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Responsibility,  
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# Fireside Chat : “AI Policy In The Next 4 Years”

Join us for a post-election Fireside Chat, where we'll discuss the future directions of tech policy in a 45-minute conversation with expert insights and lively Q&A—don't miss it!

RSVP by scanning the code! →



November 13th, 4-5:30 PM



Joukowsky Forum @  
Old Watson Institute



Dr. Nicholas Ziegler  
(International and Public Affairs)



Dr. Suresh Venkatasubramanian  
(Director of CNTR)



In conversation with Emma Huang '25  
(CNTR Undergraduate Coordinator)

# We want to hear about **your** high school digital safety education experience! **Scan me!**

Help CS1953A students explore:

- The **current state of digital safety education** in U.S. high schools
- The **prevalence of tech-facilitated gender-inequities** (e.g. nonconsensual intimate imagery, explicit deepfakes, reproductive health surveillance)



# Doubly Linked List (1/3)

- Is there an easier/faster way to get to previous node while removing a node?
  - with Doubly Linked Lists, nodes have references both to next and previous nodes
  - can traverse list both backwards and forwards – Linked List still stores reference to front of list with `head` and back of list with `tail`
  - modify `Node` class to have *two* pointers: `next` and `prev`
  - eliminates pointer-chasing loop because `prev` points to predecessor of every `Node`, at cost of second pointer
    - classic space-time tradeoff!

# Doubly Linked List (2/3)



- For Singly Linked List, processing typically goes from first to last node, e.g. [search](#), finding place to insert or delete
- Sometimes, particularly for sorted list, need to go in the opposite direction
  - e.g., sort CS15 students on their final grades in ascending order. Find lowest numeric grade that will be recorded as an “A”. Then ask: who has a lower grade but is closer to the “A” cut-off, i.e., in the grey area, and therefore should be considered for “benefit of the doubt”?

# Doubly Linked List (3/3)

- This kind of backing-up can't easily be done with the Singly Linked List implementation we have so far
  - could build our own *specialized search* method, which would scan from the *head* and be, at a minimum,  $O(n)$
- It is simpler for Doubly Linked Lists:
  - find student with lowest "A" using search
  - use *prev* pointer, which points to the predecessor of a node ( $O(1)$ ), and back up until hit end of B+/A- grey area

# Lecture 19

## Stacks, Queues, and Trees



# Stacks and Queues



Abstractions that are Wrappers for [MyLinkedList](#)

# Outline

- Stacks and Queues
- Trees



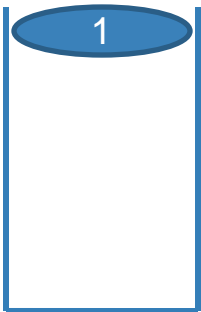
# Stacks

- **Stack** has special methods for insertion and deletion, and two others for size
  - **push** and **pop**
  - **isEmpty**, **size**
- Instead of being able to insert and delete nodes from anywhere in the list, can only add and delete nodes from top of **Stack**
  - **LIFO (Last In, First Out)**
- We'll implement a stack with a linked list

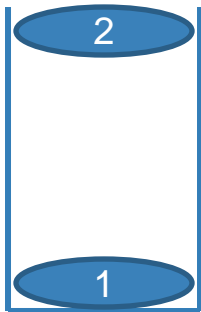


# Methods of a Stack

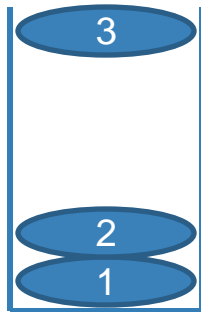
- Add element to top of `stack` `public void push(Type el)`
- Remove element from top of `stack` `public Type pop()`
- Returns whether `stack` has any elements `public boolean isEmpty()`
- Returns number of elements in `stack` `public int size()`



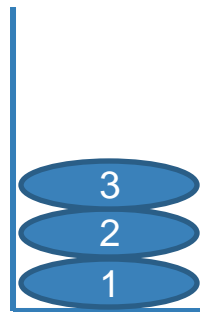
**push(1)**



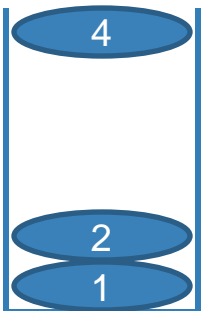
**push(2)**



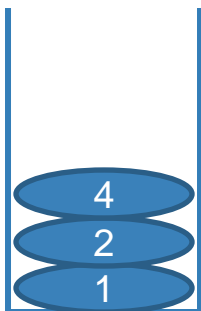
**push(3)**



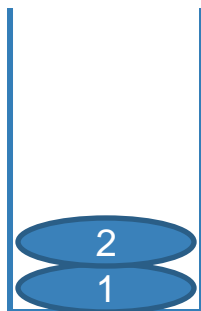
**pop()**



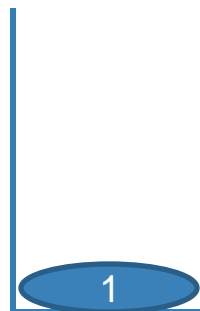
**push(4)**



**pop()**



**pop()**



**pop()**

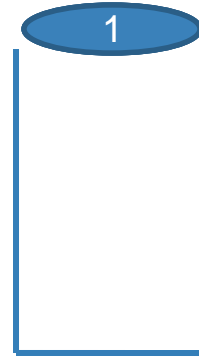
# Stack Constructor

```
public class Stack<Type> {  
    private MyLinkedList<Type> list;  
    public Stack() {  
        this.list = new MyLinkedList<>();  
    }  
    /* other methods elided */  
}
```

- When generic **Stack** is instantiated, it contains an empty **MyLinkedList**
- When using a stack, you will replace the generic **Type** with type of object your **Stack** will hold – enforces homogeneity
- Note: **Stack** uses classic “wrapper” pattern to modify functionality of the wrapped data structure, **MyLinkedList**, and to add other methods

# Implementing Push

```
//in the Stack<Type> class ...  
public Node<Type> push(Type newData) {  
    return this.list.addFirst(newData);  
}
```

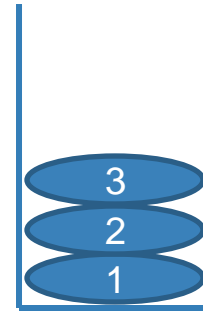


- Let's see behavior...
- When element is **pushed**, it is always added to front of list
- Thus, **Stack** delegates to the **MyLinkedList**, **this.list** to implement **push**

# Implementing Pop

- Let's see what this does...
- When popping element, it is always removed from top of **Stack**, so call **removeFirst** on **MyLinkedList** – again, delegation
- **removeFirst** returns element removed, and **Stack** in turn returns it
- Remember that **removeFirst** method of **MyLinkedList** first checks to see if list is empty

```
//in the Stack<Type> class ...  
public Type pop() {  
    return this.list.removeFirst();  
}
```



# isEmpty

- `Stack` will be empty if the `MyLinkedList`, `list`, is empty - delegation
- Returns `true` if `Stack` is empty; `false` otherwise

```
//in the Stack<Type> class ...  
public boolean isEmpty() {  
    return this.list.isEmpty();  
}
```

# size

- Size of `Stack` will be number of elements that the `MyLinkedList`, `list` contains – delegation
- Size is updated whenever `Node` is added to or deleted from `list` during `push` and `pop` methods

```
//in the Stack<Type> class ...  
public int size() {  
    return this.list.size();  
}
```

# TopHat Question

Look over the following code:

```
Stack<HeadTA> myStack = new Stack<>();  
myStack.push(htaSarah);  
myStack.push(htaGrace);  
myStack.pop();  
myStack.push(htaKarim);  
myStack.pop();
```

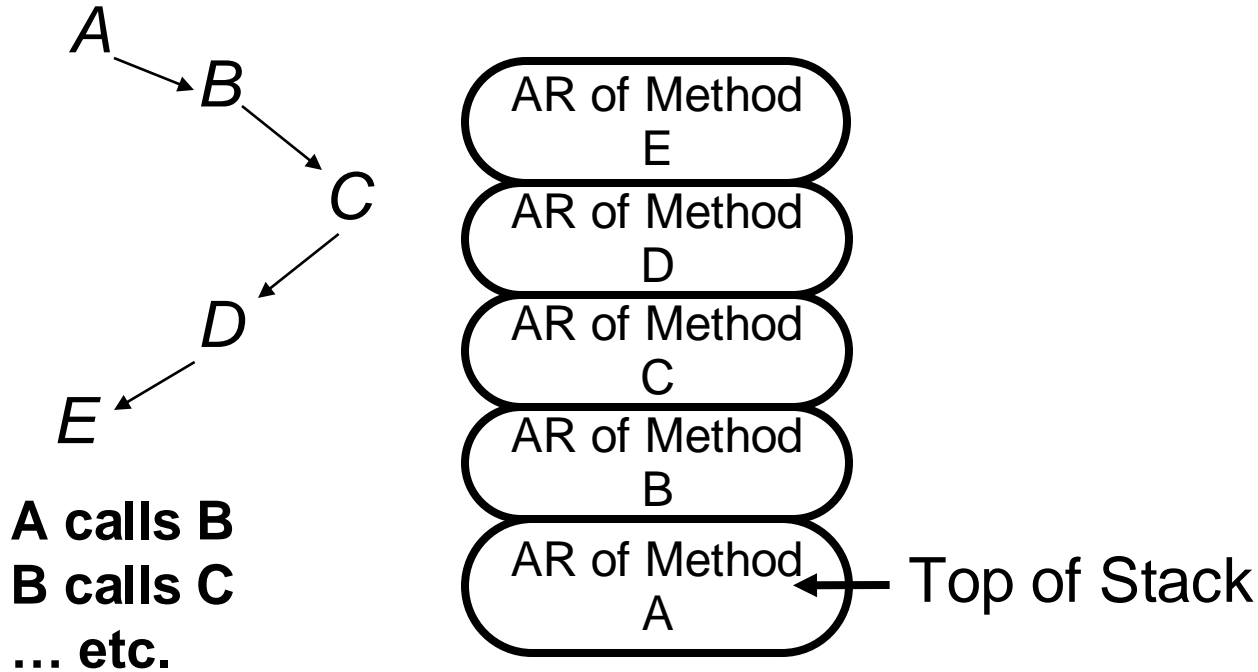
Who's left in the stack?

- A. htaSarah
- B. htaGrace
- C. htaKarim
- D. none of them!

# Example: Execution Stacks

- Each method has an Activation Record (AR) – recall recursion lecture
  - contains execution pointer to next instruction in method
  - contains all local variables and parameters used by method
- When methods execute and call other methods, Java uses a **Stack** to keep track of the order of execution: “stack trace”
  - when a method calls another method, Java adds activation record of called method to **Stack**
  - when new method is finished, its AR is removed from **Stack**, and previous method is continued
  - method could be different or a recursively called clone, when execution pointer points into same immutable code, but different values for variables/parameters

# Execution Stacks



When E finishes, its AR is popped. Then D's AR is popped, etc. Note this handles the tracking of invocations (clones) in recursion automatically

# Stack Trace

- When an exception is thrown in a program, get a long list of methods and line numbers known as a **stack trace**

```
Exception in thread "main" java.lang.NullPointerException  
    at DoodleJump.scroll(DoodleJump.java:94)  
    at DoodleJump.updateGame(DoodleJump.java:44)  
    ...
```

- A stack trace prints out all methods currently on execution stack
- If exception is thrown during execution of recursive method, prints all calls to recursive method



# Bootstrapping Data Structures

- This implementation of the stack data structure uses a **wrapper** of a contained **MyLinkedList**, but user has no knowledge of that
- Could also implement it with an **Array** or **ArrayList**
  - **Array** implementation could be more difficult--**Array**'s have fixed size, so would have to copy our **Array** into a larger one as we push more objects onto the **Stack**
  - User's code should not be affected even if the implementation of **Stack** changes (true for methods as well, if their semantics isn't changed) – **loose coupling!**
- We'll use the same technique to implement a **Queue**

# What are Queues?

- Similar to stacks, but elements are removed in different order
  - information retrieved in the same order it was stored
  - **FIFO: First In, First Out** (as opposed to stacks, which are **LIFO: Last In, First Out**)
- Examples:
  - standing in line for merch at the Eras Tour
  - waitlist for TA hours after randomization



*Server at Seattle restaurant reminding herself what order customers get served in*

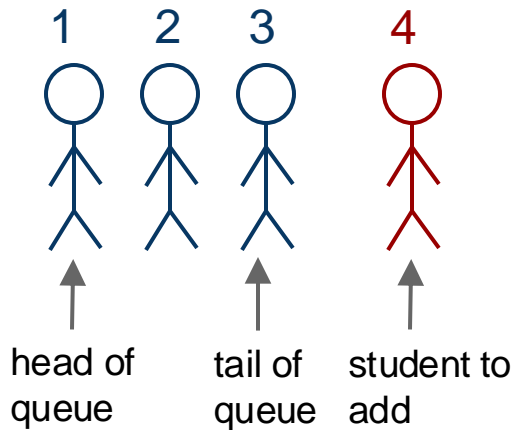
# Methods of a Queue

- Add element to end of `queue` `public void enqueue(Type el)`
- Remove element from beginning of `queue` `public Type dequeue()`
- Returns whether `queue` has any elements `public boolean isEmpty()`
- Returns number of elements in `queue` `public int size()`

