# The Arrow Distributed Directory Protocol

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**Abstract.** Most practical techniques for locating remote objects in a distributed system suffer from problems of scalability and locality of reference. We have devised the *Arrow distributed directory protocol*, a scalable and local mechanism for ensuring mutually exclusive access to mobile objects. This directory has communication complexity optimal within a factor of (1 + MST - stretch(G))/2, where MST - stretch(G) is the "minimum spanning tree stretch" of the underlying network.

# 1 Introduction

Many distributed systems support some concept of *mobile objects*. A mobile object could be a file, a process, or any other data structure. For an object to be mobile, we require only that it can be transmitted over a network from one node to another. A mobile object "lives" on only one node at a time, and it moves from one node to another in response to explicit requests by *client* nodes. A *directory* service allows nodes to keep track of mobile objects. A directory must provide the ability to locate a mobile object (*navigation*), as well as the ability to ensure mutual exclusion in the presence of concurrent requests (*synchronization*).

This paper describes the arrow distributed directory protocol, a novel directory protocol being implemented as part of the Aleph toolkit [10], an distributed shared object system currently under development at Brown. The arrow directory protocol is designed to avoid scalability problems inherent in many directory services currently used in distributed shared memory systems. In this paper, we focus on proving the correctness of the protocol, and on analyzing its communication complexity. The service's data structures and protocols are extremely simple, and yet its communication complexity compares well to more complicated asymptotic schemes in the literature.

# 2 Motivation

Perhaps the simplest way to implement a directory service is to have each node broadcast each access request, and await the response from the node currently holding the object. Indeed, this approach is common in bus-based multiprocessor systems (e.g., [9]), where broadcasting can be accomplished efficiently. In

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distributed systems, however, broadcast is impractical, so a directory structure must provide a way to navigate and synchronize by point-to-point communication. It is equally impractical to have each node store each object's current location, since all nodes must be notified when an object moves. Additionally, since requests occur simultaneously, maintaining consistency in such schemes becomes difficult.

The most common directory protocol used in existing distributed shared memory systems is a *home-based* structure. Each mobile object is associated with a fixed node, termed that object's "home". The home keeps track of the object's location and status (e.g., busy or idle). When a client node requests an object, it sends a message to the object's home. The home sends a message to the client currently holding the object, and that client forwards the object to the requesting client node. The home can also enforce the necessary synchronization, by queuing concurrent requests. Home-based schemes are simple and easy to implement, and they have been observed to work well for small-to-medium scale systems. Nevertheless, such schemes suffer from problems of scalability and locality. As the number of nodes grows, or if an object is a "hot spot", that object's home is likely to become a synchronization bottleneck, since it must mediate all access to that object. Moreover, if a client is far from an object's home, then it must incur the cost of communicating with the home, even if the node currently holding the object is nearby.

One way to alleviate these problems is to allow an object's home to move. For example, Li and Hudak [15] proposed a protocol in which each object is initially associated with a particular node, but as an object is moved around, it leaves a virtual trail of *forwarding pointers*, starting from the original home. A limitation of this approach is that many requests for an object may still go through the original home, or end up chasing an arbitrarily long sequence of pointers. Additionally, if the object is close but the home is far, the client may still have to incur the large communication costs.

Our approach is also based on the idea of a trail of pointers, although we use them in a different way. Objects have no fixed homes, and synchronization and navigation are integrated into a single simple protocol. When a client requests an object from another client, all messages are sent through direct or nearly-direct paths, preserving locality, and permitting us to give explicit worst-case bounds on the protocol's communication complexity.

# 3 The Directory Structure

In this section, we give an informal definition of the arrow directory protocol, together with examples illustrating the interesting aspects of its behavior. A more formal treatment appears in Section 4.

For brevity, we consider a directory that tracks the location of a single object. We model a distributed system in the usual way, as a connected graph G = (V, E), where |V| = n. Each vertex models a node, and each edge a two-way reliable communication link. A node can send messages directly to its neighbors, and indirectly to non-neighbors along a path. Each edge is weighted with a communication *cost*. The cost of sending a message from one node to another along a particular path is just the sum of that path's edge costs. The *distance*  $d_G(x, y)$  is the cost of the shortest path from x to y in G. The network is *asynchronous* (steps are interleaved in an arbitrary order) but *reliable* (every node eventually takes a step and every message is eventually delivered). We assume the network provides a *routing service* [7, 21] that allows node v to send a message to node u with cost  $d_G(u, v)$ .

