ENGN 1570 Homework 3

Problem 1

For the signal $x(t)$ above, compute $y = x \ast h$ using the convolution formula and sketch $y(t)$ for each signal $h$ below. You should express the result of the convolution as a piece-wise function of $t$, with a closed form algebraic expression for each interval of $t$.

(a)  

$h(t) = \begin{cases} 
0 & t < 0 \\
2 & 0 < t < 1 \\
0 & t > 1 
\end{cases}$

(b) $h(t) = \delta(t) + \delta(t - 1)$

Problem 2

Let $C_h$ be the system that has output $x \ast h$ on input $x$. For each of the choices for $h$ below, determine if $C_h$ is invertible. If so give the signal $h'$ such that $C_h'$ is the inverse of $C_h$. If not give two input signals that map to the same output signal.

(a) $h(t) = e^{-j2t}$

(b) $h(t) = j\delta(t)$
Problem 3

Let \( y = x * h \). Let \( y', x' \) and \( h' \) denote \( y, x \) and \( h \) delayed by 1 respectively. Determine which of the statements below are true and which are false. Justify your answers.

(a) If \( x[n] = 0 \) for \( n < N_1 \) and \( h[n] = 0 \) for \( n < N_2 \) then \( y[n] = 0 \) for \( n < N_1 + N_2 \).

(b) If \( x[n] = 0 \) for \( n > N_1 \) and \( h[n] = 0 \) for \( n > N_2 \) then \( y[n] = 0 \) for \( n > N_1 + N_2 \).

(c) \( y' = x' * h' \)

(d) \( y' = x' * h \)

(e) \( y' = x * h' \)

Problem 4

The following are impulse responses of LTI systems. Determine if the systems are causal and/or stable. Justify your answers.

(a) \( h[n] = (1/5)^n u[n] \)

(b) \( h[n] = (0.8)^n u[n + 2] \)

(c) \( h[n] = (-1/2)^n u[n] + (1.01)^n u[n - 1] \)

(d) \( h(t) = e^{-4t} u(t - 2) \)

(e) \( h(t) = e^{-6|t|} \)

Problem 5

It is often important to transform between continuous and discrete signals. We can define a discrete signal from a continuous signal by “sampling” the continuous signal at integer times. The problem of defining a continuous signal \( y(t) \) from a discrete signal \( x[n] \) is more complicated because we have to fill in the missing values. A common approach involve using spline basis functions.
One of the simplest spline functions is the “triangular shaped” signal defined by

\[
h(t) = \begin{cases} 
0 & t < -1 \\
 t + 1 & -1 < t < 0 \\
1 - t & 0 < t < 1 \\
0 & t > 1 
\end{cases}
\]

(a) Sketch \( h(t) \)

Let \( x[n] \) be a discrete signal. We can define a continuous signal \( y(t) \) by taking linear combinations of shifted and scaled versions of \( h(t) \).

\[
y(t) = \sum_{k=-\infty}^{\infty} x[k] h(t - k)
\]

Sketch the resulting \( y(t) \) for the following cases:

(b) \( x[n] = 2 \)
(c) \( x[n] = n \)
(d) \( x[n] = \begin{cases} 
3 & n = 0 \\
0 & n \neq 0 
\end{cases} \)
(e) \( x[n] = \begin{cases} 
0 & n \leq 0 \\
5 & n = 1 \\
2 & n = 2 \\
2 & n = 3 \\
-1 & n = 4 \\
0 & n \geq 5 
\end{cases} \)

(f) In general, what can you say about the resulting \( y(t) \) at integer values of \( t \)? What about non-integer values of \( t \)?