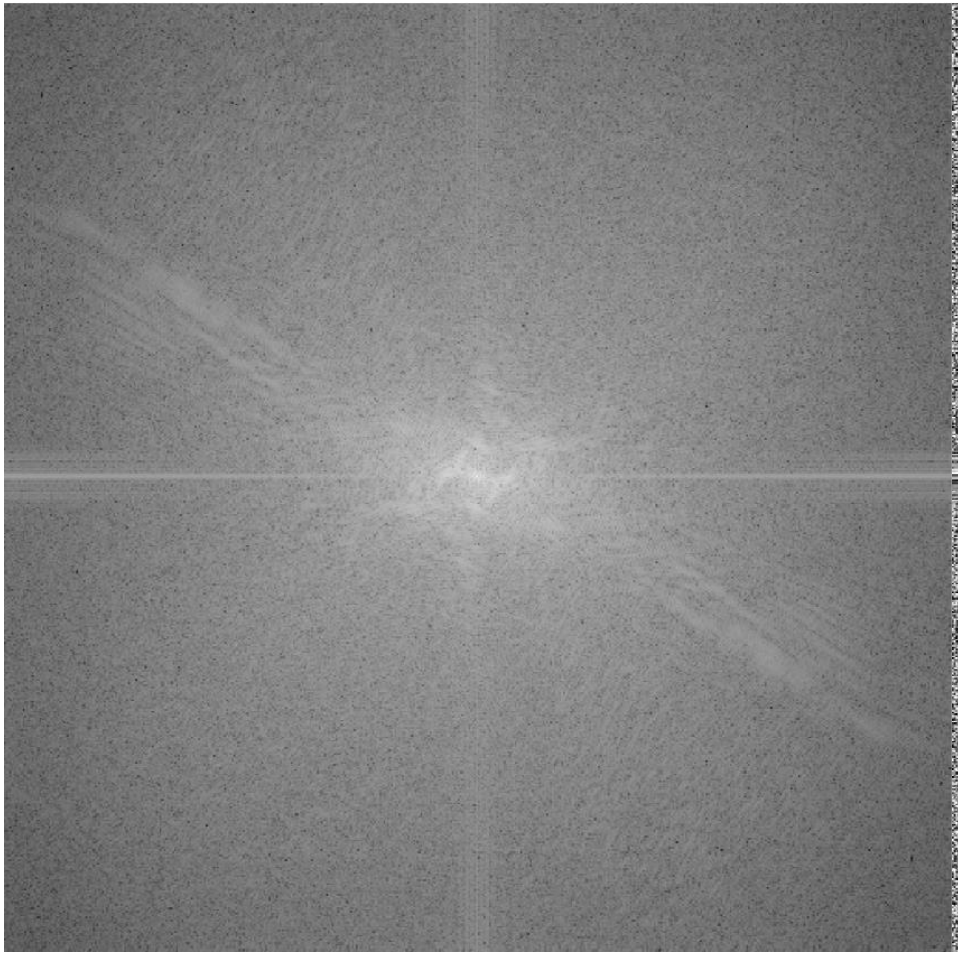
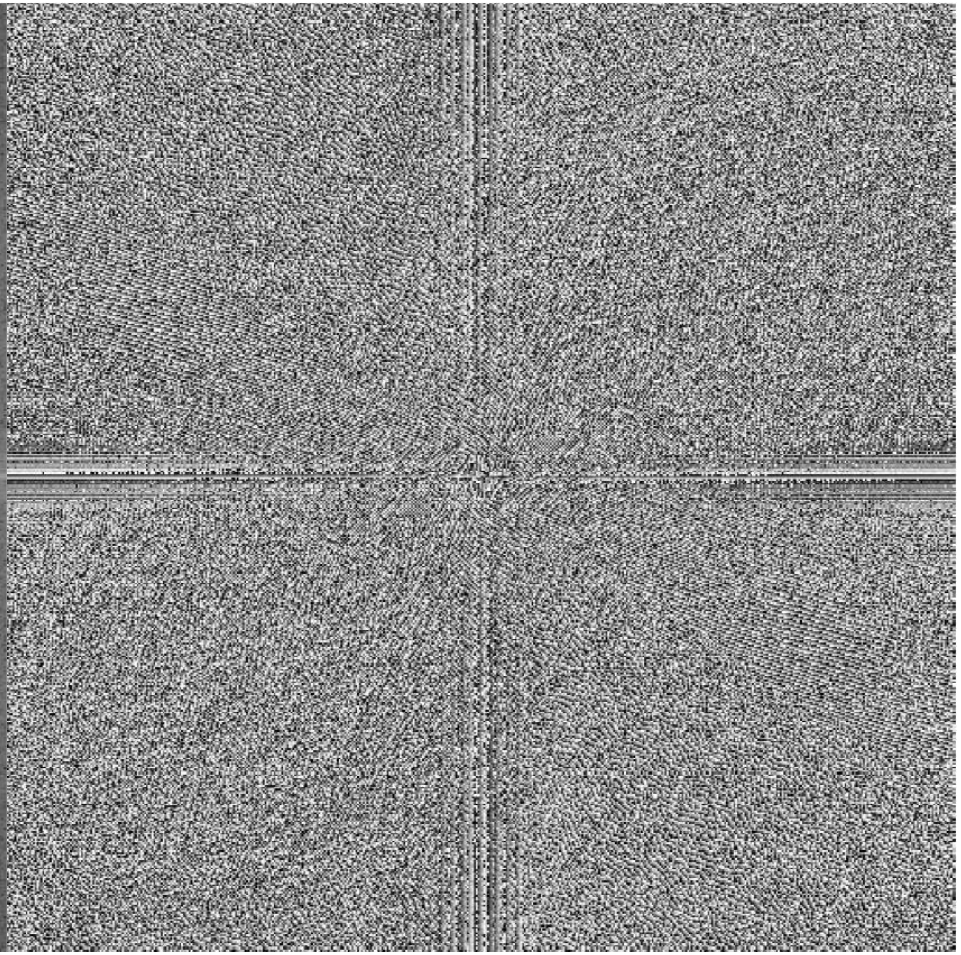
Amplitude



