CSCI 1951-G – Optimization Methods in Finance
Part 00:
Course Logistics
Introduction to Finance Optimization Problems

January 26, 2018
All information is available in the syllabus

**Instructor:** Matteo Riondato

**TA:** Won Jun “June” Kang

**Time and Place:** Fri, 2–4.30pm (with 15 mins break), CIT 316

**Office Hours:** Tue, 4–6pm, CIT 207

**Website:** [https://cs.brown.edu/courses/cs1951g/](https://cs.brown.edu/courses/cs1951g/)
- Syllabus, Diary, Slides, Assignments, Solutions, ...

**Mailing list:** cs1951g.2017-18.s@lists.cs.brown.edu
- You will be subscribed if you are enrolled into banner (?)

**Write to the staff:** cs1951gtas@cs.brown.edu
Help us help you!

Active participation → **inclusive and productive** environment for learning … and teaching.

**What can you do**

**Ask lots of questions:** we all will learn more  
**Ask to explain again:** we all will learn more

This is the **second time** the course is offered.

**What can you do**

**Help polishing the materials:** future students will learn more  
**Be patient:** we are all doing our best
Course goals

Direct goals

• Learn about *different classes of optimization problems*, and the *theory and algorithms* for solving them efficiently.
• Become familiar with *financial instruments* and problems from *computational finance*, and understand how to solve them using *optimization*.
• Learn how to use popular optimization software packages and modeling languages.

At a higher level, develop

• algorithmic intuition
• mathematical and scientific writing skills
• theory-to-practice transfer skills.
Preliminary list of topics and schedule

**Linear Programming:** duality, geometry of optimal solution, the simplex and the dual simplex algorithm, short-term financing and asset pricing. **Weeks 1 to 4**

**Integer Programming:** mixed integer linear programming, branch and bound, cutting planes, constructing an index fund. **Week 4 to 6**

**Convex Optimization:** Newton’s method, steepest descent, stochastic gradient descent, generalized reduced gradient, volatility estimation. **Week 6 to 8**

**Quadratic Programming:** Interior point methods, portfolio optimization: mean-variance optimization and maximizing the Sharpe ratio. **Week 8 to 10**

**Non-convex optimization:** optimizing difference of convex functions.

**Stochastic Programming:** Two stage problems, risk measures, asset/liability management. **Week 12**

**Robust Optimization:** Uncertainty sets, robust portfolio selection. **Week 13**


**Additional readings:** a small number of additional notes will be posted on the course website.
## Assessments and grading

### Weekly Homework Assignments
- On website after class, due the following Friday, before class.
- “Theoretical” and programming exercises.
- Collaboration and late assignment policies in syllabus.
- Midterm is non-collaborative homework assignment.
- Final is take-home, non-collaborative.

### Grading
- We care about details: more detail in your answer is better than less, but too much is bad.
- Correctness of code is more important than performances, but code that is too slow is bad.
- Final course grade is **weighted**:
  - Homework assignments (including midterm): 60%
  - Final: 40%
Why “in finance”?

Modern finance is a mathematical and computational discipline.

Goal

Given my current wealth, maximize my future wealth.

Formalization requires a mathematical definitions of market models, financial instruments, constraints, risk measures, …

Computational challenges due to:

- the scale of modern investment strategies; and
- the speed of the markets; and
- the inherent complexity of some formalizations of the goal.
Workflow for performing financial investments

1. Formulate a *parametric mathematical model* of the prices of instruments and how they relate to each other, as functions of the parameters;

2. Use historical data to *estimate the parameters of the model* (statistics / machine learning);

3. *Optimize the model* given your goals/.constraints;

4. Analyze the optimal solution and, if it is *cost-effective*, perform the investments.
Some terminology

**Asset:** anything, tangible or intangible, with positive economic value. E.g., your car (if not too old).

**Liabilities:** the amounts you own to others. E.g., the balance on your credit card.

**Equity:** The difference between your assets and your liabilities.

**Security:** a tradeable financial asset.

**Stock/share:** a fraction of the equity of an entity.

**Bonds:** a debt security. The issuer of the bond owes money to the holder of the bond, and must pay them interest at a specific date.