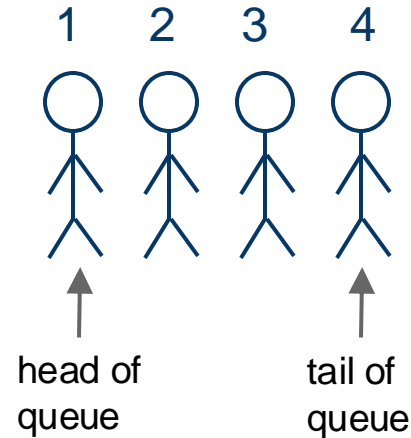
# Enqueuing and Dequeuing

- Enqueuing: adds a node
- Dequeuing: removes a node

Before Enqueuing



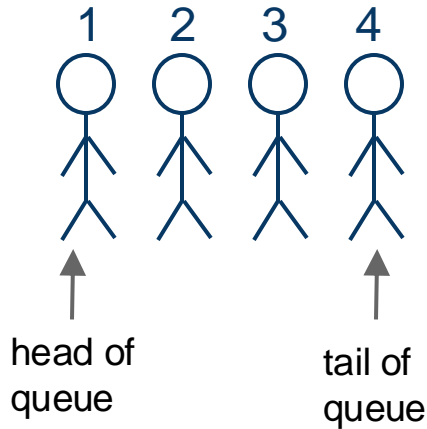
After Enqueuing



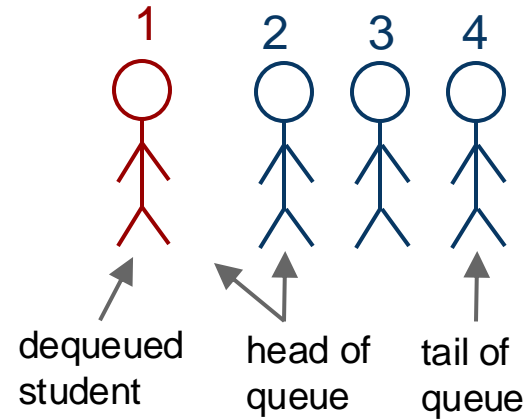
# Enqueuing and Dequeuing

- Enqueuing: adds a node to the back
- Dequeuing: removes a node from the front

Before Dequeuing



After Dequeuing



# Our Queue

- Again use a wrapper for a contained `MyLinkedList`. As with `Stack`, we'll hide most of MLL's functionality and provide special methods that delegate the actual work to the MLL

```
public class Queue<Type> {  
    private MyLinkedList<Type> list;  
  
    public Queue() {  
        this.list = new MyLinkedList<>();  
    }  
    // Other methods elided  
}
```

- Contain a `MyLinkedList` within `Queue` class
  - `enqueue` will add to the end of `MyLinkedList`
  - `dequeue` will remove the first element in `MyLinkedList`

# enqueue

- Just call `list`'s `addLast` method – delegation

```
public void enqueue(Type newNode) {  
    this.list.addLast(newNode);  
}
```

- This will add `newNode` to end of list



# dequeue

- We want first node in `list`
- Use `list`'s `removeFirst` method – delegation

```
public Type dequeue() {  
    return this.list.removeFirst();  
}
```

- What if `list` is empty? There will be nothing to dequeue!
- Our `MyLinkedList` class's `removeFirst()` method returns `null` in this case, so `dequeue` does as well

# isEmpty() and size()

- As with **Stacks**, very simple methods; just delegate to our wrapped **MyLinkedList**

```
public int size() {  
    return this.list.size();  
}  
  
public boolean isEmpty() {  
    return this.list.isEmpty();  
}
```

# TopHat Question

In order from head to tail, a **queue** contains the following: **katara**, **sokka**, **aang**, **momo**. We remove each avatar from the **queue** by calling **dequeue()** and then immediately **push()** each dequeued avatar onto a **stack**.

At the end of the process, what is the order of the **stack** from top to bottom?

- A. katara, sokka, aang, momo
- B. katara, momo, sokka, aang
- C. momo, aang, sokka, katara
- D. It's random every time.

# Outline

- Stacks and Queues
- Trees



# Trees



# Searching in a Linked List (1/2)

- Searching for element in `LinkedList` involves pointer chasing and checking consecutive `Nodes` to find it (or not)
  - it is **sequential access**
  - $O(N)$  – can stop sooner for element not found if list is sorted
- Getting  $N^{\text{th}}$  element in an `Array` or `ArrayList` by index is **random access** (which means  $O(1)$ ), but (content-based) searching for particular element, even with index, remains **sequential**  $O(N)$
- Even though `LinkedLists` support indexing (dictated by Java's `List` interface), getting the  $i^{\text{th}}$  element is also done (under the hood) by pointer chasing and hence is  $O(N)$

# Searching in a Linked List (2/2)

- For  $N$  elements, search time is  $O(N)$ 
  - **unsorted**: sequentially check **every** node in list until element (“search key”) being searched for is found, or end of list is reached
    - if in list, for a uniform distribution of keys, average search time for a random element is  $N/2$
    - if not in list, it is  $N$
  - **sorted**: average\* search time is  $N/2$  if found,  $N/2$  if not found (the win!)
  - we ignore issue of duplicates
- No efficient way to access  $N^{\text{th}}$  node in list (via index)
- Insert and remove similarly have average search time of  $N/2$  to find the right place

*\*Actually more complicated than this – depends on distribution of keys*

# Searching, Inserting, Removing

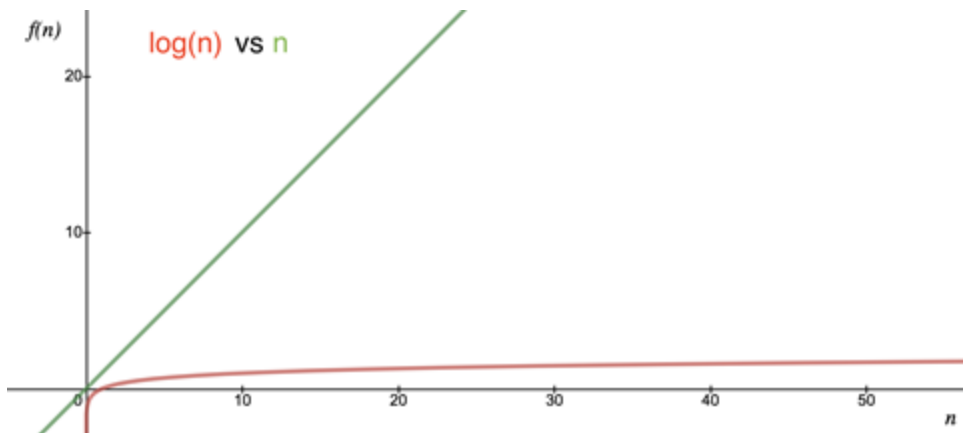
	Search if unsorted	Search if sorted	Insert/remove after search
Linked list	$O(N)$	$O(N)$	$O(1)$
Array	$O(N)$	$O(\log N)$ [coming next]	$O(N)$

# Binary Search (1/4)

- Searching **sorted linked list** is **sequential access**
- We can do better with a **sorted array** that allows **random access** at any index to improve sequential search
- Remember merge sort with search  $O(\log_2 N)$  where we did “bisection” on the array at each pass
- If we had a sorted array, we could do the same thing
  - start in the middle
  - keep bisecting array, deciding which portion of the sub-array the search key lies in, until we find that key or can't subdivide further (not in array)
  - For  $N$  elements, search time is  $O(\log_2 N)$  (since we reduce number of elements to search by half each time), very efficient!

# Binary Search (2/4)

- $\log_2 N$  grows much more slowly than  $N$ , especially for large  $N$



*\*relatively small  $n$  in this graph, but imagine how large the difference is as  $n$  increases*

N	(int) log(N)
1	0
10	3
100	7
1,000	10
10,000	13
1,000,000	17
10,000,000	20
100,000,000	23
1,000,000,000	27

# Binary Search (3/4)

- A sorted `array` can be searched quickly using bisection because arrays are indexed
- `ArrayLists` (implemented in Java using arrays) are indexed too, so a sorted `ArrayList` shares this advantage! But inserting and removing from `ArrayLists` is slow (except for insertion and removal at either end)!
  - Inserting into or deleting from an arbitrary index in `ArrayList` causes all successor elements shift over. Thus insertion and deletion have same worst-case run time  $O(N)$
- Advantage of `linkedList`s is insert/remove by manipulating pointer chain is faster [ $O(1)$ ] than shifting elements [ $O(N)$ ], but search can't be done with bisection 😞, a real downside if search is done frequently

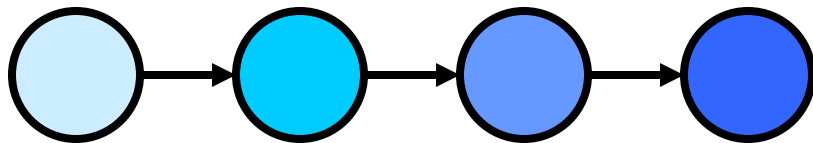
# Binary Search (4/4)

- Is there a data structure that provides both search speed of sorted arrays and [ArrayLists](#) and insertion/deletion efficiency of linked lists?
- Yes, indeed! [Trees](#)! They provide much faster searching than linked lists and much faster insertions than arrays!



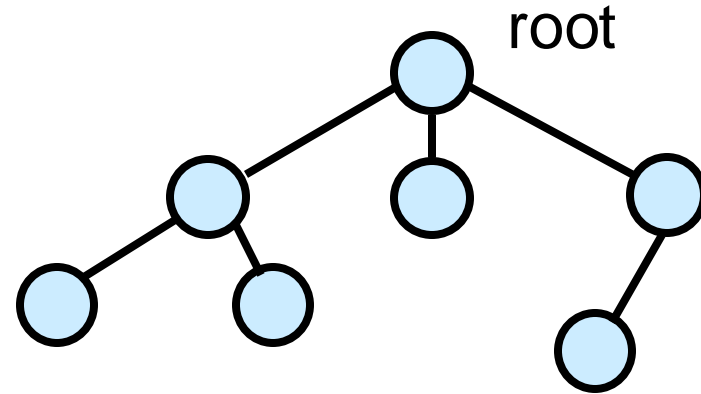
# Trees vs Linked Lists (1/2)

- Singly linked list – collection of nodes where each node references **only one neighbor**, the node's successor:



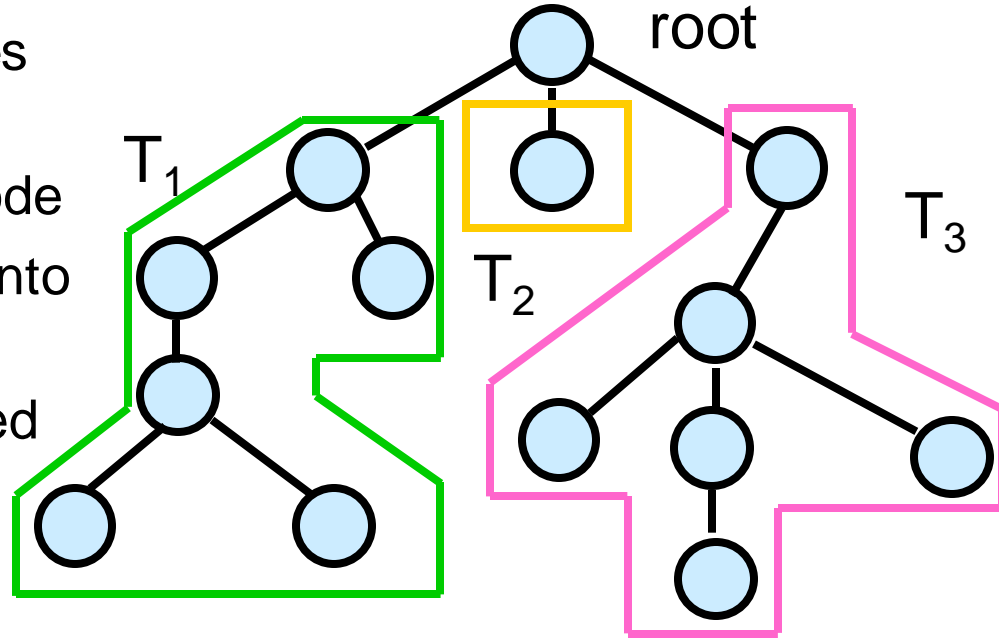
# Trees vs Linked Lists (2/2)

- Tree – also collection of nodes, but each node may reference **multiple successors/children**
- Trees can be used to model a **hierarchical organization** of data



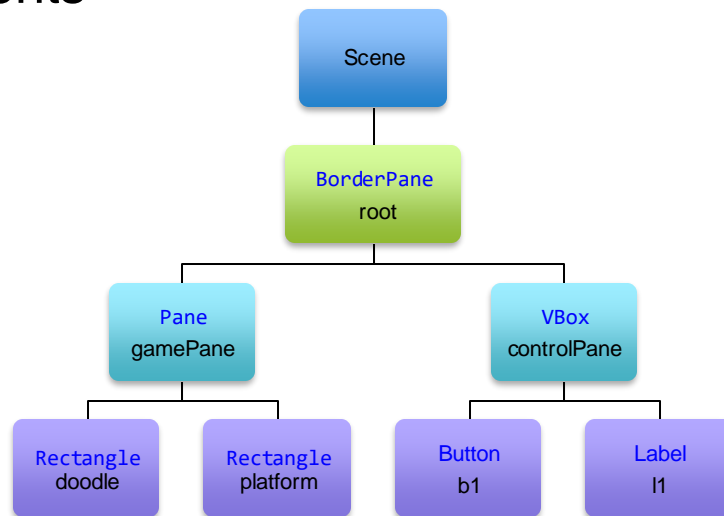
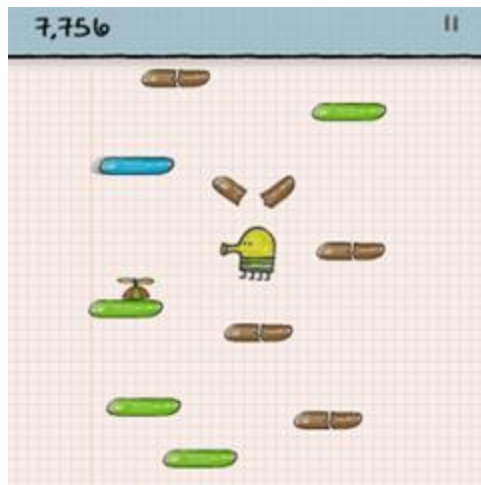
# Technical Definition of a Tree

- Finite set,  $T$ , of one or more nodes such that:
  - $T$  has one designated root node
  - remaining nodes partitioned into disjoint sets:  $T_1, T_2, \dots, T_n$
  - each  $T_i$  is also a self-contained tree, called **subtree** of  $T$
- Look at the image on the right—where have we seen such hierarchies like this before?



