Simulating The Electoral College

Adapted from slides by David Meyer

Who will win the presidential election?

- Inputs: Numerous poll results taken in all the states
- Impossible to predict the precise outcome
  - Will Trump be re-elected in 2024?
  - Will it rain tomorrow?
- Output: Pr[Clinton wins]

How to win a presidential election

- Need a majority of electoral votes
  - Total = 538 = Representatives (435) + Senators (100) + 3 for DC
  - 538 / 2 = 269, so to win, need at least 270
- Most states winner-take-all
  - Maine & Nebraska are exceptions, dividing up their electoral votes by congressional districts

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- Inputs: Numerous poll results taken in all the states
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- Output: Pr[Clinton wins] = $\Pr[C \geq 270]$

Simplified story

- Clinton has constant probability *p* of winning in each state

$$\Pr(C \geq 26) = \sum_{k=26}^{51} \binom{51}{k} p^k (1-p)^{51-k}$$

- Hard to calculate exactly, but not so hard to simulate!

Simulation

- For each state *s*, draw a random number $n \in [0, 1]$
- If $n < p$, Clinton wins *s*: i.e., set $X_s = 1$
- Clinton wins overall if $\sum_s X_s \geq 26$
- Repeat many (e.g., 10^5) times

p	Exact	Simulated
0.1	1.982×10^{-13}	0
0.2	8.129×10^{-7}	0
0.3	0.0014	0.0014
0.4	0.0735	0.0735
0.5	0.5	0.4970

Less simplified story

- Simplified story

$$\Pr(C \geq 26) = \sum_{k=26}^{51} \binom{51}{k} p^k (1-p)^{51-k}$$

- Closer to the real deal

$$\Pr(C \geq 270) = \sum_{t=270}^{538} \sum_{\substack{S \subset [51] \\ \Sigma v_S = t}} \prod_{s \in S} p_s \prod_{s \notin S} (1 - p_s)$$

Simulation

- Let EV_s be the number of electoral votes ascribed to state *s*
- For each state *s*, draw a random number $n \in [0, 1]$
- If $n < p_s$, Clinton wins *s*: i.e., set $X_s = 1$
- Clinton wins overall if $\sum_s (X_s)(EV_s) \geq 270$
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Simulation

- Let EV_s be the number of electoral votes ascribed to state s
- For each state s, draw a random number n $\in$ [0, 1]
- If $n < p_s$, Clinton wins s: i.e., set $X_s = 1$
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Problem with this general approach: $p_s \neq 1 - p_s$

What about Johnson, Stein, etc.?

Estimating p_s from Polling Data

- October 30, 2016 Mitchell Research & Communications poll
  - 953 likely voters in Michigan
  - Clinton: 47%, Trump: 41%, Johnson: 6%, Stein: 2%
  - Margin of error 3.2%

- Assume % Clinton is normal: $N(\mu_C = 47, \sigma_C = 3.2)$

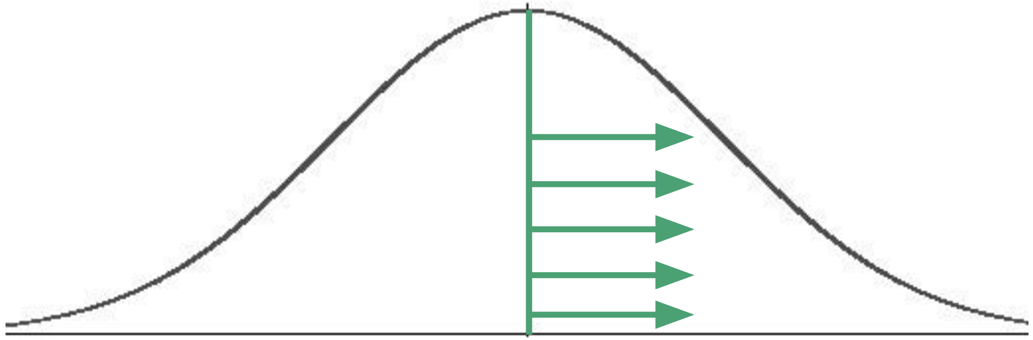
- Assume % Trump is normal: $N(\mu_T = 41, \sigma_T = 3.2)$

