Problem 1

(a) Sketch the graph of $x(t) = 2e^{-t/4}$.

(b) Calculate the value of $E_\infty$ and $P_\infty$ for the signal $x(t) = 2e^{-t/4}$.

Problem 2

For the signal $x(t)$ above, sketch $x \ast h$ when

(a) $h(t) = 0$

(b) $h(t) = \delta(t - 1)$

(c) $h(t) = \begin{cases} 
0 & t < 0 \\
1 & 0 < t < 1 \\
0 & t > 1
\end{cases}$

Problem 3

Let $C_h$ be the system that has output $x \ast h$ on input $x$. For each of the choices for $h$ below, determine if $C_h$ is invertible. If so give the signal $h'$ such that $C_{h'}$ is the inverse of $C_h$. If not
give two input signals that map to the same output signal.

(a) \( h[n] = 2\delta[n] \)

(b) \[
h[n] = \begin{cases} 
0 & n < 0 \\
1 & n = 0, 1 \\
0 & n > 1 
\end{cases}
\]

(c) \[
\begin{align*}
h[n] &= \begin{cases} 
0 & n < -1 \\
-1 & n = -1 \\
2 & n = 0 \\
-1 & n = 1 \\
0 & n > 1 
\end{cases} 
\end{align*}
\]

**Problem 4**

Determine if the following statements are true or false. Justify your answers.

(a) If \( x(t) \) and \( h(t) \) are odd signals, then \( y(t) = x(t) * h(t) \) is an even signal.

(b) If an LTI system has an impulse response \( h[n] \) of finite duration and amplitude, then the system is stable.

(c) For an LTI system, causal \( \Rightarrow \) stable.

**Problem 5**

Let \( h_1, h_2, h_3, h_4 \) be discrete signals such that \( h_1 * h_2 = \delta \) and \( h_3 * h_4 = \delta \).

Determine \( h_3 * ((h_2 * h_4) * h_1) \).

**Problem 6**

One important use of inverse systems is in situations in which one wishes to remove distortions of some type. A good example is the problem of removing echoes from acoustic
signals. For example, if a room has echo, then an initial acoustic impulse will be followed by attenuated versions of the sound at regularly spaced intervals. A model for this is an LTI system with impulse response given by a train of impulses

$$h(t) = \sum_{k=0}^{\infty} h_k \delta(t - kT)$$

Here the echoes occur $T$ seconds apart, and $h_k$ represents the gain factor on the $k$-th echo. We can think of $T$ and the $h_k$’s as parameters describing the acoustics of a room.

Suppose that $x(t)$ represents an original acoustic signal and that $y(t) = x(t) * h(t)$ is the signal recorded by a microphone. Note that $y(t)$ is the superposition of delayed versions of $x(t)$, scaled by the coefficients $h_k$.

The system defined by convolution with $h$ is invertible; therefore we can remove the echoes from $y(t)$ by convolving it with another signal $g(t)$. The signal $g(t)$ is also a train of impulses

$$g(t) = \sum_{k=0}^{\infty} g_k \delta(t - kT)$$

Suppose that $h_0 = 1, h_1 = 1/2$, and $h_i = 0$ for all $i \geq 2$. Determine the value for the $g_k$’s in this case.