# Graphical Containment Hierarchies as Trees

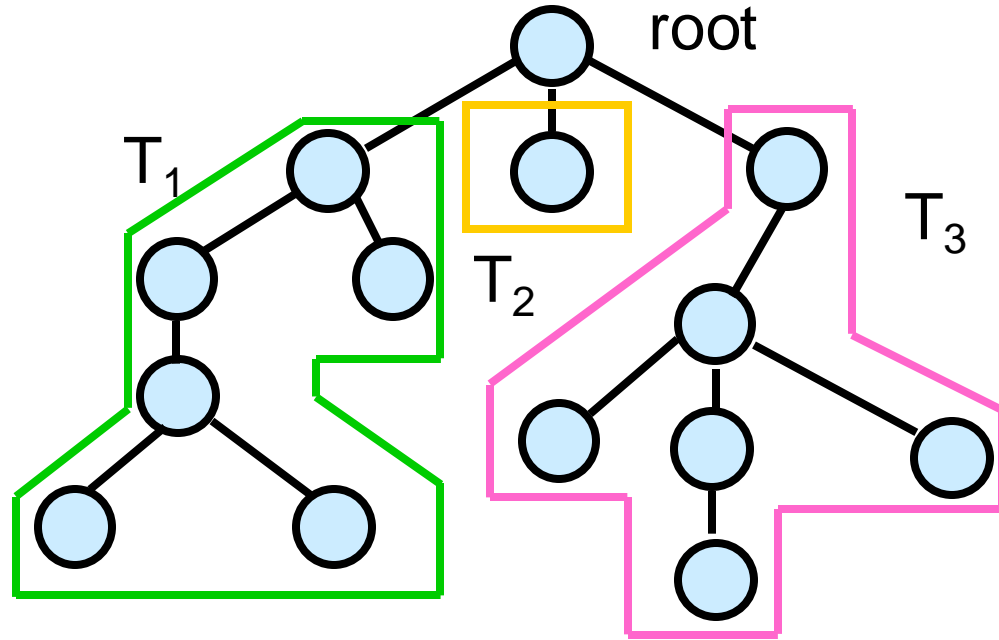
- Levels of containment of **GUI** components



- Higher levels contain more components
- Lower levels contained by all above them
  - Panes contained by **root** pane, which is contained by Scene

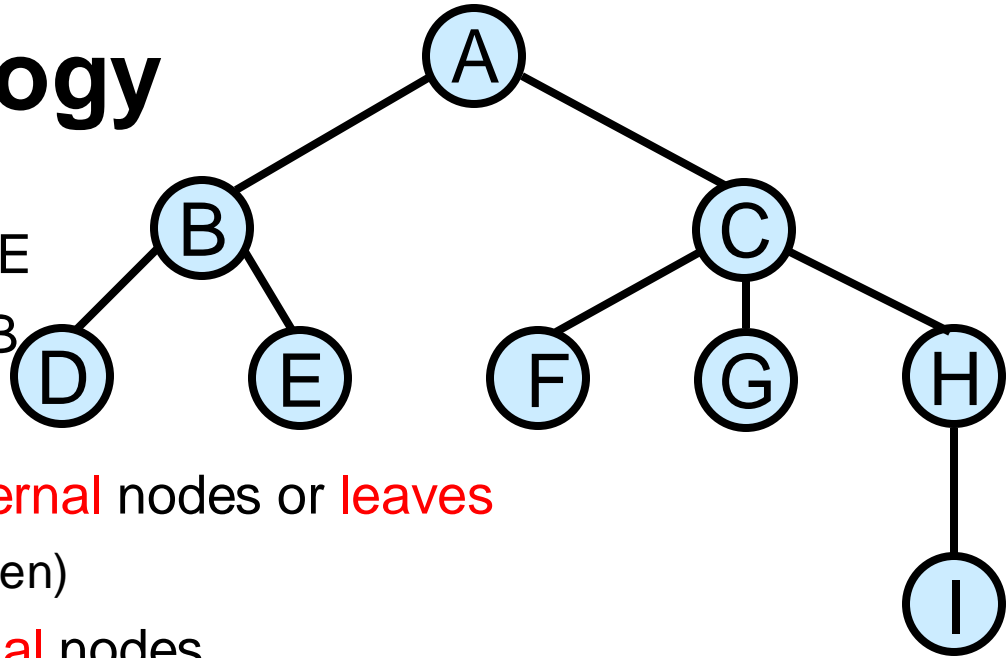
# Tree Structure

- Note that the tree structure has meaning
  - any **subtree** of  $T$ ,  $T_i$ , is also a tree with specific values
- Can be useful to only examine specific **subtrees** of  $T$



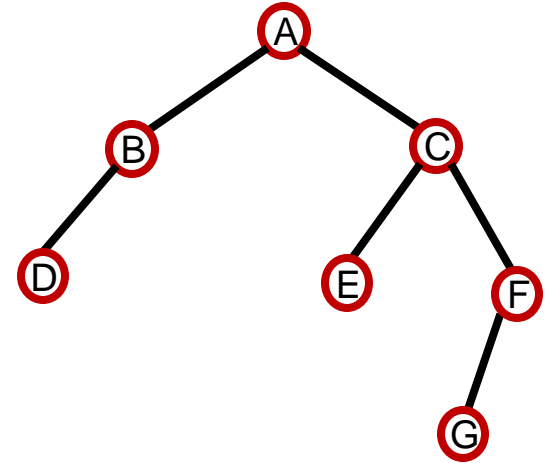
# Tree Terminology

- A is the **root** node
- B is the **parent** of D and E
- D and E are **children** of B
- (C — F) is an **edge**
- D, E, F, G, and I are **external** nodes or **leaves**
  - (i.e., nodes with no children)
- A, B, C, and H are **internal** nodes
- **depth** (level) of E is 2 (number of edges to root)
- **height** of the tree is 3 (max number of edges in path from root)
- **degree** of node B is 2 (number of children)



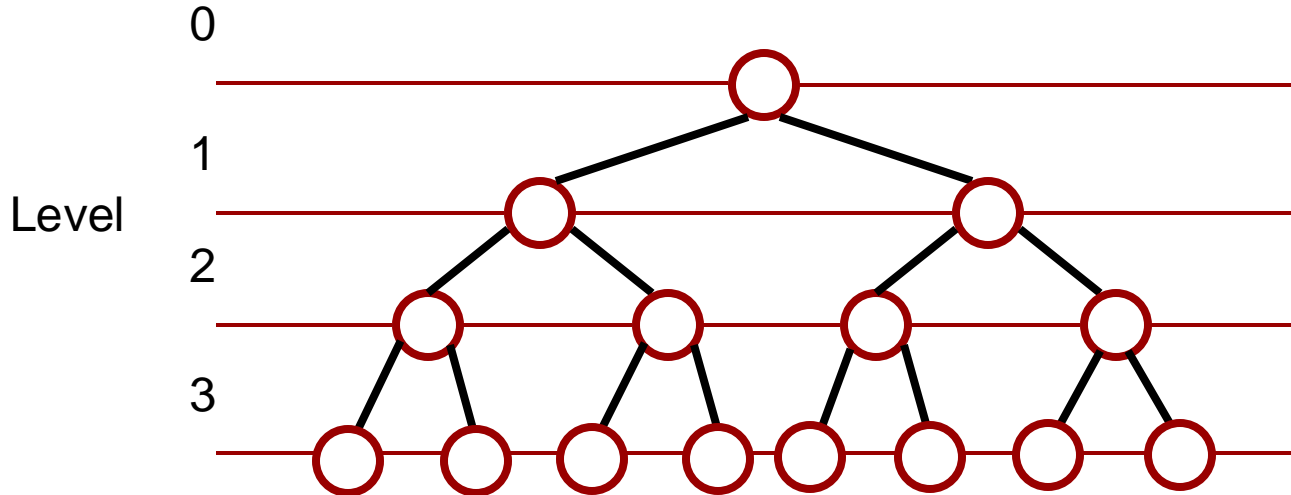
# Binary Trees

- Each internal node has a maximum of 2 successors, called **children**
  - i.e., each internal node has **degree** 2 at most
- Recursive definition of binary tree: A binary tree is either an:
  - external node (**leaf**), or
  - internal node (**root**) with one or two binary trees as children (**left subtree**, **right subtree**)
  - empty tree (represented by a null pointer)
- *Note:* These nodes are similar to the linked list nodes, with one data and two child pointers – we show the data element inside the circle



# Properties of Binary Trees (1/2)

- A binary tree is **full** when each node has exactly zero or two children
- Binary tree is **perfect** when, for every level  $i$ , there are  $2^i$  nodes (i.e., each level contains a complete set of nodes)
  - thus, adding anything to the tree would increase its height

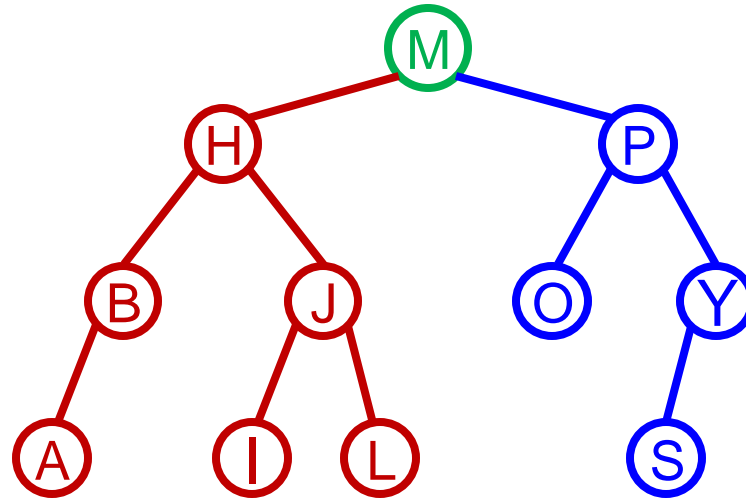


# Properties of Binary Trees (2/2)

- *In a full Binary Tree: (# leaf nodes) = (# internal nodes) + 1*
- *In a perfect Binary Tree: (# nodes at level  $i$ ) =  $2^i$*
- *In a perfect Binary Tree: (# leaf nodes)  $\leq 2^{(\text{height})}$*
- *In a perfect Binary Tree: ( $\text{height}$ )  $\geq \log_2(\# \text{ nodes}) - 1$*

# Binary Search Tree a.k.a BST (1/2)

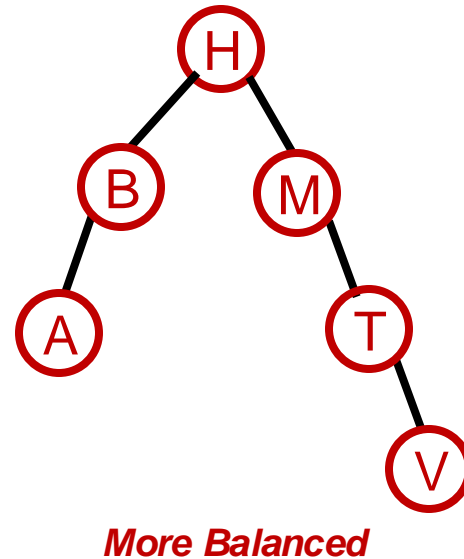
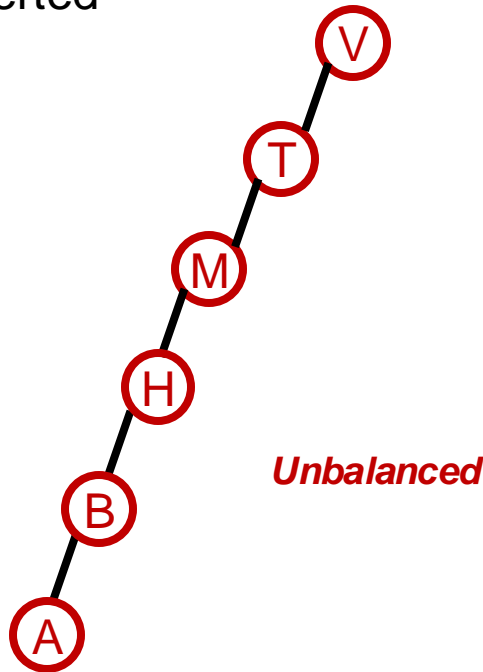
- Binary search tree stores keys in its nodes such that, for every **node**, keys in **left subtree** are **smaller**, and keys in **right subtree** are **larger**



