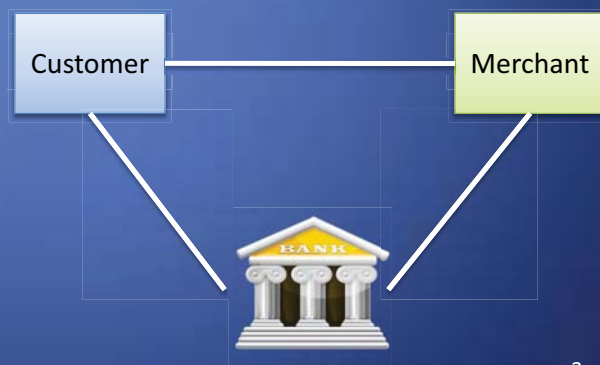


# Payment Systems

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## Electronic Payment Schemes

- Schemes for electronic payment are **multi-party protocols**
- Payment instrument modeled by **electronic coin** that has a fixed value and can be exchanged with a traditional monetary instrument
- Parties include:
  - Payer (customer)
  - Payee (merchant)
  - Bank

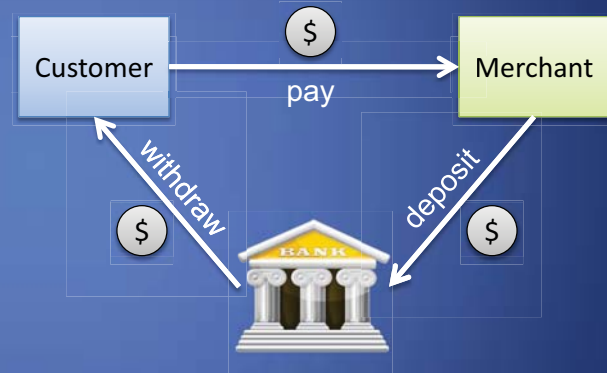


2

# Transactions

- Transactions in an electronic payment scheme typically include:

- **Withdrawal** of coins by customer from the bank
- **Payment** of coins by customer to merchant
- **Deposit** of coins by merchant into bank



- Online scheme:
  - The bank participates in the payment transaction
- Offline scheme
  - The bank does not participate in the payment transaction

3

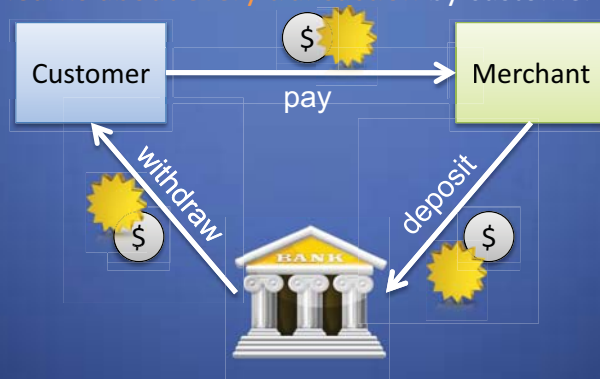
# Goals

- Integrity
  - Coins cannot be forged
  - Legitimate transactions are honored
- Accountability
  - Transactions cannot be later denied
  - Disputes can be efficiently settled
- Privacy
  - The identity of some parties is not revealed to other parties
  - Coins cannot be traced to the payer and/or payee (digital cash)

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# Payment with Digital Signatures

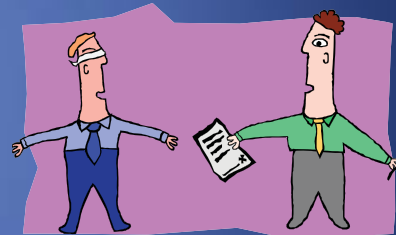
- Coins are random identifiers digitally signed by the bank at the time of withdrawal
- The merchant verifies the signature by the bank
- The bank honors deposit of valid coins
- Security and privacy issues:
  - Customer can copy coin and **double spend**
  - The **bank learns about every transaction** by customer and merchant



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# Private Payment Scheme

- A **blind signature** allows the signed to sign a message without knowing the message itself
- Basic digital cash scheme:
  - The bank does a blind signature on the coins withdrawn by the customer
  - The merchant verifies the signature and deposits the coins
  - The bank cannot link the coins to the customer



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# Blind Signatures with RSA

- The RSA cryptosystem supports a simple and efficient blind signature scheme
- Consider an RSA signing scheme with
  - Public modulus  $N$
  - Public encryption exponent  $e$  and public cryptographic hash function  $h$
  - Secret decryption exponent  $d$
- The bank can create a signature on any item without knowing it
- Bank must have assurance that it is signing a valid coin of the correct amount