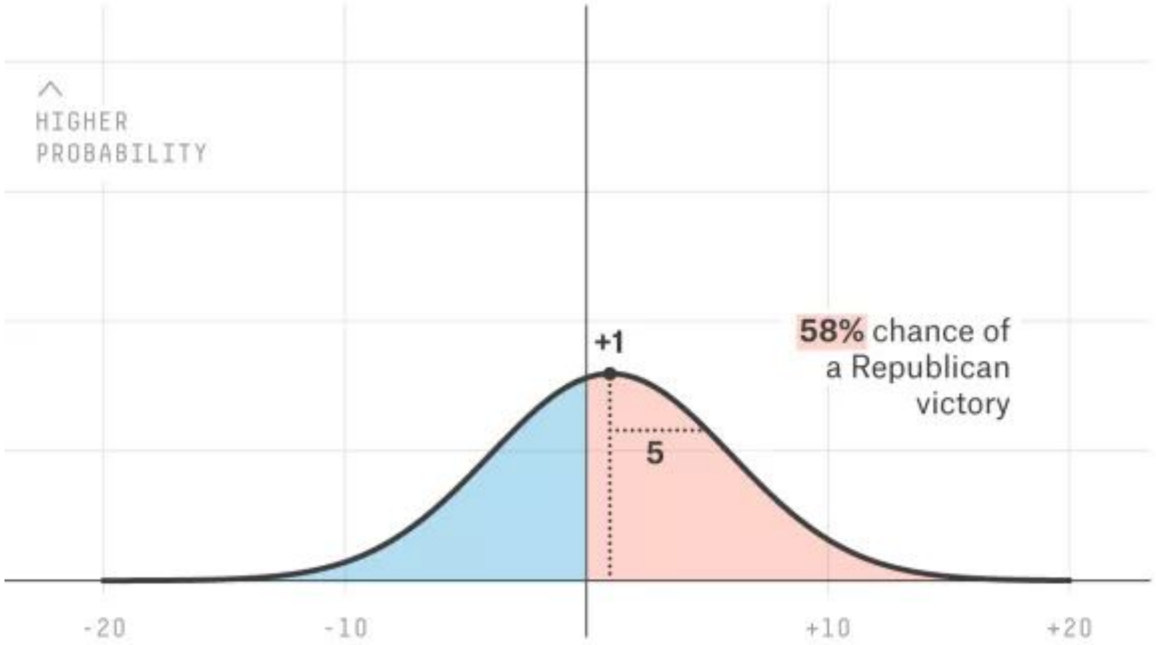
- % Difference is also normal: $N(\mu_C - \mu_T, \mathrm{sqrt}(\sigma_C^2 + \sigma_T^2))$
  - These are the parameters, assuming the percentages are independent.
  - We adjust the standard deviation because they are not:
    - $N(47 - 41, f\ \mathrm{sqrt}(3.2^2 + 3.2^2))$, $f \in \{1, 2, 4\}$

Estimating p_s from Polling Data, cont'd

- Clinton wins state s if $D_s > 0$

$$p_s = \Pr(\text{Clinton wins state } s) = \frac{1}{\sqrt{2\pi}\sigma}\int_0^\infty e^{-(x-\mu)^2/2\sigma^2}\,\mathrm{d}x$$





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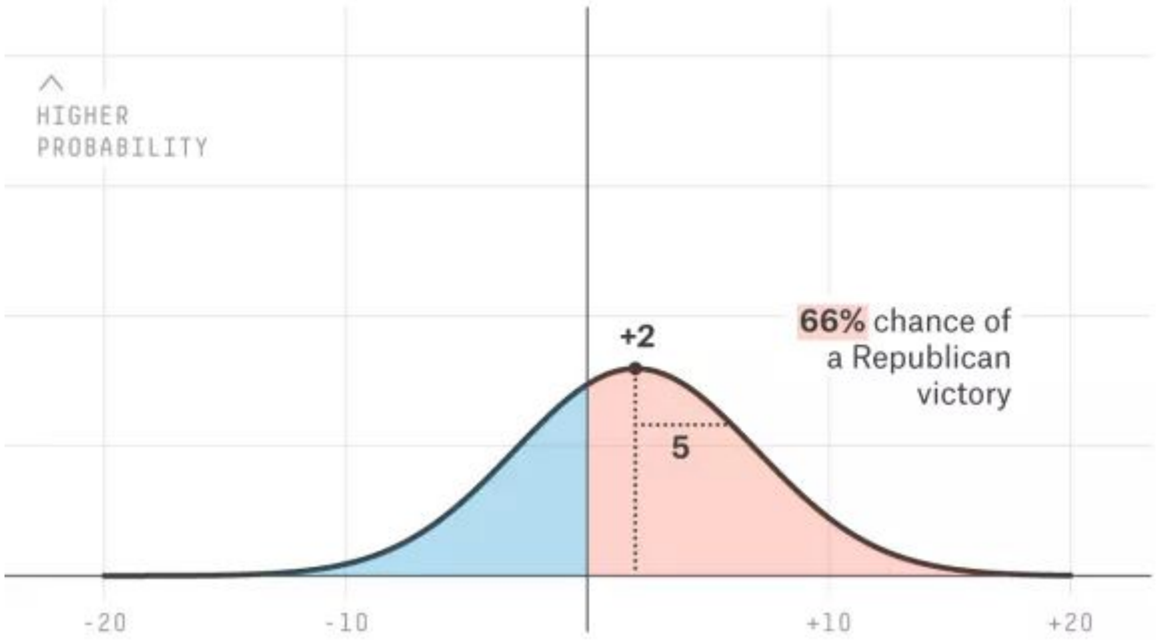