*Note: the keys here are sorted alphabetically!*

# BST (2/2)

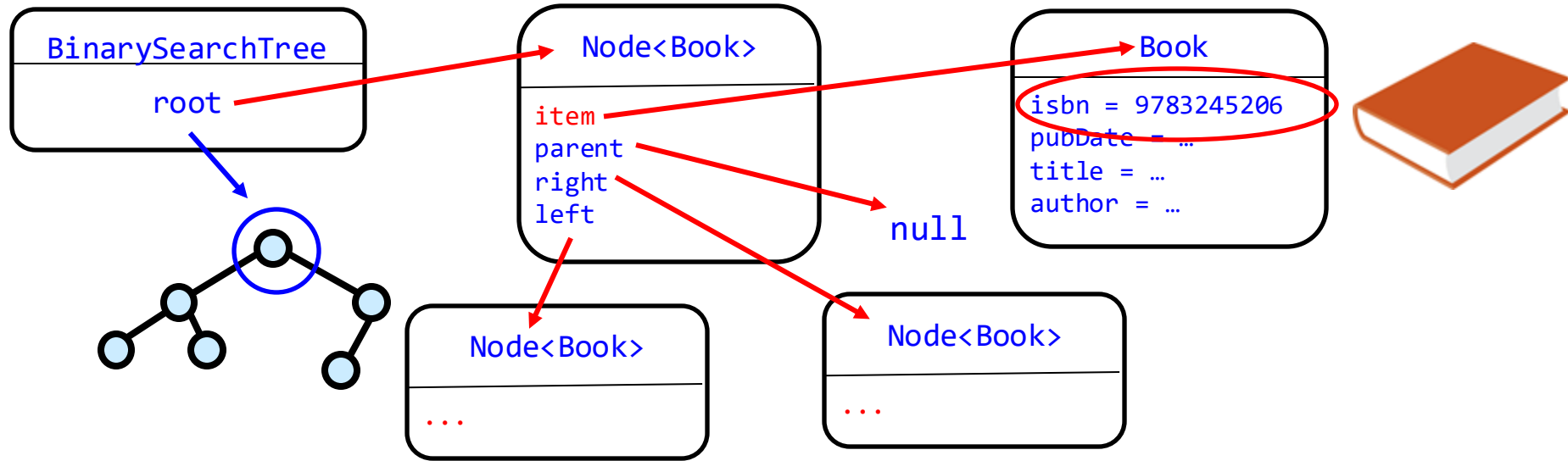
- Below is also BST but much less **balanced**. Gee, it looks like a linked list!
- The shape of the trees is determined by the order in which elements are inserted



# BST Class (1/4)

- What do BSTs know how to do?
  - much the same as sorted linked lists: *insert*, *remove*, *size*, *empty*
  - BSTs also have their own search method – a bit more complicated than simply iterating through its nodes
- What would an implementation of a BST class look like...
  - in addition to data, left, and right child pointers, we'll add a parent “back” pointer for ease of implementation (for the **remove** method – analogous to the **previous** pointer in doubly-linked lists!)
  - you'll learn more about implementing data structures in CS200!

# Nodes, data **items**, and keys



- **item** is a composite that can contain many properties,
- one of which is a **key** that **Nodes** are sorted by (here, ISBN #)

# Java's `Comparable<Type>` interface (1/3)

- Previously we used `==` to check if two things are equal
  - this only works correctly for primitive data types (e.g., `int`), or when we are comparing two variables referencing the exact same object
  - to compare `Strings`, need a different way to compare things
- We can implement the `Comparable<Type>` generic interface provided by Java
- It specifies the `compareTo` method, which returns an `int`
- Why don't we just use `==`, even when using something like ISBN, which is an `int`?
  - can treat ISBNs as `ints` and compare them directly, but more generally we implement the `Comparable<Type>` interface, which could easily accommodate comparing `Strings`, such as author or title, or any other property

# Java's `Comparable<Type>` interface (2/3)

- The `Comparable<Type>` interface is specialized (think of it as parameterized) using generics

```
public interface Comparable<Type> {  
    int compareTo(Type toCompare);  
}
```

- Call `compareTo` on a variable of same type as specified in implementator of interface (`Book`, in our case)
  - `currentBook.compareTo(bookToFind);`

# Java's `Comparable<Type>` interface (3/3)

- `compareTo` method must return an `int`
  - **negative** if element on which `compareTo` is called is *less* than element passed in as the parameter of the search
  - **0** if element is *equal* to element passed in
  - **positive** if element is *greater* than element passed in
  - sign of `int` returned is all-important, magnitude is not and is implementation dependent
- `compareTo` not only used for numerical comparisons—it could be used for alphabetical or geometric comparisons as well—depends on how you implement `compareTo`

# “Comparable” Book Class

- Recall format for `compareTo`:
  - `elementA.compareTo(elementB)`
- Book class now implements `Comparable<Book>`
  - this means we can compare books, using `bookA.compareTo(bookB)`
- `compareTo` is defined according to these specifications
  - returns number that is `<0`, `0` or `>0`, depending on the ISBN numbers
  - `< 0` if stored `this.isbn < toCompare`
  - `== 0` if `this.isbn == toCompare`
  - `> 0` if `this.isbn > toCompare`

```
public class Book implements Comparable<Book> {
    // variable declarations, e.g., isbn, elided
    public Book(String author, String title,
                int isbn){
        //variable initializations elided
    }

    public int getISBN(){
        return this.isbn;
    }

    //other methods elided

    //compare isbn of book passed in to stored one

    @Override
    public int compareTo(Book toCompare){
        return (this.isbn - toCompare.getISBN());
    }
}
```

# BST Class (2/4)

- Using keyword `extends` in this way ensures that `Type` implements `Comparable<Type>`
  - note nested `<>`; shows it modifies `Type` and not the class
  - for generics, `extends` is used instead of `implements` for interfaces
- All elements stored in `MyLinkedList` must now have `compareTo` method for `Type`; }  
thus restricts generic

- In our example, use `Book` as `Type`
- In generics, `extends` is used both for 'extends' and 'implements'

```
public class BinarySearchTree<Type extends  
    Comparable<Type>> {  
  
    private Node<Type> root;  
  
    public BinarySearchTree(Type item) {  
        //Root of the tree  
        this.root = new Node(item, null);  
    }  
  
    // other methods shown next slide
```

# BST Class (3/4)

```
public class BinarySearchTree<Type extends
    Comparable<Type>> {

    private Node<Type> root;

    public BinarySearchTree(Type item) {
        //Root of the tree
        this.root = new Node(item, null);
    }

    public void insert(Type newData) {
        // . . .
    }
```

```
//class continued
    public void remove(Type dataToRemove) {
        // . . .
    }

    public Node<Type> search(Type dataToFind) {
        // . . .
    }

    public int size() {
        // . . .
    }

} // end of class
```

# BST Class (4/4)

- Our implementations of `LinkedLists`, `Stacks`, and `Queues` are “smart” data structures that chain “dumb” nodes together
  - the lists did all the work by maintaining `previous` and `current` pointers and did the operations to search for, insert, and remove information – thus, nodes were essentially data containers
- Now we will use a “dumb” tree with “smart” nodes that will delegate using `recursion`
  - tree will delegate action (such as searching, inserting, etc.) to its root, which will then delegate to its appropriate child, and so on
  - creates specialized `Node` class that stores its item, parent, and children, and can perform operations such as `insert` and `remove`

# BST: Node Class (1/3)

- “Smart” **Node** includes the following methods:

```
// pass in entire data item, containing key, so compareTo() will work
```

```
public Node<Type> search(Type itemToFind);
```

```
public Node<Type> insert(Type newItem);
```

```
/* remove deletes Node pointing to dataToRemove, which contains key;  
removing Node also will remove the matched data element instance unless  
there's another reference to it */
```

```
public Node<Type> remove(Type itemToRemove);
```

- Plus **set**ters and **get**ters of instance variables, defined in the next slides ...

# BST: Node Class (2/3)

- **Nodes** have a maximum of two non-`null` children that hold data implementing `Comparable<Type>`
  - four instance variables: `item`, `parent`, `left`, and `right`, with each having a `get` and `set` method.
  - `item` represents the data that **Node** stores. It also contains the key attribute that **Nodes** are sorted by – we'll make a **Tree** that stores **Books**
  - `parent` represents the direct parent (another **Node**) of **Node**—only used in `remove` method
  - `left` represents **Node**'s left child and contains a subtree, all of whose data is **less** than **Node**'s data
  - `right` represents **Node**'s right child and contains a subtree, all of whose data is **greater** than **Node**'s data
  - arbitrarily select which child should contain data **equal** to **Node**'s data

# BST: Node Class (3/3)

```
public class Node<Type implements Comparable<Type>> {  
    private Type item;  
    private Type parent;  
    private Node<Type> left;  
    private Node<Type> right;  
    public Node(Type item, Node<Type> parent){ //construct a leaf node as default  
        this.item = item;  
        this.parent = parent;  
        //child ptrs null for leaf nodes; set for internal nodes when child is created  
        this.left = null;  
        this.right = null;  
    }  
    // will define other methods in next slides..  
}
```

# Smart Node Approach

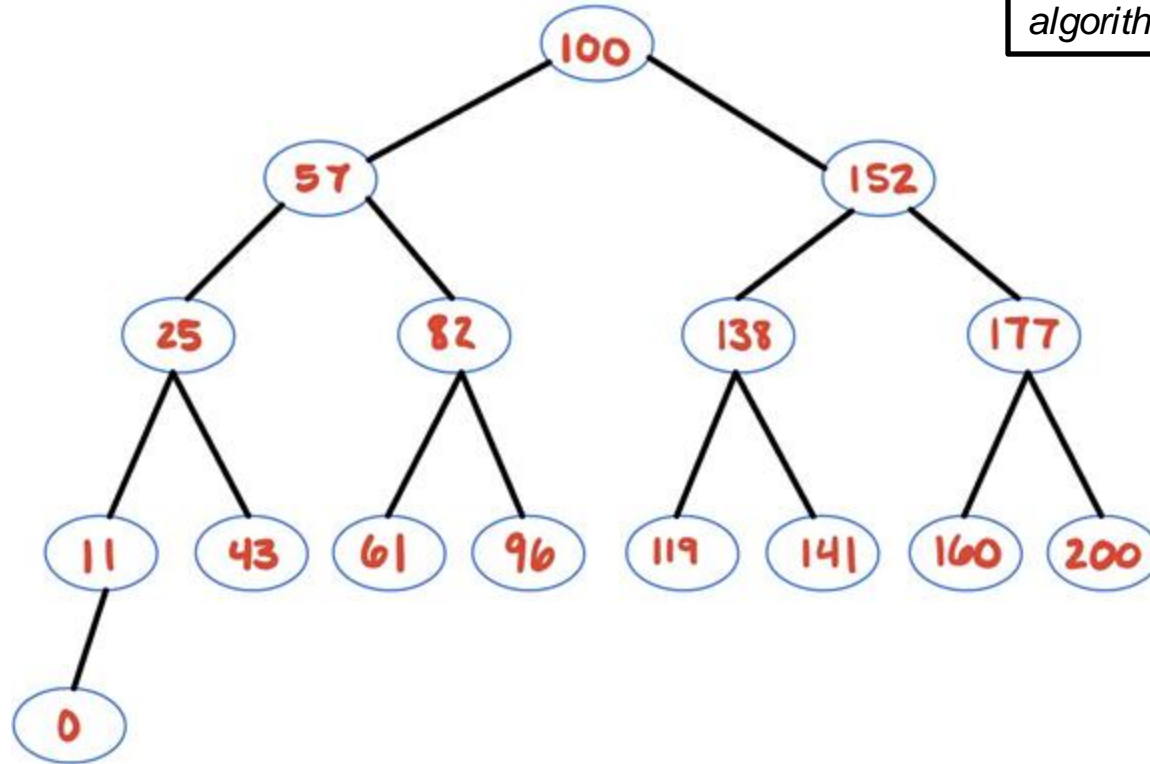
- `BinarySearchTree` is “dumb,” so it delegates to root, which in turn will delegate recursively to its left or right child, as appropriate

```
// search method for entire BinarySearchTree:  
public Node<Type> search(itemToFind) {  
    return this.root.search(itemToFind);  
}
```

- Smart node approach makes our code clean, simple and elegant
  - non-recursive method is much messier, involving explicit bookkeeping of which node in the tree we are currently processing
    - we used the non-recursive method for sorted linked lists, but trees are more complicated, and recursion is easier – a tree is composed of subtrees!

# Let's Search a BST

*For a step-by-step walkthrough of this algorithm, see slide 82*



# TopHat Question

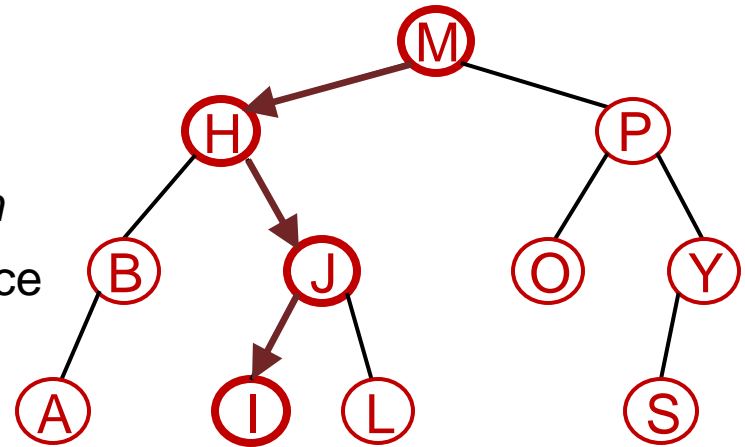
What's the runtime of (recursive) search in a BST and why?