Phase



John Brayer, Uni. New Mexico

- “We generally do not display PHASE images because most people who see them shortly thereafter succumb to hallucinogenics or end up in a Tibetan monastery.”

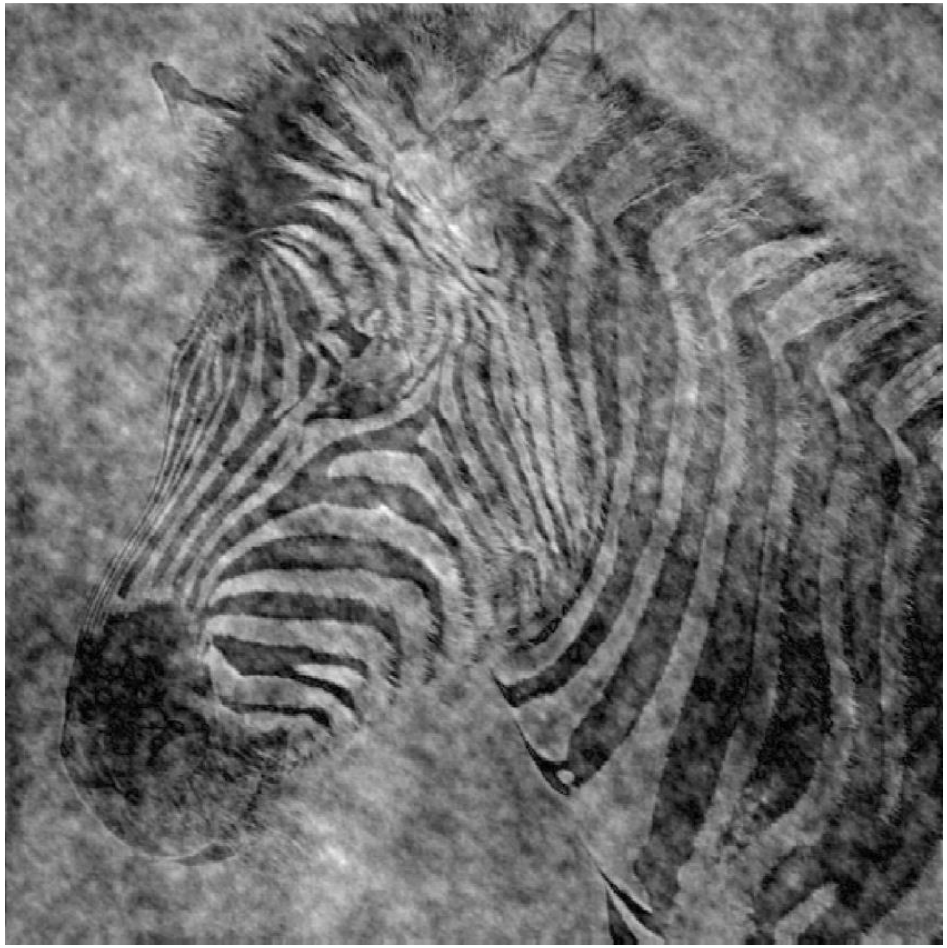
- https://www.cs.unm.edu/~brayer/vision/fourier.html

