Enhanced Approach Towards Network Reliability Using Boolean Algebra

Alex Balan

Lafayette College, Easton PA

Motivation

- Computers are often connected to networks of various sizes
- Distributed Computer Systems (DCS) require reliable inter-computer communication
- Performance of such a network can be measured by determining the reliability of the overall network

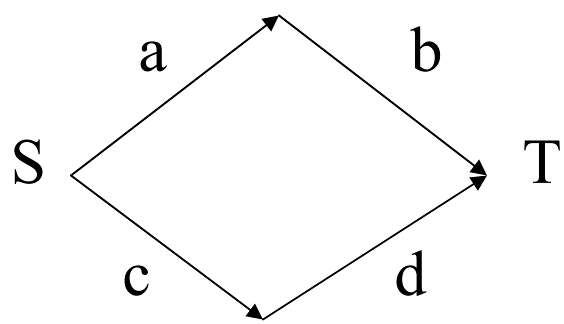
Problem

- The reliability analysis problem consists of determining the probability that a stochastic binary system operates
  - Example: a network for which each component/edge has an associated probability of working
- A coherent binary system is given by the set of minpaths P_i
  - Example: the set of minimal paths between two nodes whose functionality is sufficient for communication

Approach

- Use Boolean algebra, and in particular an SDP (sum of disjoint products) algorithm
- P_i can be viewed as a probabilistic event
  - Example: the probability that path P_i is working
- Make the events P_i disjoint and then add their corresponding probabilities to obtain the reliability of the entire system

Example of a simple network



- P_1 = a b, representing the event a ∧ b
- P_2 = c d, representing the event c ∧ d

Background

- A very simple method of making the events P_1, P_2, ..., P_n disjoint:
  - P_1, $\neg P_1 P_2$, $\neg P_1 \neg P_2 P_3$, ..., $\neg P_1 \neg P_2 \neg \ldots \neg P_{n-1} P_n$, where $\neg P_i$ means that P_i is failing
  - The order in which the minpaths P_i are listed can and does have an impact on the performance of any algorithm that is being used because this non-symmetrical pattern

Existing techniques

- SVI (single-variable inversions) [1]
  - to make two products disjoint, individual variables are negated
  - Eg: $\neg P_1 \, P_2 = \neg a \, c \, d + a \, \neg b \, c \, d$
- MVI (multiple-variable inversions) [2]
  - groups of variables can be negated together (disjunction of negations)
  - Eg: $\neg P_1 \, P_2 = \neg(a \, b) \, c \, d$

Proposed Approach

- GSDP (generalized sum of disjoint products)
  - Variables may be grouped together with any kind of Boolean expressions inside
  - No variable appears more than once in a product, to preserve the statistical independence of the groups
- Assumptions
  - Component states are mutually statistically independent
  - Each component of the network is either operational or failing

Example

$$P_1 = a\ b\ c\ x; \quad P_2 = a\ c\ d\ y; \quad P_3 = x\ y\ z$$

SVI: $\neg P_1 \neg P_2 P_3 = \neg a\ x\ y\ z + a \neg c\ x\ y\ z + a\ c \neg b \neg d\ x\ y\ z$

- 3 terms

MVI: $\neg P_1 \neg P_2 P_3 = \neg(a\ c)\ x\ y\ z + a\ c \neg b \neg d\ x\ y\ z$

- 2 terms

GSDP: $\neg P_1 \neg P_2 P_3 = x\ y\ z\ (\neg(a\ c) \vee (\neg b \neg d))$

- 1 term

Results and Discussion

- The Union of Products Problem (UPP), is NP-hard
- Two measures will be used to analyze the quality of the algorithm:
  - *computational efficiency*: practical run-time
  - resulting *number of disjoint products*: easy numerical evaluation and reduced computation and rounding errors.
- It is expected that this will reduce the size of the resulting reliability formula because several MVI terms can be combined into one GSDP term.

References

[1] J.A. Abraham, "An Improved Algorithm for Network Reliability", IEEE Trans. Reliability, vol R-28, no. 1 1979 Apr, pp 58-61.

[2] K.D. Heidtmann, "Smaller Sums of Disjoint Products by Subproduct Inversion", IEEE Trans. Reliability, vol. 38, no. 3, 1989 Aug, pp 305-311.