The arrow directory is given by a minimum spanning tree T for G. Each node v stores a directory entry link(v), which is either a neighbor of v in T, or v itself. The meaning of the link is essentially the following: if link(v) = v, then the object either resides at v, or will soon reside at v. Otherwise, the object currently resides in the component of the spanning tree containing link(v). Informally, except for the node that currently holds an object, a node knows only in which "direction" that object lies. If T has maximum degree  $\Delta$ , then the directory requires  $n \cdot \log(\Delta)$  bits of memory to track the object (some techniques for memory reduction are discussed below).

The entire directory protocol can be described in a single paragraph. The directory tree is initialized so that following the links from any node leads to the node where the object resides. When a node v wants to acquire exclusive access to the object, it sends a find(v) message to  $u_1 = link(v)$  and sets link(v) to v. When node  $u_i$  receives a find(v) message from node  $u_{i-1}$ , where  $u_{i+1} = link(u_i)$ , it immediately "flips"  $link(u_i)$  to  $u_{i-1}$ . If  $u_{i+1} \neq u_i$ , then  $u_i$  forwards the message to  $u_{i+1}$ . Otherwise,  $u_i$  buffers the request until it is ready to release the object to v. Node  $u_i$  releases the object by sending a move(v) message containing the object directly to v, without further interaction with the directory.

Despite the protocol's simplicity, it has a number of non-trivial properties that we believe to be of both practical and theoretical interest. Before analyzing this behavior, it is helpful to review some simple examples. Figure 1 shows a properly initialized directory, where directed edges correspond to the directory links. The object resides at node u (indicated by a square). When node v requests the object, it sends a find(v) message to  $u_1 = link(v)$ , and then sets link(v) to v. When  $u_1$  receives the message from v, it forwards it to  $u_2 = link(u_1)$ , and sets  $link(u_1)$  to v. Continuing in this way, each vertex thus "flips" the link to point in the director from which the find(v) message arrived. Figure 2 illustrates the directory state after three steps. Links changed by the protocol are shown in gray: v's link points to itself,  $u_1$  points to v and  $u_2$  points to  $u_1$ .

As shown in Figure 3, the find(v) message continues to flip links until it arrives at u, where the object resides (marked by link(u) = u). When u receives the find(v) message, it responds by sending the object in a move((v) message directly to v. This response takes the shortest path, and does not affect the directory structure. As illustrated in Figure 4, after v has received the object, all links in the graph again lead to the object's new location, so any subsequent find messages will be directed there. The cost of acquiring access to the object is

$$d_T(u,v) + d_G(v,u).$$



**Fig. 2.** Three steps after u issues find(u) message

The term  $d_T(u, v)$  is the cost of sending the find(v) message from u to v through the directory tree, and the term  $d_G(v, u)$  is the cost of sending the move(v)message directly through the network. Notice the locality of this interaction: the message traffic affects only the nodes between u and v in the directory tree and network.

We now turn our attention to a concurrent example in which multiple nodes try to acquire the object at the same time. As described above, as a *find* message traverses the directory tree, it flips each link to point back to the neighbor from which the *find* originated. If two *find* messages are issued at about the same time, one will eventually cross the other's path, and be "diverted" away from the object and toward its competitor. Figure 2 illustrates this behavior. If any of the nodes on the right half of the tree requests the object, then its *find* message will follow the links to v, whereas nodes on the left half of the tree will follow the links to u.

For example, suppose w requests the object while v's find is still in progress.



Fig. 4. Move message received

This second *find* will be diverted to v. When v receives w's message, it will not respond until it is has received the object and is ready to release it. This directory state is illustrated in Figure 5, where w's request is blocked at v.

Now suppose that node z issues a *find* which arrives at u, where it is buffered (u is not finished with the object). The *find* message from v is then diverted to z, as illustrated in Figure 6. This example illustrates how the arrow directory integrates synchronization and navigation in a natural way. In a quiescent state (when no messages are in transit), the directory imposes a distributed queue structure on blocked *find* requests. When z completes its *find*, after flipping its links, it blocks at u. Similarly, v blocks at z, and w at v. These blocked requests create a distributed queue where each *find* is buffered at its predecessor's node. When u releases the object, it goes directly to z, then v, then w. This distributed queue structure is valuable from a practical perspective, for several reasons. First, it ensures that no single node becomes a synchronization bottleneck. Second, if there is high contention for an object, then each time that



Fig. 6. Deflected find

object is released, that node will already have a buffered *find* request. In the limit, the cost of delivering *find* messages is hidden by the local computation times, and the protocol's performance approaches optimal (repeated local computation followed by direct object delivery). Third, the queue structure ensures locality: each *find* message takes a direct path through the spanning tree to its predecessor's node, with no detour to a home node.

