Problem 1

In this problem we will consider what happens when we demodulate an AM signal using a sinusoidal signal with frequency that is similar but doesn’t exactly match the carrier frequency. Let $y(t)$ be the AM modulation of $x(t)$ with carrier frequency $w_c$,

$$y(t) = x(t) \cos(w_c t).$$

Suppose we demodulate $y(t)$ by multiplying it with a sinusoidal of frequency $w_d$ followed by lowpass filtering,

$$w(t) = y(t) \cos(w_d t),$$

$$z(t) = w(t) * h(t).$$

Here $h(t)$ is an ideal lowpass filter with cutoff frequency $W$. Denote the difference between the frequencies in the modulator and demodulator by $\Delta w = w_d - w_c$. Suppose $x(t)$ is band limited with $X(w) = 0$ for $|w| \geq w_M$ and assume the cutoff frequency $W$ is such that

$$w_M + \Delta w < W < 2w_c + \Delta w - w_M$$

(a) Show that the demodulated signal $z(t)$ is proportional to $x(t) \cos(\Delta wt)$.

(b) Suppose $x(t)$ is an audio signal. What does the demodulated signal sound like?
Problem 2

AM modulation and demodulation requires multiplication of signals. Multiplication can be hard to implement in practice and some systems use alternative nonlinear operations instead of multiplications.

One example for AM modulation involves transforming the input signal $x(t)$ by squaring the sum of $x(t)$ and the carrier signal, and then bandpass filtering the result to obtain the amplitude modulated signal $z(t)$. That is, we take $y(t) = (x(t) + \cos(w_c t))^2$ and then use a bandpass filter on $y(t)$ to obtain $z(t)$.

Assume that $x(t)$ is band limited with $X(w) = 0$ for $|w| \geq w_M$. Determine what the Fourier transform of the bandpass filter should look like so that $z(t) = x(t) \cos(w_c t)$. Specify any necessary constraints (if any) on $w_c$ and $w_M$. 