- A.  $O(n)$  – because you only iterate once
- B.  $O(2n)$  – because you go visit both the left and right subtrees
- C.  $O(n/2)$  – because you incorporate the idea of “bisection” to eliminate half the number of nodes to search at each recursion
- D.  $O(\log_2 n)$  - because you incorporate the idea of “bisection” to eliminate half the number of nodes to search at each recursion
- E.  $O(n^2)$  – because recursion makes your runtime quadratic

# Searching a BST Recursively Is $O(\log_2 N)$

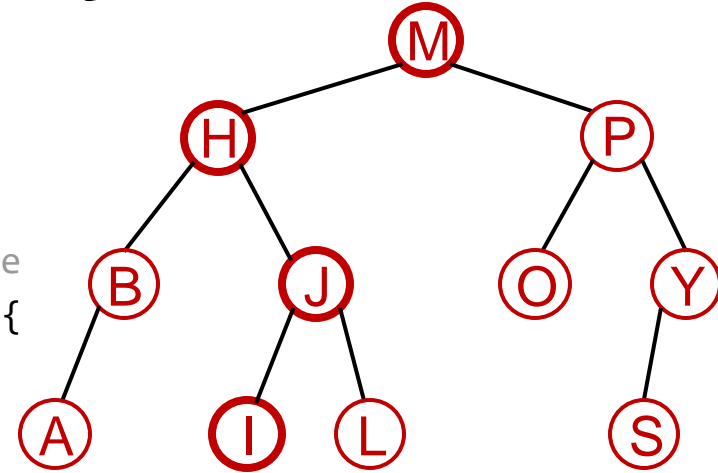
- Search path: start with root **M** and choose path to **I** (for a reasonably balanced tree, **M** will be more or less “in the middle,” and left and right subtrees will be roughly the same size)

- **structurally**, the height of a reasonably balanced tree with  $n$  nodes is about  $\log_2 n$
- at most, we visit each level of the tree once
- so, **runtime performance** of searching is  $O(\log_2 N)$  as long as tree is reasonably balanced, which will be true if entry order is reasonably random
- $O(\log_2 N)$  is much less than **N**, this is thus much more efficient!



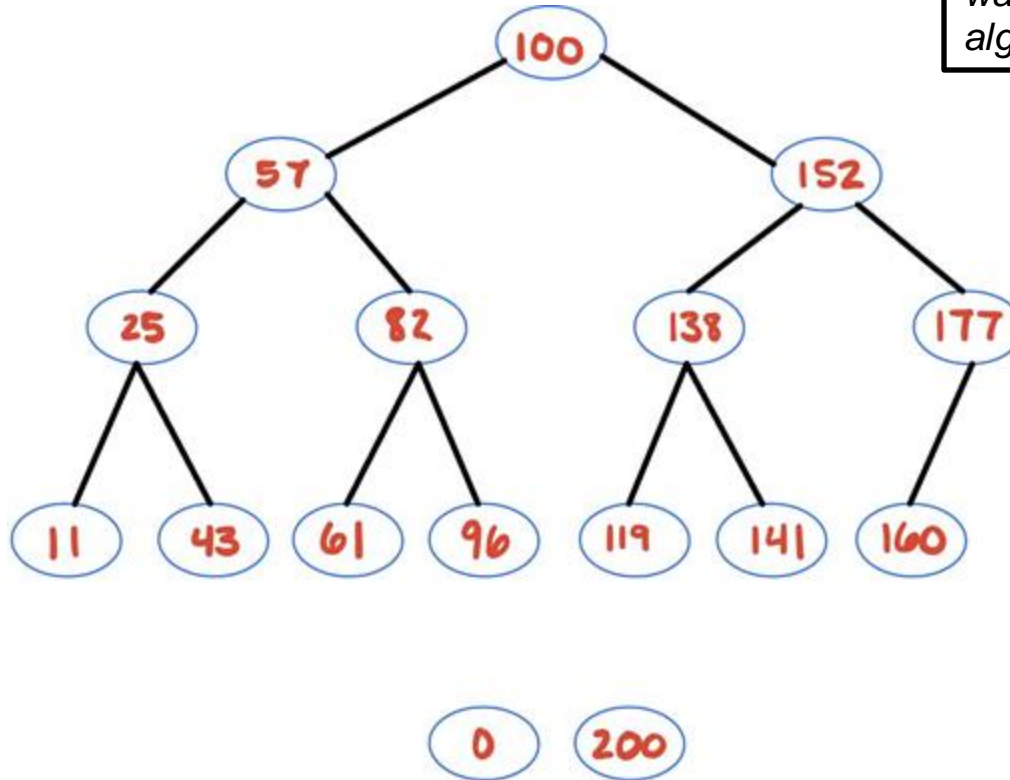
# Searching a BST Recursively

```
public Node<Type> search(Type itemToFind) {  
    //if item is the thing we're searching for  
    if(this.item.compareTo(itemToFind) == 0) {  
        return this.item;  
    }  
    //if item > itemToFind, can only be in left tree  
    } else if(this.item.compareTo(itemToFind) > 0) {  
        if(this.left != null) {  
            return this.left.search(itemToFind);  
        }  
    }  
    //if item < itemToFind, can only be in right tree  
    } else if (this.right != null) {  
        return this.right.search(itemToFind);  
    }  
}  
//Only get here if itemToFind isn't in tree, otherwise would've returned sooner  
return null;  
}
```



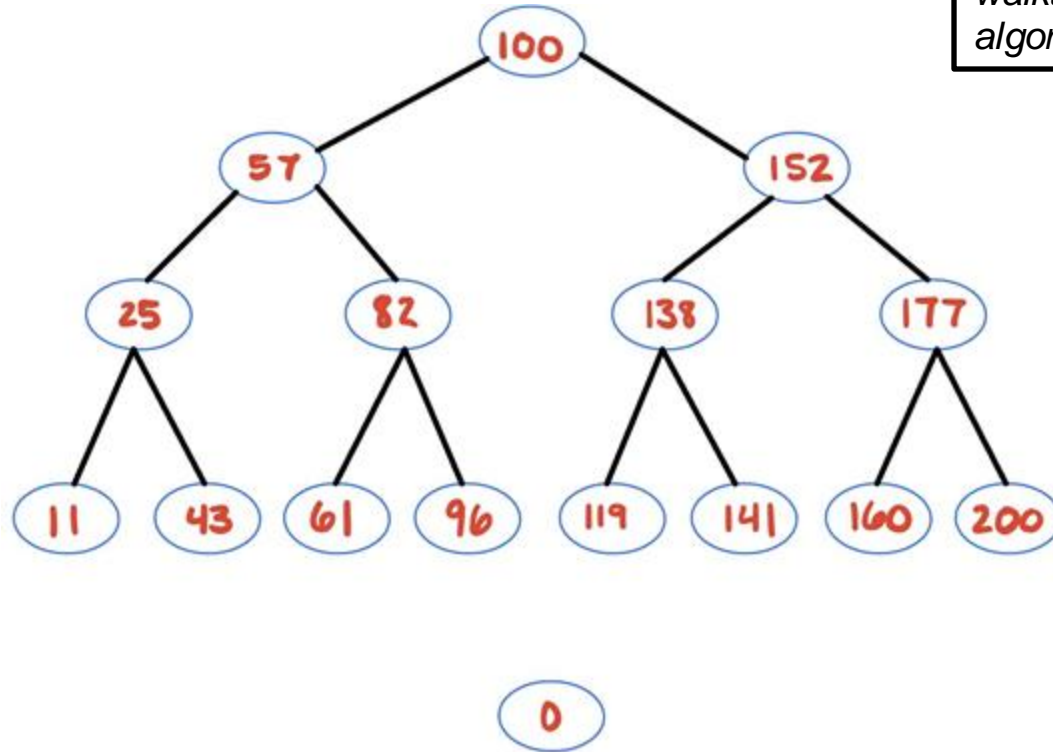
# Let's Add to a BST (1/3)

*For a step-by-step walkthrough of this algorithm, see slide 90*



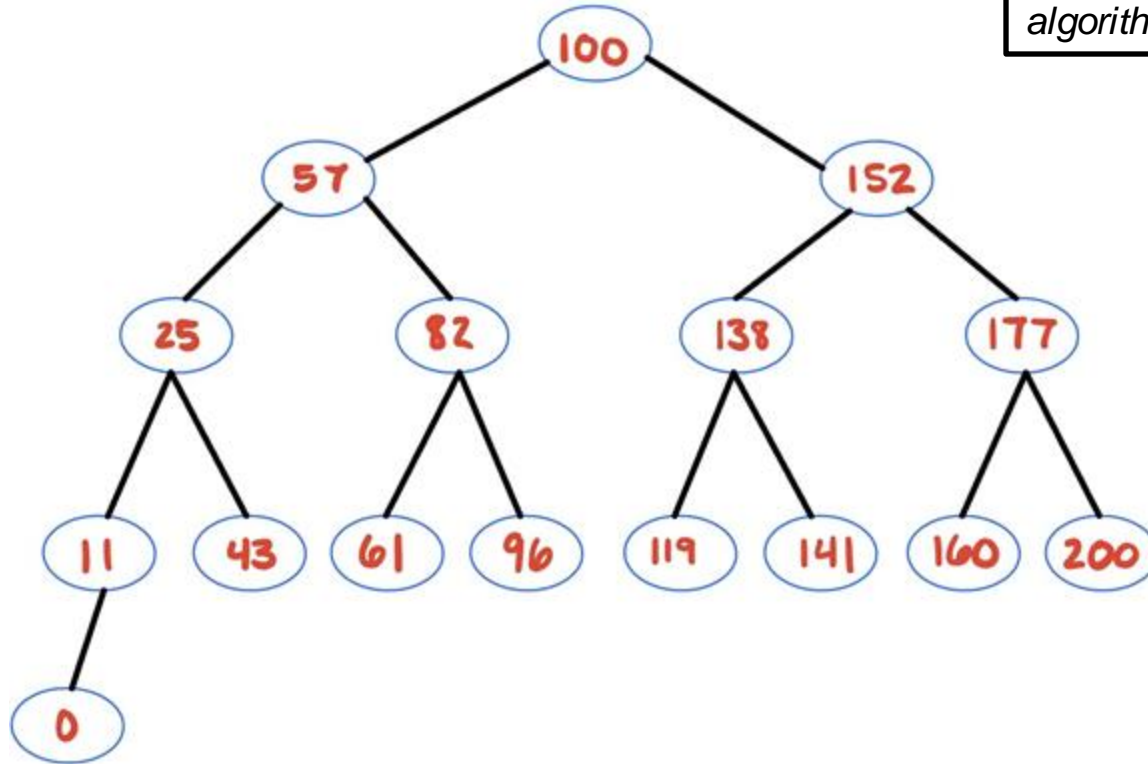
# Let's Add to a BST (2/3)

*For a step-by-step walkthrough of this algorithm, see slide 90*



# Let's Add to a BST (3/3)

*For a step-by-step walkthrough of this algorithm, see slide 90*



# Insertion into a BST

- Search BST starting at root until we find where the data to insert belongs
  - insert data when we reach a **Node** whose appropriate L or R child is **null**
- That **Node** makes a new **Node**, sets the new **Node**'s **data** to the data to insert, and sets child reference to this new **Node**
- Runtime is  **$O(\log_2 N)$** , yay!
  - $O(\log_2 N)$  to search the nearly balanced tree to find the place to insert
  - constant time operations to make new **Node** and link it in

# Insertion Code in **BST**

- Again, we use a “Smart Node” approach and delegate

```
//Tree's insert delegates to root
public Node<Type> insert(Type newItem) {
    //if tree is empty, make first node. No traversal necessary!
    if(this.root == null) {
        this.root = new Node(newItem, null); //root's parent is null
        return this.root;
    } else {
        //delegate to Node's insert() method
        return this.root.insert(newItem);
    }
}
```

# Insertion Code in **Node**

```
public Node<Type> insert(Type newItem) { //insert method continued!
    if (this.item.compareTo(newItem) > 0) { //newItem should be in left subtree
        if(this.left == null) { //left child is null - we've found the place to insert!
            this.left = new Node(newItem, this);
            return this.left;
        } else { //keep traversing down tree
            return this.left.insert(newItem);
        }
    } else { //newItem should be in right subtree
        if(this.right == null) { //right child is null-we've found the place to insert!
            this.right = new Node(newItem, this);
            return this.right;
        } else { //keep traversing down tree
            return this.right.insert(newItem);
        }
    }
}
```

*Reference to the new **Node** is passed up the tree so it can be returned by the tree*

# Notes on Trees (1/2)

- Different insertion order of nodes results in different trees
  - if you insert a node referencing data value of 18 into empty tree, that node will become root
  - if you then insert a node referencing data value of 12, it will become left child of root
  - however, if you insert node referencing 12 into an empty tree, it will become root
  - then, if you insert one referencing 18, that node will become right child of root
  - even with same nodes, **different insertion order makes different trees!**
  - on average, for reasonably random (unsorted) arrival order, trees will look similar in depth so order doesn't play a major role in runtime

# Notes on Trees (2/2)

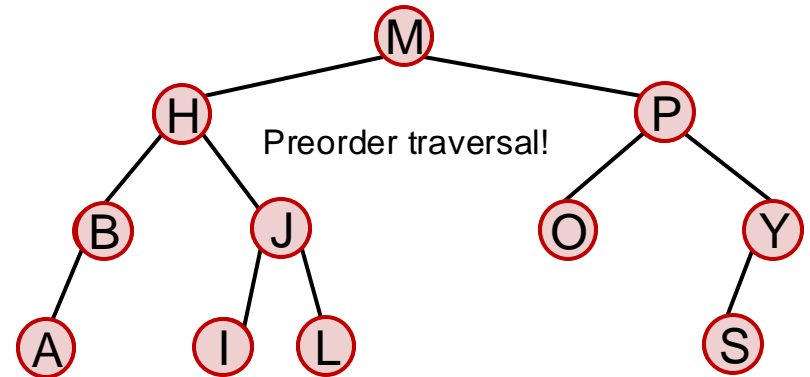
- When searching for a value, reaching another value that is greater than the one being searched for **does not mean that the value being searched for is not present in tree** (whereas it does in linked lists!)
  - it may well still be contained in left subtree of node of greater value that has just been encountered
  - thus, where you might have given up in linked lists, **you can't give up here until you reach a leaf** (but depth is roughly  $\log_2 N$  for a nearly balanced tree, which is much smaller than  $N/2$ !)