The communication complexity of the concurrent execution in which the object goes from u to z to v to w is

$$(d_T(z, u) + d_G(u, z)) + (d_T(v, z) + d_G(z, v)) + (d_T(w, v) + d_G(v, w)).$$

The first expression in each parenthesized term is the cost of sending the *find* message from each node to its predecessor via the directory tree, and the second is the cost of sending the *move* message directly from each node to its successor. There are two important points to notice about this communication cost. First, there is no synchronization overhead: the communication cost is the same as a

serial execution in which each node issues its find only after its predecessor is ready to release the object. Second, the inherent cost of the transfer from u to z is  $d_G(z, u) + d_G(u, z)$ , the inherent cost of delivering matching find and move messages. The arrow directory replaces the first term with  $d_T(z, u)$ , suggesting that the navigation cost of the arrow directory approaches optimality to the degree that  $d_T(z, u)$  approaches  $d_G(z, u)$ . We elaborate on these observations in the next section.

### 4 Analysis

In this section, we describe the algorithm using a simplified version of the I/O automaton formalism of Lynch and Tuttle [17], we sketch safety and liveness issues, and evaluate the algorithm's cost.

#### 4.1 Description

Recall that the network is a connected graph G = (V, E), where |V| = n. There are two kinds of messages: node v issues a find(v) message to request access to an object, and move(u) to transfer ownership to u. Each node v has following attributes: link(v) is a node, queue(v) is either a node or  $\bot$ , and owner(v) is a boolean. Each pair of nodes (u, v) has a an associated pending(u, v), which is a set of messages that have been sent but not delivered (i.e., in transit, or queue at the sender or receiver).

A node v or link link(v) is terminal if link(v) = v. The directory is initialized so that there is exactly one node  $v_0$  such that  $v_0$  is terminal and  $owner(v_0)$  is true, the remaining non-terminal *link* pointers form an oriented tree, and all *pending*(u, v) sets are empty. Message delivery is reliable, but not necessarily FIFO. A node initiates a *find* operation by sending a *find* message to itself For brevity, we assume that a node w has only one find(w) message in the network at a time.

We use T to denote the undirected tree, and L to denote the directed graph induced by non-terminal links. The algorithm has the property that T does not change, but L does. The algorithm is defined by a number of transitions. The first part of each transition is a precondition that must be satisfied for the transition to occur, and the second part describes the state of the directory after the transition. A primed component indicates how a component's state changes after the transition.

Several transitions require a node atomically to remove a message from a network link, undergo an internal transition, and insert the message in another link. In practice, this atomicity is realized simply by requiring each node to finish processing one message before it starts processing the next.