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# RSA Blind Signature Protocol

- The customer picks secret random values  $x$  and  $r$ 
  - Coin identifier  $x$
  - Number  $r$  in  $\mathbf{Z}_N$  relatively prime to  $N$
- The customer sends to the bank value
$$y = r^e h(x) \bmod N$$
- The bank creates signature  $\sigma(y)$  on  $y$ 
$$\sigma(y) = y^d \bmod N$$
- The customer derives from  $\sigma(y)$  signature  $\sigma(x)$  on  $x$ 
$$\sigma(x) = \sigma(y) / r \bmod N$$

- Proof

$$\sigma(y) / r \bmod N = r^{ed-1} h(x)^d \bmod N = h(x)^d \bmod N = \sigma(x)$$

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## Blindly Signing a Valid Coin

- The customer generates  $k$  coins and submits to the bank commitments (cryptographic hashes) for all the coins
- The bank randomly selects  $k - 1$  coins
- The customer reveals to the bank the selected  $k - 1$  coins
- The bank verifies the commitments on the selected  $k - 1$  coins
- The bank creates a blind signature on the remaining coin
- The coin signed by the bank is valid with probability

$$1 - 1/k$$

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## Defenses Against Double Spending

- Online protocol
  - The bank is online during the payment transaction to revoke spent coins
- Offline protocol
  - Withdrawn coins embed encrypted customer identity
  - Deposited coins embed also encrypted merchant identity
  - Double spending caused the identity of the cheating party to be revealed

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# Secret Splitting into Shares

- A secret string  $x$  can be split into random values  $y$  and  $z$  as follows
  - Pick a random value  $y$
  - Set  $z = y \oplus x$
- String  $x$  can be reconstructed from  $y$  and  $z$  by setting
  - $x = y \oplus z$
- Both shares  $y$  and  $z$  are random values and are referred to as **shares** of  $x$
- Neither share reveals any information about secret  $x$

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# Coins

- Let  $h$  be a cryptographic hash function
- Given a secret string  $x$ , a **commitment pair** for  $x$  is a pair  $(a, b)$  such that
  - $a = h(y)$
  - $b = h(z)$
  - $y$  and  $z$  are random shares of  $x$
- Let  $ID$  be a string identifying the customer (e.g., name, address, etc.)
- The coin issued by the bank to the customer consists of
  - Coin identifier  $x$
  - Vector of  $n$  commitments pairs  $(a_1, b_1), \dots, (a_n, b_n)$  for  $ID$
- The coin does not reveal the identity of the customer

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# Withdrawal

- The customer generates and submits  $k$  coins to the bank
- The bank randomly selects  $k - 1$  coins
- The customer reveals to the bank the shares associated with the commitments pairs of the selected coins
- The bank creates a blind signature on the remaining coin
- The coin signed is valid with probability  $1 - 1/k$

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# Payment

- The customer gives to the merchant a coin  $\{ x ; [(a_1, b_1), \dots, (a_n, b_n)] \}$
- The merchant verifies the signature on the coin
- The customer gives to the customer a random binary vector  $s_1, \dots, s_n$ , called **selector**
- The customer reveals to the merchant the shares indicated by the selector, i.e., it sends to the merchant a vector of strings  $P_1, \dots, P_n$  such that

$$h(P_i) = a_i \text{ if } s_i = 0$$

$$h(P_i) = b_i \text{ if } s_i = 1$$

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# Deposit and Security Properties

- Deposit
  - The merchant deposits with the bank the coin and strings  $P_1, \dots, P_n$
  - The bank verifies the signature and keeps track of coins and associated strings
- Security properties
  - The probability that the selectors provided by two merchants are identical is  $1/2^n$
  - Thus, if the customer double spends a coin, then the bank finds out the identity of the customer with probability  $1 - 1/2^n$
  - A merchant can double spend a coin without being detected by the bank only if it can find a collision of the hash function
- The scheme does not prevent double spending but detects it and identifies the culprit with high probability

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# References

- The electronic cash scheme presented in this lecture is based on the work by David Chaum <http://www.chaum.com/>
- D. Chaum, A. Fiat, and M. Naor. *Untraceable Electronic Cash*, in Proc. CRYPTO 1988. <http://citeseer.ist.psu.edu/421212.html>
- S. Goldwasser and M. Bellare. *Lecture Notes on Cryptography* [Section 12.5] <http://www-cse.ucsd.edu/users/mihir/papers/gb.html>

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