Multi-Parameter Mechanism Design

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This paper serves as the foundation for a series of notes for the second half of a new Brown CS course on Mechanism Design. While the first half covered single-parameter,\textsuperscript{1} sealed-bid auctions, in the second half we focus on various multi-parameter settings; for each, we attempt to design welfare-maximizing, incentive compatible\textsuperscript{2} ascending auctions.\textsuperscript{3} In the sealed-bid, single parameter case, a result known as Myerson’s Lemma gives, in general form, a welfare-maximizing allocation as well as payments that incentivize truthful bidding. In the multi-parameter, ascending auction case, we find that we are able to satisfy many, though not all, of Myerson’s guarantees. We outline a general step-by-step process to prove that an ascending auction is welfare-maximizing and incentive compatible, and then tackle increasingly general auction settings to see how far this process can take us. Additionally, we address an open question by generalizing the demand of each bidder, multiplying it by an item-dependent value denoting that item’s ‘quality’.

\textsuperscript{1}In a single-parameter auction, only one type of good is auctioned off at a time.

\textsuperscript{2}Meaning that every bidder is incentivized to bid truthfully, i.e. exactly what they value the item at.

\textsuperscript{3}For example, the typical ‘English auction’, where bidders compete by naming increasingly high prices. Contrast this with the ‘sealed bid’ auction, where everyone submits their single bid on a slip of paper.