The first transition says that when find(w) is sent from node u to a nonterminal node v, v flips its *link* pointer to u and forwards the message to its old link.

```
\begin{array}{lll} \text{pre:} & find(w) \in pending(u,v) \\ & link(v) \neq v \\ \text{post:} & pending'(u,v) = pending(u,v) - \{find(w)\} \\ & pending'(v,link(v)) = pending(v,link(v)) \cup \{find(w)\} \\ & link'(v) = u \end{array}
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If, however, v is a terminal node, then w is enqueued behind v.

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pre: find(w) \in pending(u, v)

link(v) = v

post: pending'(u, v) = pending(u, v) - \{find(w)\}

link'(v) = u

queue'(v) = w
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We do not explicitly model whether the object is actively in use. Instead, we treat the owner's receipt of a *find* and its corresponding *move* as distinct events, separated by an arbitrary but finite delay. If the current owner's queue is non-empty, then it may relinquish ownership and move the object directly to the waiting node.

 $\begin{array}{ll} \text{pre:} & owner(v) = true \\ & queue(v) \neq \bot \\ \text{post:} & pending'(v, queue(v)) = pending(v, queue(v)) \cup \{move(queue(v))\} \\ & owner'(v) = false \\ & queue'(v) = \bot \end{array}$ 

When a waiting node receives the object, that node becomes the new owner.

pre:  $move(v) \in pending(u, v)$ post: owner'(v) = true $pending'(u, v) = pending(u, v) - \{move(v)\}$ 

#### 4.2 Safety

Our proof of the directory's safety properties is an invariance argument. At all times, we show that there exists a well-defined *path* in L from any node that leads either to the object's current owner, or to another node that has requested the object. Because *find* messages follow these paths, any message that reaches the end of a path will either acquire ownership of the object, or join the queue waiting for the object.

First, we need to check that such paths exist.

**Lemma 1.** The following property is invariant:

$$find(w) \in pending(u, v) \Rightarrow link(u) \neq v \land link(v) \neq u.$$
(1)

*Proof.* Initially, the property holds vacuously, because no messages are in transit. Consider the first transition to violate the property. There are two cases to consider. In the first case, suppose the directory enters a state where

$$find(w) \in pending'(u, v) \land link'(u) = v$$

Immediately before this transition, we claim that

$$find(w) \in pending(v, u) \land link(u) = v,$$

so the property was already violated. First, find(w) is in pending(v, u), because otherwise link'(u) would not be set to v. Second, link(u) must be v, because otherwise the message would not be forwarded to v.

In the second case, suppose the directory enters a state where

 $find(w) \in pending'(u, v) \land link'(v) = u.$ 

Immediately before this transition, we claim that

$$find(w) \in pending(v, u) \land link(v) = u,$$

so the property was already violated. First, find(w) is in pending(v, u), because otherwise link'(u) would not be set to v. Second, link(v) must be u, because the transition does not change link(v).

**Lemma 2.** The directed graph L induced by the non-terminal links is always acyclic.

*Proof.* Because T is a tree, it suffices to show the directory cannot enter a state where link(u) = v and link(v) = u for distinct nodes u and v. Consider the first transition that sets link'(v) = u while link(u) = v. Any such transition is enabled only if  $find(w) \in pending(u, v)$  and link(u) = v, which is impossible by Lemma 1.

**Definition 3.** The *path* from a node is the directed path in L from that node to a terminal node.

**Definition 4.** The *target* of v, denoted target(v), is the terminal node at the end of its path. If find(w) is a message in pending(u, v), then that message's target is target(v).

Lemma 2 guarantees that these notions are well-defined. A node is a *waiter* if it has requested an object, but has not yet become the owner of that object.

We are now ready to prove our principal safety result:

**Theorem 5.** The path from any node always leads either to the current owner or a waiter.

*Proof.* Any node v partitions the set of nodes G into  $A_v$  and  $B_v$ , where  $A_v$  consists of nodes whose paths include v, and  $B_v$  consists of the rest. For every u in  $A_v$ , target(u) = target(v).

Consider the directory state immediately before a transition "flipping" link(v)to u. By Lemma 2,  $u \in B_v$ . After the transition, for every w in  $B_v$ , target'(w) = target(w), so target(w) remains an owner or a waiter. For every z in  $A_v$  (including v itself), target'(z) = target'(u) = target(u), so target(z) remains an owner or a waiter.

#### 4.3 Liveness

Liveness is also based on invariance arguments. Liveness requires that transitions occur fairly: any transition whose precondition remains true will eventually occur. (For example, any message in pending(u, v) will eventually be removed.) We will show that when a find message is in transit, that message "traps" its target in a component  $L_1$  of the link graph L: other find and move messages can move the target within  $L_1$ , but the target cannot leave  $L_1$ . Each transition that advances the find message shrinks  $L_1$  by at least one node, so after at most n advances,  $L_1$  consists of a single node, and the find message must reach its target.

**Theorem 6.** Each find(w) message will be delivered to its target in n steps or fewer.

*Proof.* Suppose find(w) is in pending(u, v). Deleting the edge (u, v) from the undirected tree T splits T into two disjoint trees, before(w) (containing u) and after(w) (containing v). Lemma 1 states that as long as find(w) is in pending(u, v), then  $link(v) \neq u$  and  $link(u) \neq v$ , so the message's target will remain in after(w). (Other find requests may still move the target within after(w).)