Think-Pair-Share

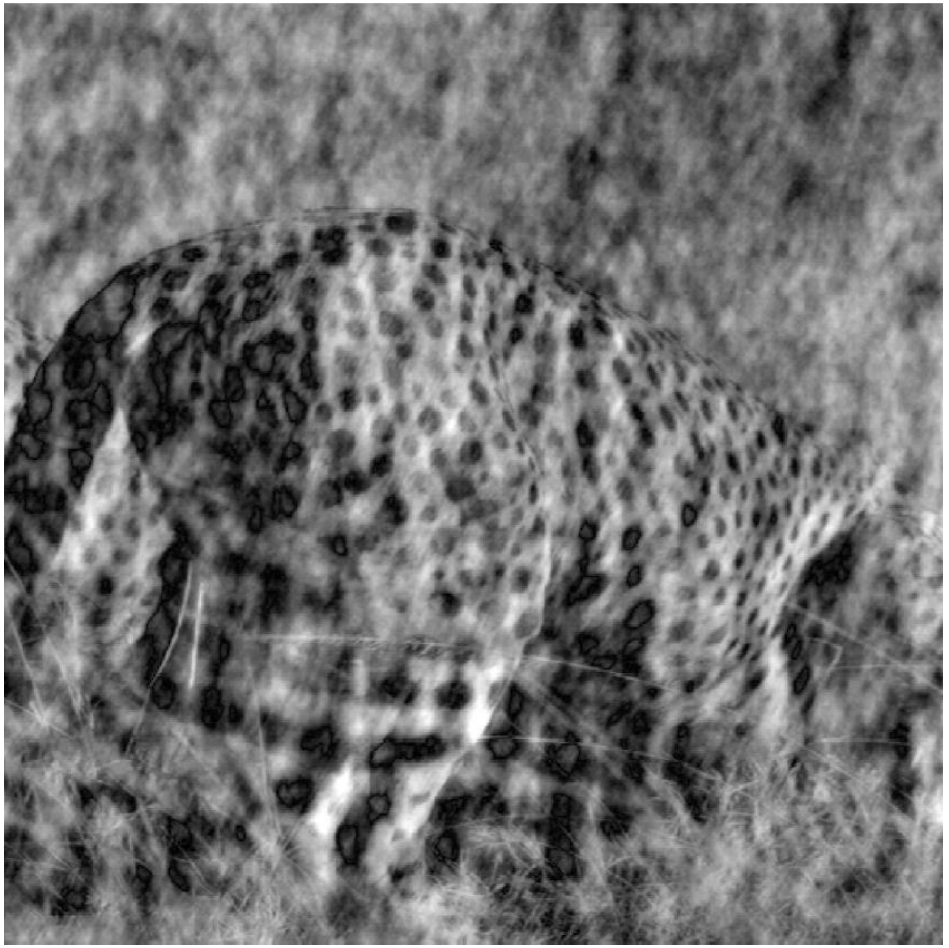
- In Fourier space, where is more of the information that we see in the visual world?
  - Amplitude
  - Phase

Cheebra

Zebra phase, cheetah amplitude



Cheetah phase, zebra amplitude



- The frequency amplitude of natural images are quite similar
  - Heavy in low frequencies, falling off in high frequencies
  - Will *any* image be like that, or is it a property of the world we live in?
- Most information in the image is carried in the phase, not the amplitude
  - Not quite clear why

What is the relationship to phase in audio?

- In audio perception, frequency is important but phase is not.
- In visual perception, both are important.

- ??? :(

- Proj 0 & 1 written due today 9pm

- Juniors in the room who want to take the class
  - No promises, but come to front