# Preorder Traversal of BST

- **Preorder** traversal

- “pre-order” because self is visited before (“pre-”) visiting children
- again, use recursion!

```
public void preOrder() {  
    //Check for null children elided  
    System.out.println(curr.item);  
    this.left.preOrder();  
    this.right.preOrder();  
}
```



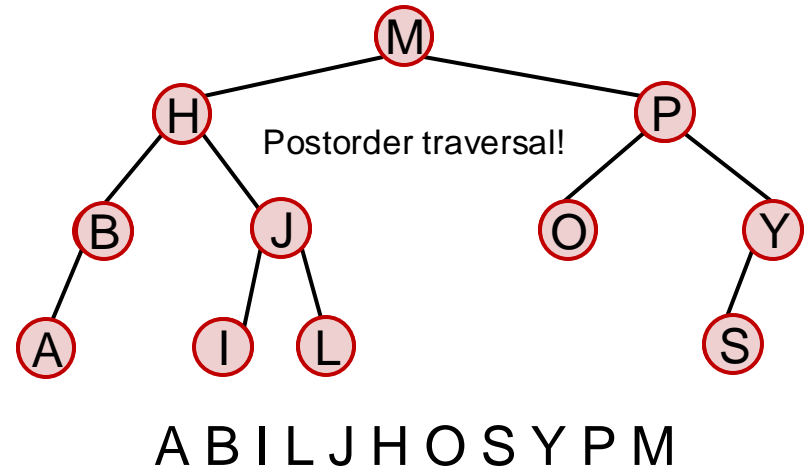
M H B A J I L P O Y S

# Postorder Traversal of BST

- **Postorder** traversal

- “post-order” because self is visited after (“post-”) visiting children
- again, use recursion!

```
public void postOrder() {  
    //Check for null children elided  
    this.left.postOrder();  
    this.right.postOrder();  
    System.out.println(curr.item);  
}
```

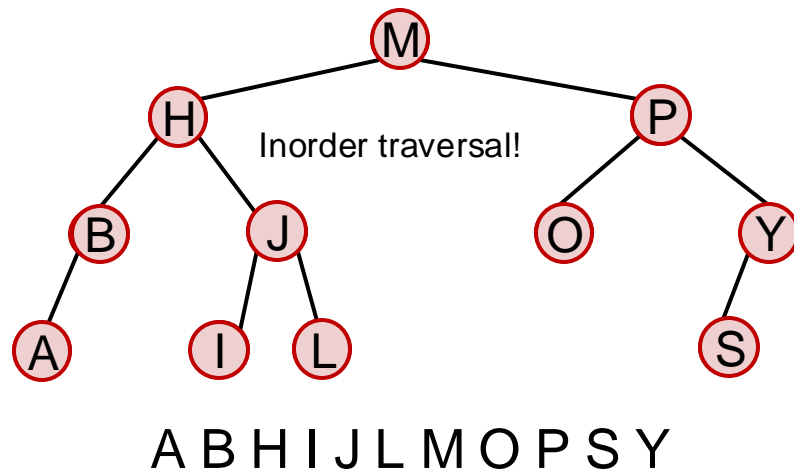


# Inorder Traversal of BST

- **Inorder** traversal

- “in-order” because self is visited between (“in-”) visiting children
- again, use recursion!

```
public void inOrder() {  
    //Check for null children elided  
    this.left.inOrder();  
    System.out.println(curr.item);  
    this.right.inOrder();  
}
```



To learn more about the exciting world of trees, take CS200 (CSCI0200): **Program Design with Data Structures and Algorithms!**

# Tree Runtime

- Binary Search Tree has a search of  $O(\log_2 n)$  runtime, can we make it faster?
- Could make a ternary tree! (each node has at least 3 children)
  - $O(\log_3 n)$  runtime
- Or a 10-way tree with  $O(\log_{10} n)$  runtime
- Let's try the runtime for a search with 1,000,000 nodes
  - $\log_{10} 1,000,000 = 6$
  - $\log_2 1,000,000 < 20$ , so shallower but broader tree
- Analysis: the logs are not sufficiently different and the comparison (basically an n-way nested if-else-if) is far more time consuming, hence not worth it
- Furthermore, binary tree makes it easy to produce an ordered list

# Prefix, Infix, Postfix Notation for Arithmetic Expressions (1/2)

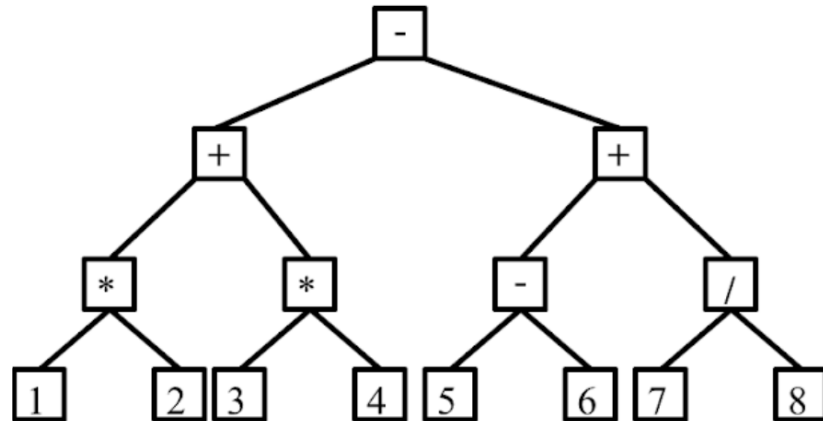
- When you type an equation into a spreadsheet, you use Infix; when you type an equation into many Hewlett-Packard calculators, you use Postfix, also known as “Reverse Polish Notation,” or “RPN,” after its inventor Polish Logician Jan Lukasiewicz (1924)
- Easier to evaluate Postfix because it has no parentheses and evaluates in a single left-to-right pass
- Use Dijkstra’s 2-stack shunting yard algorithm to convert from user-entered Infix to easy-to-handle Postfix – compile or interpret it on the fly

# Prefix, Infix, Postfix Notation for Arithmetic Expressions (2/2)

- Infix, Prefix, and Postfix refer to where the operator goes relative to its operands

- Infix: (fully parenthesized)
  - $((1 * 2) + (3 * 4)) - ((5 - 6) + (7 / 8))$
- Prefix:
  - $- + * 1 2 * 3 4 + - 5 6 / 7 8$
- Postfix:
  - $1 2 * 3 4 * + 5 6 - 7 8 / + -$

- Graphical representation for equation:



# Announcements

- Tetris deadlines
  - late handin: Wednesday 11/13
  - keep an eye out for Tetris code debriefs
- **COME TO LECTURE THURSDAY TO LEARN ABOUT FPs AND HAVE A FUN TIME!**
- HTA Hours Friday 3-4pm (as always!) in CIT 209
  - come talk to us about which FP to do!
- Reminder to resubmit all non-MF projects by end of semester
- Reminder that you cannot use late days on FPs, and can only use max 2 on Tetris no matter how many you have left
- DoodleJump form on Ed
- Final mentor meetings – keep an eye on your emails - talk about registering for classes (coming up soon! come talk to us about 200!)

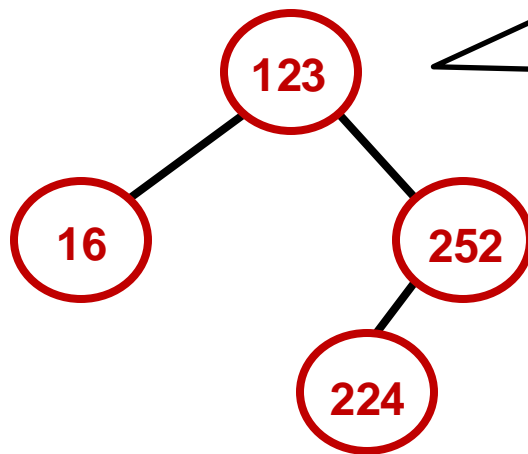
# Appendix

- Searching Simulation
- Insertion Demonstration
- Remove

# Searching Simulation (animated)

- What if we want to know if **224** is in Tree?
- Tree says:

“Hey Root! Ya got **224**?”

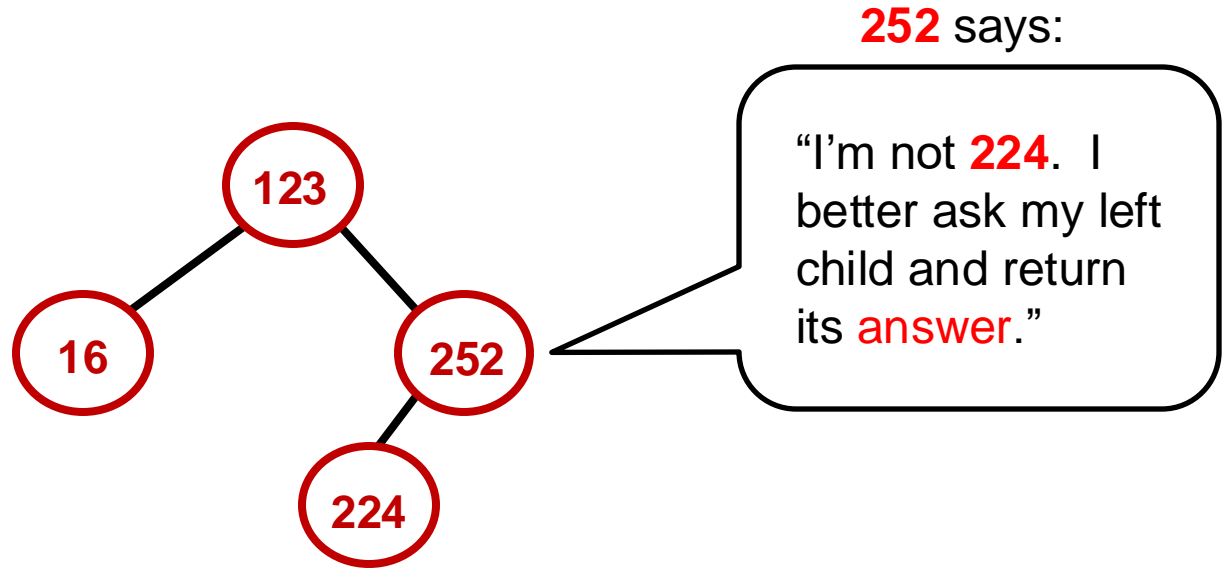


**123** says:

“Let’s see. I’m not **224**. But if **224** is in tree, since it’s larger, it would be to my right. I’ll ask my right child and return its **answer**.”

# Searching Simulation (animated)

- What if we want to know if **224** is in Tree?

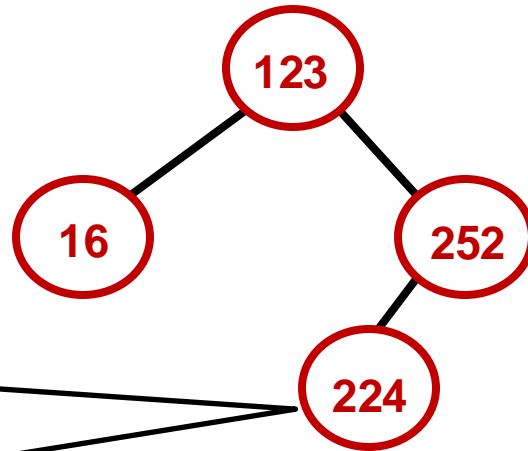


# Searching Simulation (animated)

- What if we want to know if **224** is in Tree?

**224** says:

“**224**? That’s me!  
Hey, caller (**252**)  
here’s your  
answer.”

