Introduction to Computer Vision

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Finish PCA and classification
Start motion estimation
In an attempt to quell rumors regarding the health of North Korea's leader Kim Jong-Il, the North Korean government released a series of photographs showing a healthy and active Kim Jong-Il. Shortly after their release the BBC claimed that the photographs were doctored. The article pointed to purported visual incongruities, which were claimed to be the result of photo tampering.

The BBC was wrong. Because judgments of photo authenticity are often made by eye, we wondered how reliable the human visual system is in detecting discrepancies that might arise from photo tampering. We describe three experiments that show that the human visual system is remarkably inept at detecting simple geometric inconsistencies in shadows, reflections, and planar perspective distortions. We also describe computational methods that can be applied to detect the inconsistencies that seem to elude the human visual system.

If you’re curious, here is a video clip of Hany on Nova Science Now describing some of this work: http://www.pbs.org/wgbh/nova/sciencenow/0301/03.html
Goals

• Finish probability and classification
• Clear up any questions about PCA and assign 2 parts 1 and 2.

• Introduce motion estimation
Example: Covariance
Points in PCA space

\[
[U,D] = \text{eig}(C)
\]

\[
U = \begin{bmatrix}
-0.7876 & 0.6162 \\
0.6162 & 0.7876
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
0.0004 & 0 \\
0 & 0.0617
\end{bmatrix}
\]

\[
\text{plot}(r(:,1), r(:,2), '.');
\]
[U, D] = eig(C)
% project points onto basis
coeffs = r * U;
plot(coeffs(:,1), coeffs(:,2), '.');
axis true

\[ \tilde{x}^n = \bar{x}^n \mathbf{U} \]

\[ \bar{x}^n \text{ zero mean} \quad \text{cov}(\tilde{x}) = D \]
Mahalanobis distance

\[ p(\tilde{x}) = \frac{1}{(2\pi)^{D/2} |C|^{1/2}} \exp \left( -\frac{1}{2} (\tilde{x} - \tilde{\mu})^T C^{-1} (\tilde{x} - \tilde{\mu}) \right) \]

\[ \tilde{x} = \tilde{x} - \tilde{\mu} \]

\[ d(\tilde{x}) = (\tilde{x}^T C^{-1} \tilde{x}) \]

\[ C = USU^T \]
Mahalanobis Distance

\[ d(\tilde{x}) = (\tilde{x}^T C^{-1} \tilde{x}) \]
\[ = \tilde{x}^T (USU^T)^{-1} \tilde{x} \]
\[ = \tilde{x}^T US^{-1} U^T \tilde{x} \]
\[ = y^T S^{-1} y \quad \text{Linear coefficients} \]
\[ = \sum_{i=1}^{D} \frac{y_i^2}{\lambda_i} \]
\[ \approx \sum_{i=1}^{M} \frac{y_i^2}{\lambda_i} \]
Mahalanobis distance

\[ p(\bar{x}) = \frac{1}{(2\pi)^{D/2} |C|^{1/2}} \exp\left( -\frac{1}{2} (\bar{x} - \bar{\mu})^T C^{-1} (\bar{x} - \bar{\mu}) \right) \]

\[ p(\bar{x}) = \frac{1}{(2\pi)^{D/2} |S|^{1/2}} \exp\left( -\frac{1}{2} y^T S^{-1} y \right) \]

\[ p(\bar{x}) = \prod_{i=1}^{D} \frac{1}{\sqrt{2\pi\lambda_i}} \exp\left( -\frac{1}{2} \frac{y_i^2}{\lambda_i} \right) \]
Classification
Covariance

Multivariate Gaussian (Normal)

\[
p(\tilde{x}) = \frac{1}{(2\pi)^{D/2} |C|^{1/2}} \exp \left( -\frac{1}{2} (\tilde{x} - \tilde{\mu})^T C^{-1} (\tilde{x} - \tilde{\mu}) \right)
\]
Covariance Ellipse

hyperellipsoids of constant Mahalanobis distance $\Delta^2$

Note the ellipse is axis-aligned. Why?
What about Not Mouths

Note ellipse is not axis aligned. Why?
Plot them together
Posterior
Discriminant Function

Classify feature vector as a mouth if:

\[ g_{\text{mouth}}(\vec{x}) > g_{\text{\neg mouth}}(\vec{x}) \]

Take \( g \) to be the posterior

\[ g_{\text{mouth}}(\vec{x}) = p(\text{mouth} | \vec{x}) \propto p(\vec{x} | \text{mouth}) p(\text{mouth}) \]

Holds true for monotonic functions of \( g \) (e.g. log). Define

\[ g_{\text{mouth}}(\vec{x}) = \log(p(\vec{x} | \text{mouth})) + \log(p(\text{mouth})) \]
Decision boundary
Decision boundary

In higher dimensions we should do even better.
Decision boundary

In higher dimensions we should do even better.
Goal: Introduce Motion

• So far we’ve looked at static images.
• The world is more complex and interesting.
• We need to understand movement.
Optical Flow

J. J. Gibson, The Ecological Approach to Visual Perception
Motion Field

Motion field = 2D motion field representing the projection of the 3D motion of points in the scene onto the image plane.
Optical flow = 2D velocity field describing the apparent motion in the images.
Optical Flow Field

Image irradiance at time $t$ and location $\mathbf{x} = (x, y)$

$I(x, y, t)$

$u(x, y)$  Horizontal component

$v(x, y)$  Vertical component
Problem
Thought Experiment 1

Lambertian (matte) ball rotating in 3D

What does the 2D motion field look like?

What does the 2D optical flow field look like?

Image source: http://www.evl.uic.edu/aej/488/lecture12.html
Special Cases: Lambertian

Perfect matte surface
* reflects all light
* reflects equally in all directions
* patch appears equally bright from all viewing directions
* diffuse reflectance
Thought Experiment 2

Stationary Lambertian (matte) ball, moving light source.

What does the 2D motion field look like?

What does the 2D optical flow field look like?

Image source: http://www.evl.uic.edu/aej/488/lecture12.html