limits of computation: homework 7 solutions

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march 13th, 2001

problem 9.11

show that the language maxclique=\{⟨G, k⟩| the largest clique of G has k vertices\} is in np^sat.

• a graph G has a maximum clique of size k if and only if it has a clique of size k and it does not have a clique of size k + 1.

• the language clique is in np, and the language overline{clique} is in conp. since np ∈ psat and conp ∈ psat, maxclique is in psat.

• since psat ⊆ np^sat, maxclique is in np^sat.

problem 9.13

prove that for time constructible functions t(n), if a language A is recognized in time O(t(n)), then A is decided in time O(t(n)log t(n)).

• suppose a machine M recognizes a language A in a time bound t(n), where t(n) is time computable. the following machine decides A.
  on input “w”,
  1. compute the time bound t(n), and record the bound as a counter on one “track” of the tape.
  2. simulate M, but between each step, decrement the bound, and copy the entire counter one step to the right or left, as in the proof of the time heirarchy theorem. if the counter hits zero and the machine has not yet halted, reject.

• since t(n) is time constructible, step 1 of the above algorithm runs in O(t(n)) time.

• the maximum amount of time taken for step 2 is the number of steps of M times the size of the counter, plus the total number of steps taken to decrement the counter. since the counter could have at most O(log t(n)) bits, and the total cost of decrementing the counter t(n) times is O(t(n)), the total number of steps is at most O(t(n)log t(n) + t(n)) = O(t(n)log t(n)).