If a transition delivers find(w) to its target, we are done. A transition that moves the message from pending(u, v) to pending'(v, link(v)) induces a new partition before'(w) and after'(w), where  $v \notin after'(w)$ . Because each after'(w) is strictly smaller than after(w), the find(w) message will arrive at its target after at most n steps.

#### 4.4 Complexity

How should we evaluate the communication cost of this protocol? Following Peleg [20], it is natural to compare our communication complexity to that of an *optimal* directory for which synchronization and navigation are free. The *optimal* directory accepts only serial schedules (so it pays nothing for synchronization), and delivers each *find* and *move* message directly (so it pays nothing for navigation).

An execution is a sequence of atomic node steps, message sends, and message receipts. An execution is *complete* if every *find* message has a matching *move*, and an execution is *serial* if the receipt of the *i*-th *move* message precedes the sending of each (i + 1)-st *find*. The next lemma states that we can restrict our attention to complete serial executions.

**Lemma 7.** If E is any complete concurrent execution of the arrow directory protocol, then there exists a complete serial execution E' that serializes requests in the same order, sends the same messages, and leaves the directory in the same state.

Define the MST-stretch of G, denoted MST-stretch (G), to be

 $\max_{u,v\in V} \frac{d_T(u,v)}{d_G(u,v)}.$ 

This quantity measures how far from optimal a path through the minimum spanning tree can be. The MST-stretch can be n in the worst case (a ring), but is likely to be much smaller in realistic networks (such as LANs, or the Internet).

Consider a serial execution in which  $v_0, \ldots, v_\ell$  successively acquire the object. In the arrow directory, each find message traverses the spanning tree T, with cost  $d_T(v_i, v_{i+1})$ , while in the optimal directory, the message goes directly from  $v_i$  to  $v_{i+1}$ , with cost  $d_G(v_i, v_{i+1})$ . The ratio of these quantities is no more than MST-stretch(G). In both the arrow and optimal directories, the move message incurs cost  $d_G(v_i, v_{i+1})$ , so this ratio is 1. (From a practical perspective, it is worth emphasizing that we are not hiding any constants: these bounds really are MST-stretch(G) and 1, not O(MST-stretch(G)) or O(1).)

As a result, for any complete serial execution, the ratio of communication costs for the arrow directory and the optimal directory is bounded by

$$\frac{1 + MST - stretch(G)}{2}.$$

The lower the network's MST-stretch, the closer the communication cost of the arrow directory is to optimal.

# 5 Discussion

Our discussion so far has focused on the safety, liveness, and communication cost of an idealized directory scheme. We now briefly address other issues of practical interest.

One sensible way to reduce the directory's memory consumption and message traffic is to organize the directory as a two-level structure. The network is partitioned into neighborhoods of small diameter, where the arrow directory is maintained at a per-neighborhood granularity, and a home-based directory is used within each individual neighborhood. (For example, the neighborhood could be a local subnetwork, and the node its gateway.) The resulting hybrid scheme trades communication cost against memory consumption and directory tree traffic. As discussed below, Peleg [20] and Awerbuch and Peleg [3] have also proposed multi-level directory structures.

Note that only *find* messages go through the directory tree — the network's routing service handles all other traffic, such as *move* messages, remote procedure calls, etc. *Find* messages for different objects that arrive together can be combined before forwarding.

We have assumed for simplicity that every node is initialized with a directory entry for each object. It is more practical to initialize an object's directory links in the following "lazy" manner. Before a node can request an object, it must first acquire that object's name. When a node u receives a message from vcontaining the name of an unfamiliar object, it sends a message through the directory tree to v initializing the intermediate missing links. This lazy strategy preserves locality: if all references to an object are localized within the directory tree, then the other nodes need never create a directory entry for that object. So far, we have considered only exclusive access to objects. Many applications would benefit from support for shared (read-only) access as well. We now outline a simple extension of the arrow directory protocol to support read-only replication. When multiple read-only copies exist, one is designated the *primary*. Each directory entry consists of a *primary link* pointing toward the primary copy, and a set of *secondary links*, each pointing toward one or more copies. When a node requests a read-only copy of an object, it follows any secondary link from each vertex, creating a new secondary link pointing back to itself at each node it visits. When a node requests exclusive access to the object, it first follows the primary links, flipping each primary link toward itself. When the node has acquired exclusive access to the primary, it *invalidates* the read-only copies by following, flipping, and consolidating the secondary links. When all invalidations are complete, the node has acquired the object. This protocol is *linearizable* [11]. More efficient protocols might be achieved by replacing linearizability with weaker notions of correctness [1].