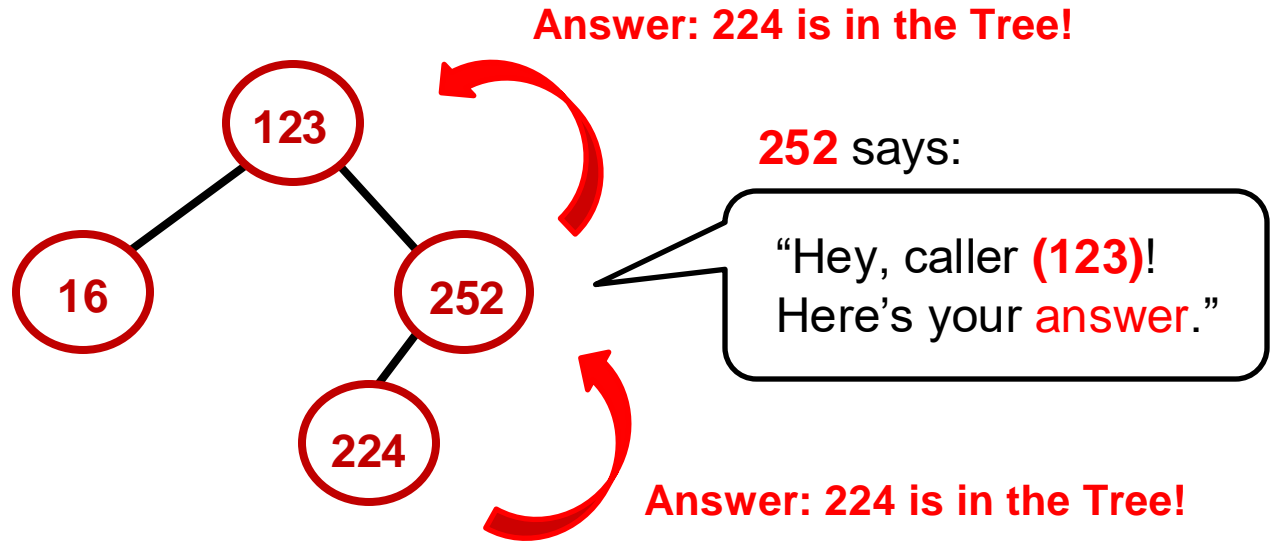


(returning node indicates that  
query is in tree)

**Answer: 224 is in the Tree!**

# Searching Simulation (animated)

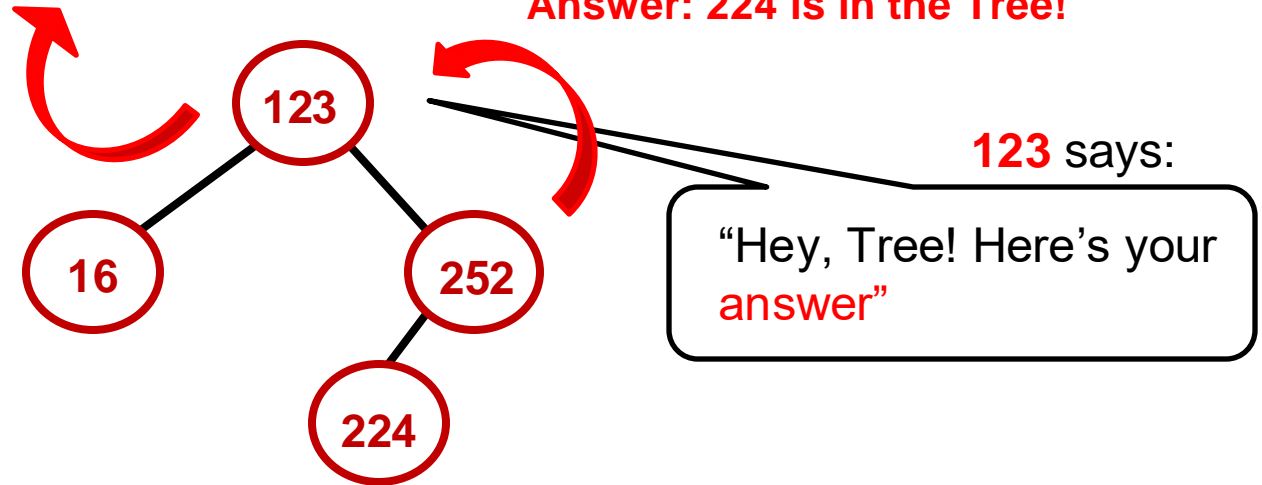
- What if we want to know if **224** is in Tree?



# Searching Simulation (animated)

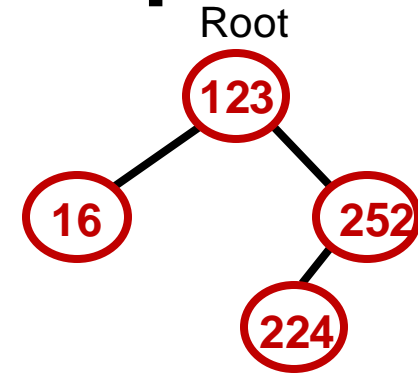
- What if we want to know if **224** is in Tree?

**Answer: 224 is in the Tree!**



# Searching Simulation - Recap

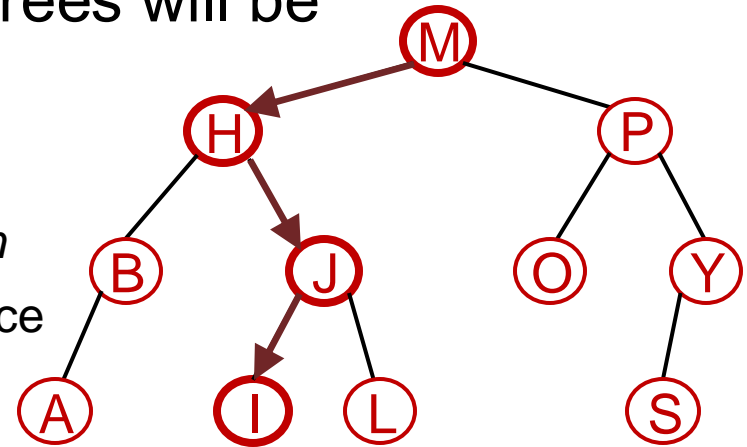
- What if we want to know if **224** is in Tree?
- Tree says “Hey Root! Ya got **224**?”
- **123** says: “Let’s see. I’m not **224**. But if **224** is in tree, it would be to my right. I’ll ask my right child and return its **answer**.”
- **252** says: “I’m not **224**, it’s smaller than me. I better ask my left child and return its **answer**.”
- **224** says: “**224**? That’s me! Hey, caller (**252**) here’s your **answer**.” (returning node indicates that query is in tree)
- **252** says: “Hey, caller (**123**)! Here’s your **answer**.”
- **123** says: “Hey, Tree! Here’s your **answer**.”



# Searching a BST Recursively Is $O(\log_2 N)$

- Search path: start with root **M** and choose path to **I** (for a reasonably balanced tree, **M** will be more or less “in the middle,” and left and right subtrees will be roughly the same size)

- **structurally**, the height of a reasonably balanced tree with  $n$  nodes is about  $\log_2 n$
- at most, we visit each level of the tree once
- so, **runtime performance** of searching is  $O(\log_2 N)$  as long as tree is reasonably balanced, which will be true if entry order is reasonably random (slide 87)



# Appendix

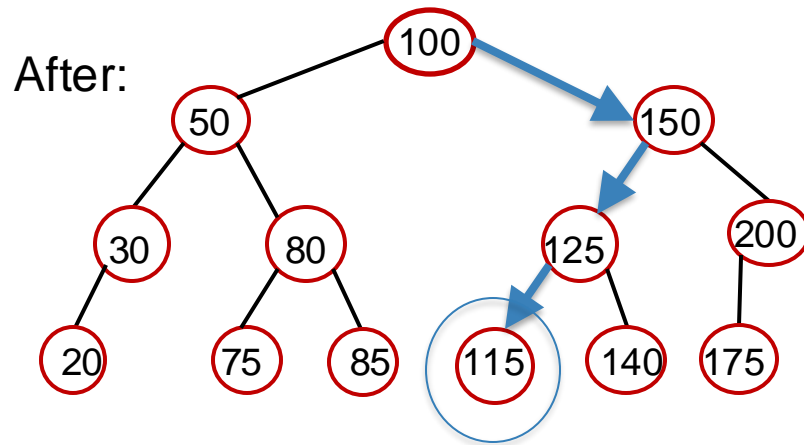
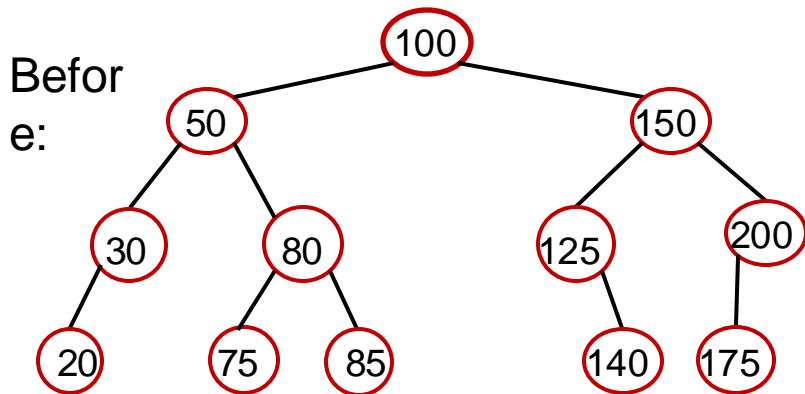
- Searching Simulation
- Insertion Demonstration
- Remove

# Insertion into a BST(1/2)

- Search BST starting at root until we find where the data to insert belongs
  - insert data when we reach a **Node** whose appropriate L or R child is **null**
- That **Node** makes a new **Node**, sets the new **Node**'s **data** to the data to insert, and sets child reference to this new **Node**
- Runtime is  **$O(\log_2 N)$** , yay!
  - $O(\log_2 N)$  to search the nearly balanced tree to find the place to insert
  - constant time operations to make new **Node** and link it in

# Insertion into a BST(2/2)

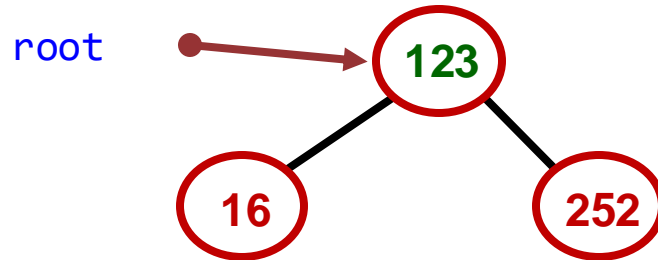
- Example: Insert **115**



# Insertion Simulation (1/4)

- Insert: **224**
- First call `insert` in BST:

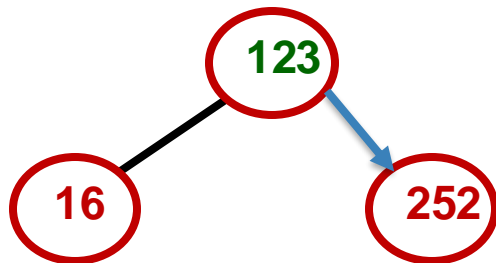
```
this.root = this.root.insert(newData);
```



# Insertion Simulation (2/4)

- **123** says: “I am less than **224**. I’ll let my right child deal with it.”

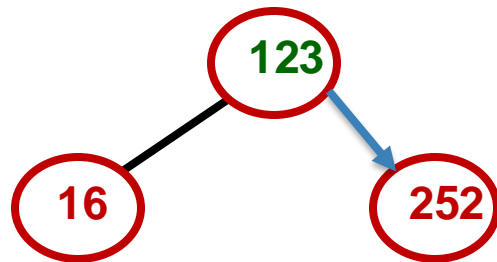
```
if (this.data.compareTo(newData) > 0) {  
    //code for inserting left elided  
} else {  
    if(this.right == null) {  
        //code for inserting with null right child elided  
    } else {  
        return this.right.insert(newData);  
    }  
}
```



# Insertion Simulation (3/4)

- **252** says: “I am greater than **224**. I’ll pass it on to my left child – but my left child is **null**!”

```
if (this.data.compareTo(newData) > 0) {  
    if(this.left == null) {  
        this.left = new Node(newData, this);  
        return this.left;  
    } else {  
        //code for continuing traversal elided  
    }  
}
```

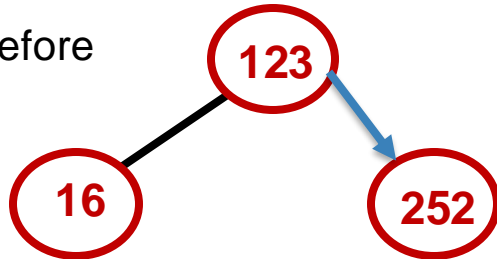


# Insertion Simulation (4/4)

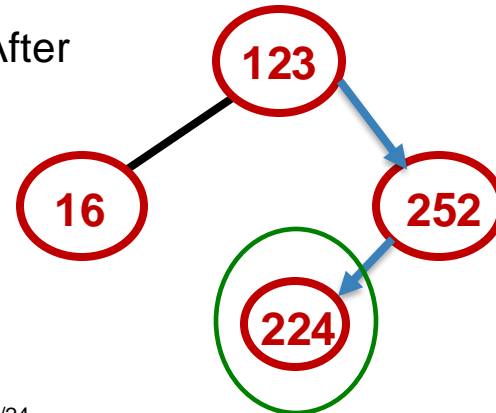
- **252** says: “You belong as my **left** child, **224**. Let me make a node for you, make this new node your home, and set that node as my left child. Lastly, I will return a pointer to the new left node”. (And each node, as its recursive invocation ends, passes the pointer to the new 224 node up to its parent, eventually up to whatever method called on the tree’s search)

```
this.left = new Node(newData, this);  
return this.left;
```

Before



After

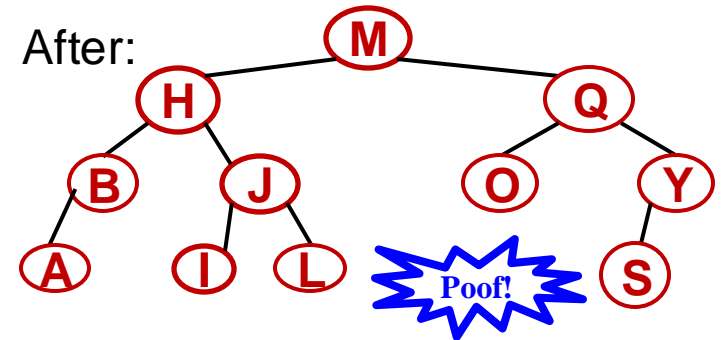
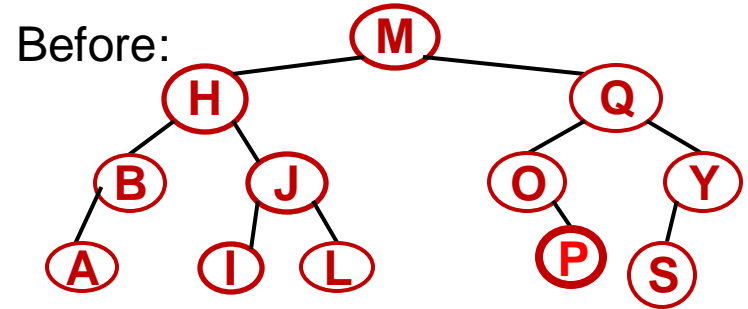