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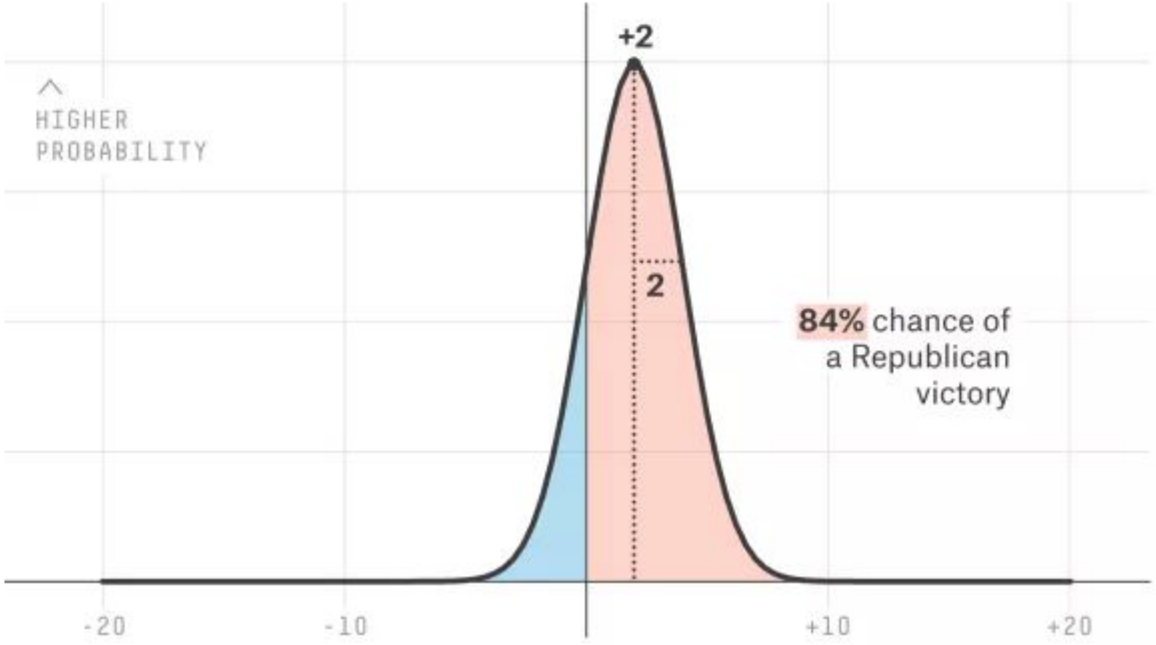

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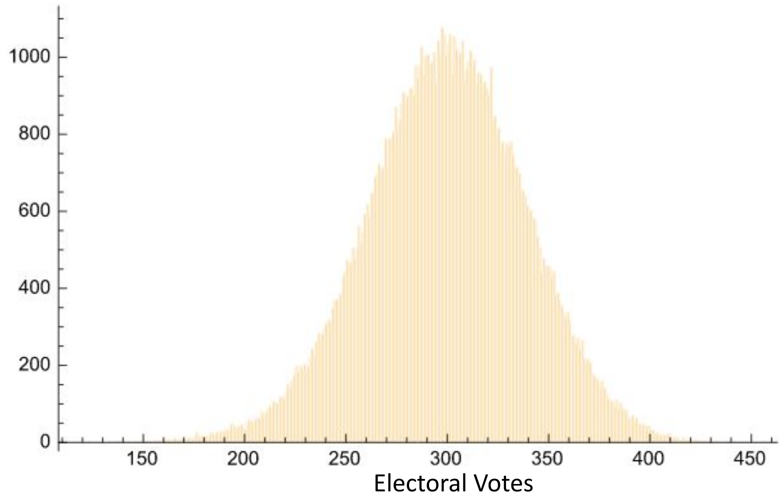
Simulation Results

- Compute p_s as described for each state
  - Treat each of Maine/Nebraska's districts as its own state for a total of 56 states
- Simulate as described using these more sensible estimates
- $E[C] \approx 299$ and $\Pr[C \geq 270] \approx 0.778$



538's results

- Doesn't use only the most recent polls. Regresses on past polls to predict future.
  - Also weighs polls by reliability, recency, sample size, etc.
- Pays attention to correlations among states

Extras

An Interesting Aside

- The electoral college has the effect of stretching mid-range probabilities
- Close races become less close as a result