We have not addressed the issue of fault-tolerance. Distributed mutual exclusion algorithms address the problem by "token regeneration", effectively electing a node to hold the new token. Here, the problem is different, since we cannot elect a new object, but we do need to recover navigational information that may be lost following a crash.

### 6 Related Work

Distributed directory management is closely related to the problem of distributed mutual exclusion. Naïmi, Tréhel, and Arnold [18] describe a distributed mutual exclusion algorithm (the *NTA* algorithm) also based on a dynamically changing distributed directed graph. In the NTA algorithm, when a node receives a message, it flips its edge to point to the node from which the request originated. In our algorithm, however, the node flips its edge to point to the immediate source of the message. As a result, our graph is always a directed subgraph of a fixed minimal spanning tree, while the NTA graph is more fluid.

The safety and liveness arguments for NTA and for our algorithm are similar, (though not identical), but the complexity models are quite different. The complexity analysis given in [18] is based on a model in which the underlying network is a clique, with unit cost to send a message from any node to any other. All nodes are assumed equally likely to request the token, and requests are assumed to occur sequentially. Under these assumptions, NTA shows that the average number of messages sent over a long period converges to  $O(\log n)$ . (Ginat [8] gives an amortized analysis of the NTA algorithm and some variants.) Our analysis, by contrast, models the underlying network as a weighted graph, where the end-to-end cost of sending a message is the sum of weights along its path. Instead of taking an average over a presumed uniform distribution, we compete against a directory with perfect instantaneous knowledge of the object's current location and no synchronization costs. The complexity analysis of Naïmi *et al.* does not apply to our algorithm, nor ours to theirs. The NTA complexity model is appropriate for the general problem of distributed mutual exclusion, while ours is intended for the more specific problem of managing distributed directories in the Internet. Each link in our spanning tree is intended to represent a direct network link between routers, which is why our algorithm does not flip edges to point to arbitrarily distant network nodes (even though the routing service renders any pair of nodes "adjacent" for communication). The NTA complexity model, while mathematically quite elegant, does not seem well-suited for distributed Internet directory management, because point-to-point communication cost is not uniform in real networks, and access requests to distributed objects are typically not generated uniformly at random, but often have strong locality properties. For these reasons, a competitive complexity model seems more appropriate for our purposes.

Another mutual exclusion protocol employing path reversal is attributed to Schönhage by Lynch and Tuttle [16]. In this protocol, the directory is a directed acyclic graph. *Users* residing at leaf nodes request the object, and requests are granted by *arbiters* residing at internal nodes. Edges change labels and orientation to reflect the object's current status and location.

The arrow directory protocol was motivated by emerging *active network* technology [14], in which programmable network switches are used to implement customized protocols, such as application-specific packet routing. Active networks are intended to ensure that the cost of routing messages through the directory tree is comparable to the cost of routing messages directly through the network.

Small-scale multiprocessors (e.g., [9]) typically rely on broadcast-based protocols to locate objects in a distributed system of caches. Existing large-scale multiprocessors and existing distributed shared memory systems are either homebased, or use a combination of home-based and forwarding pointers [4, 5, 6, 12, 13, 15, 19].

Plaxton et al. [22] give a randomized directory scheme for read-only objects. Peleg [20] and Awerbuch and Peleg [3] describe directory services organized as a hierarchical system of subdirectories based on sparse network covers [2]. The earlier paper [20] proposed the notion of *stretch* to evaluate the performance of distributed directory services. In the worst case, these directories use per-object memory logarithmic in n, while ours is linear. (As described above, we believe we can avoid worst-case memory consumption in practice.) Strictly speaking, the asymptotic communication complexities are incomparable. Their notion of an optimal concurrent find execution is less conservative than ours, but for simplicity we will treat them as equivalent. The ratio of their concurrent find and move operations to their notion of an optimal protocol is polylogarithmic in n, while the ratio of our concurrent find and move operations to our (more conservative) notion of an optimal protocol is  $(1 + MST \cdot stretch(G))/2$ . The arrow directory has lower asymptotic communication complexity when the MST-stretch of the network is less than some polylogarithm in n. Nevertheless, we feel the most important distinction is that the arrow directory protocol is much simpler than any of the hierarchical protocols, because it is explicitly designed to be implemented. We plan to report experimental results in the near future.

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