# Appendix

- Searching Simulation
- Insertion Demonstration
- Remove

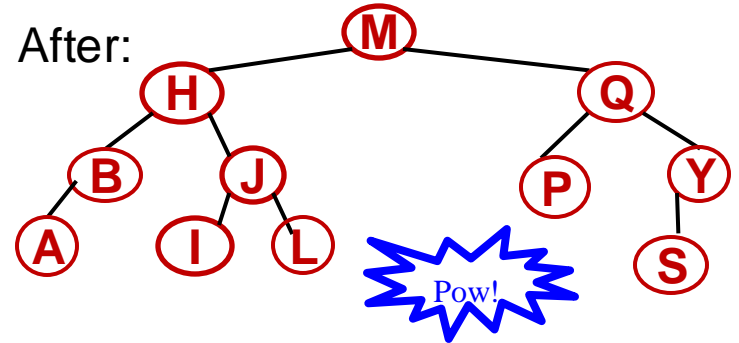
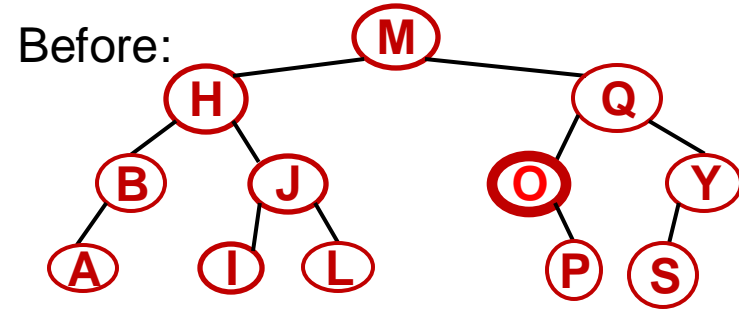
# Remove: No Child Case

- **Node** to remove has no children (is a leaf)
  - just set the parent's reference to this **Node** to `null` – no more references means the **Node** is garbage collected!
- Example: Remove **P**
  - set **O**'s right child to `null`, and **P** is gone!



# Remove: One-Child Case

- Harder case: **Node** to delete has one child
  - replace **Node** child
- Example: Remove **O**
  - **O** has one child
  - **Q** replaces **O** by replacing its left child, previously **O**, with **P**
  - we know that all of the children of **O** are less than **Q** and greater than **M**. So, making **O**'s child a child of **Q** results in a valid BST!



# Remove: Two-Children Case (1/3)

- Hard case: node to remove has two internal children
  - brute force: just flag node for removal, and rewrite tree at a later time -- bad idea, because now every operation requires checking that flag. Instead, do the work right away
  - this is tricky, because not immediately obvious which child should replace its parent
  - slow solution: re-insert each member of one of the sub-trees. Might be bad,  $O(n)$ , even for a balanced tree.
  - non-obvious solution: first swap the data in **Node** to be removed with data in a **Node** that doesn't have two children, then remove **Node** using one of simpler remove cases

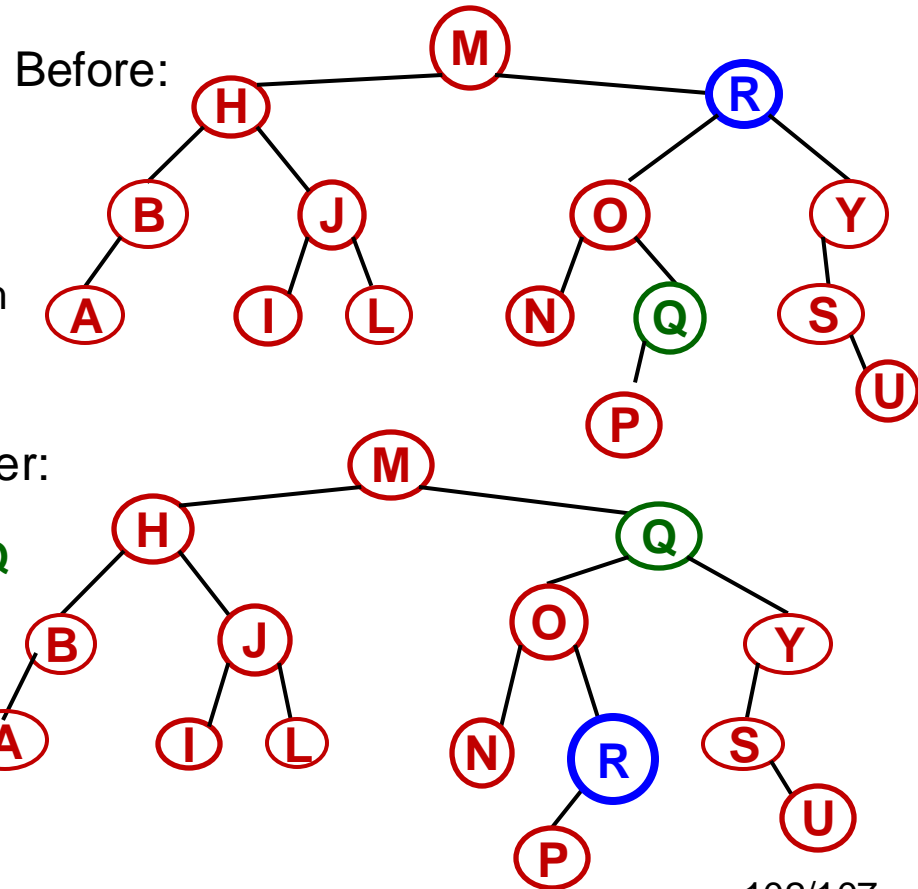
# Remove: Two-Children Case (2/3)

- Use an auxiliary method, `swapData`
  - swaps data in node to be removed with the data in the **right-most node in its left subtree**
  - this child has a key value less than all `Nodes` in the to-be removed `Node`'s right subtree, and greater than all other nodes in its left subtree
  - since it is a right-most `Node`, it has at most one child because if it is the right most child, it won't have any right children
  - this swap is temporary—we then **remove the node in the right-most position using simpler remove**

# Remove: Two-Children Case (3/3)

How do we remove **R**?

- **R** has two children
- swap **R** with the right-most **Node** in the left subtree, the largest **Node** less than **R**, which will be **Q**
- observe the following relationship, which must be maintained after the swap:
  - children in **R**'s left subtree are smaller than **Q**
  - children in **R**'s right subtree are larger than **Q**
  - **R** is in the wrong place but...
- remove **R** (in its new position) using the one-child case



# Remove: BST Code

- Starts as usual with delegating to root
- **Nodes** are “smart,” so they can remove themselves
- Need to first find the **Node** to remove; if not **null**, it removes itself
- **$O(\log_2 N)$**  because of searching in a nearly balanced tree

```
// in BinarySearchTree class:  
//BinarySearchTree's remove takes a data element  
public void remove(Type itemToRemove) {  
    Node<Type> toRemove = this.root.search(itemToRemove);  
    if (toRemove != null) {  
        //smart node's remove takes no params  
        toRemove.remove();  
    }  
}
```

# Remove: Node Code (1/3)

- In the **Node** class, remove method allows **Node** to remove itself

```
public Node<Type> remove() {  
    //Case 1 - Node to remove is a leaf node  
    //Set its parent's reference that originally refers to this Node to null  
    if(this.left == null && this.right == null) { //if it's a leaf, set appropriate parent to null  
        if(this.parent.getLeft() == this) {  
            this.parent.setLeft(null);  
        } else {  
            this.parent.setRight(null);  
        }  
    }  
    //Code for other cases on next slides...  
}
```

Note: because a node removes itself, it compares the parents' child pointers to itself via **this**

# Remove: Node Code (2/3)

```
public Node<Type> remove() { //code for case 1 elided
```

```
//In a one-child case, we replace the _parent's reference to Node with the Node's child
```

```
} else if (this.left != null && this.right == null) {
```

```
    //case 2.1 - Node only has left child
```

```
    if (this.parent.getLeft() == this) {
```

```
        this.parent.setLeft(this.left);
```

```
    } else {
```

```
        this.parent.setRight(this.left);
```

```
    }
```

```
} else if (this.left == null && this.right != null) { //case 2.2 - Node has only right child
```

```
    if (this.parent.getLeft() == this) {
```

```
        this.parent.setLeft(this.right);
```

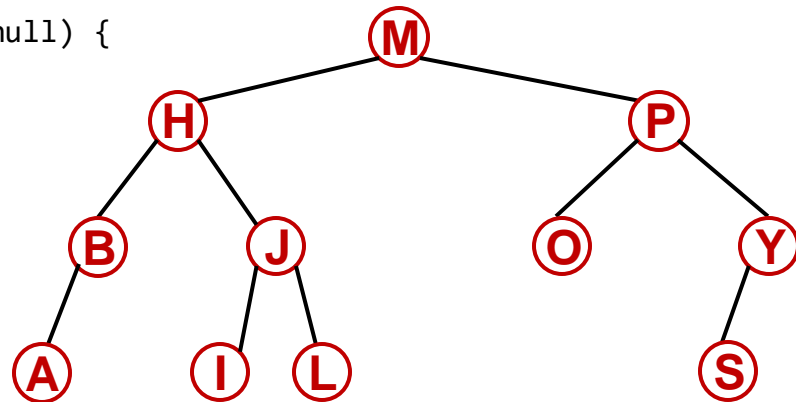
```
    } else {
```

```
        this.parent.setRight(this.right);
```

```
    }
```

```
} //Case 3 on next slide ...
```

```
}
```



# Remove: Node Code (3/3)

- Successor is guaranteed to have at most one child, so we remove with simpler remove case

```
public Node<Type> remove() {  
    //code for case 1 (no children) elided  
    //code for case 2 (one child) elided  
} else { //case 3 - both children  
    Node<Type> toSwap = this.swapItem(); //swap data with successor  
    toSwap.remove(); //now remove toSwap, which holds original Node's data  
    return toSwap; //return toSwap, since toSwap was data we removed  
}  
return this; //return this if we didn't do any swapping since Node is removed  
}  
//swapData() defined on next slide
```

# Remove: `swapItem` Code

- We find the right-most `Node` in left subtree, but we can also find the left-most `Node` in right subtree

```
public Node<Type> swapItem(){
    Node<Type> curr = this.left; //first get left child
    while(curr.getRight() != null) { //go right as far as possible
        curr = curr.getRight();
    }
    //swap data of this Node and successor
    Type tempItem = this.item;
    this.data = curr.getItem();
    curr.setItem(tempItem);
    return curr;
}
```

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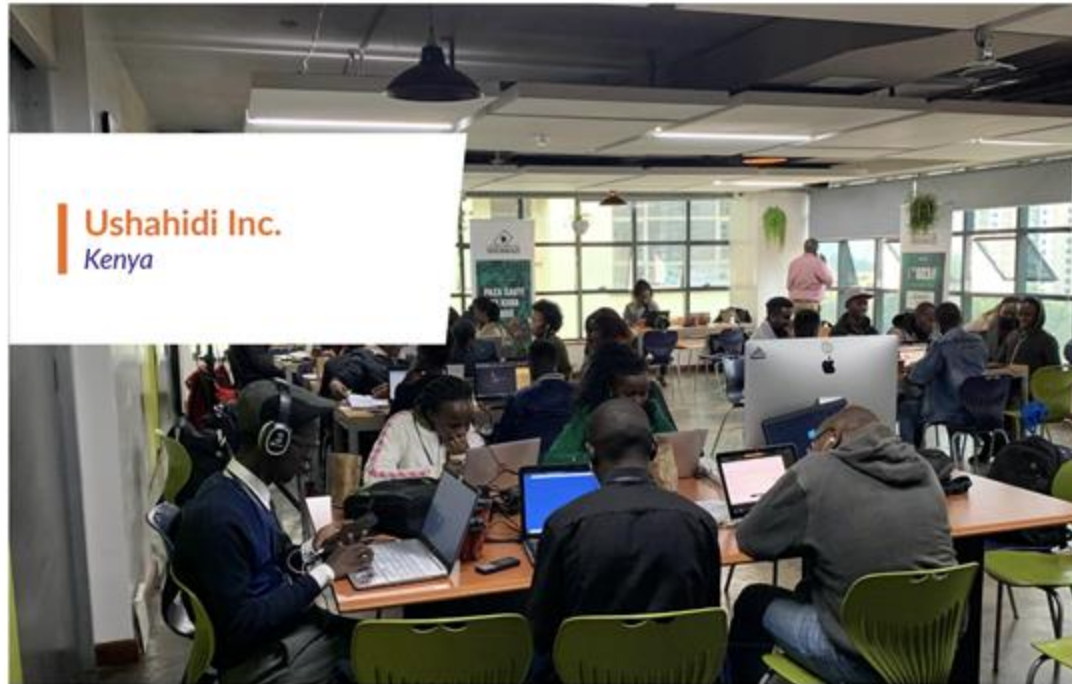
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