**Return:** the profit of an investment in securities.

**Portfolio:** a collection of investments you own.

**Cash flow:** when you receive a payment or pay something, there is a positive or negative cash flow in your wallet.
Example 1: Portfolio optimization


**Settings**

- $ to be invested in multiple securities with *random returns*.
- We “know” *expectation* and *variance* of return of each security.
- We “know” the correlation coefficient of the returns of each pair of securities.

**Task (Mean-variance optimization)**

Use $ to create a portfolio s.t.:

- its *expected* return *exceeds some fixed minimal value*;
- *variance* of its return is *minimized*.

Can be extended to include regulations, investment preferences, short sales, etc.
Optimization problems

**Optimization:** maximizing a *function of multiple variables* under a set of *constraints.*

**Optimization problem**

Given $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and $S \subseteq \mathbb{R}^n$, find $x^* \in \mathbb{R}^n$ that solves

$$
\min f(x)
$$

s.t. $x \in S$

$f$ is the *objective function*, $S$ the *feasible region*, and the components of $x$ are the *decision variables*.

A *feasible assignment* is any $x \in S.$
Optimization problems (cont.)

**Constraints**

*S* is specified through functional *constraints* on the variables:

\[ S = \{ x : g(x) = 0, g \in E, \text{and } h(x) \geq 0, h \in I \} \]

**Domains**

Each decision variable \( x_i \) may be further restricted to a domain \( D_i \) (e.g., \( D_i = \mathbb{Z} \) or \( D_i = \{0, 1\} \)).

Hence, we write a generic optimization problem as:

**Optimization problem – general form**

\[
\begin{align*}
\min & \quad f(x_1, \ldots, x_n) \\
& \quad g(x_1, \ldots, x_n) = 0, g \in E \\
& \quad h(x_1, \ldots, x_n) \geq 0, h \in I \\
& \quad x_i \in D_i, 1 \leq i \leq n
\end{align*}
\]
Example 2: asset/liability cash-flow matching

Corporations must be able to finance their short-term cash commitments (e.g., payrolls, taxes, ...) or face bankruptcy.

Task

Given

- *cash flow requirements* over a time period; and
- available *sources of funds*,

determine *how to use the sources over the time period* so that

- the cash flow requirements are always *satisfied*; and
- the wealth at the end of the time period is *maximized*. 
## Cash flow scenario (amounts in thousands of USD)

<table>
<thead>
<tr>
<th>Month</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net Cash Flow</td>
<td>-150</td>
<td>-100</td>
<td>200</td>
<td>-200</td>
<td>50</td>
<td>300</td>
</tr>
</tbody>
</table>

## Sources of funds

- a line of credit up to $100k, at an interest rate of 1% per month;
- in any of the first three months, issue 90-day commercial paper bearing a total interest of 2% for the three-month period;
- invest excess funds at an interest rate of 0.3% per month.

## Task

For each month (Jan to June), decide how to use each source of funds to satisfy the cash flow requirements (including any additional liability due to the use of the sources), and maximize the wealth at the end of June.
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How to *model* this task as an optimization problem?

We need:

- decision variables and their domains;
- objective function;
- equality and inequality constraints.
Decision variables and domains

- \( c_i \in [0, 100] \): amount from credit line in month \( i \), \( 1 \leq i \leq 5 \)
  balance on the credit line, not incremental borrowing;
- \( p_i \geq 0 \): amount of commercial paper issued in month \( i \), \( 1 \leq i \leq 3 \);
- \( e_i \geq 0 \): excess funds in month \( i \), \( 1 \leq i \leq 5 \)
- \( w \in \mathbb{R} \): wealth at the end of June

Objective

Maximize \( w \).
Constraints

**January:** We can borrow $c_1$ from the line of credit and issue $p_1$ commercial paper. After satisfying the cash requirement, we may have $e_1$ excess funds. It must hold
\[ c_1 + p_1 - 150 = e_1, \text{ i.e., } c_1 + p_1 - e_1 = 150. \]

**February:** In addition to what we could do in January,
- we must pay the interest of 1% on the amount $c_1$ we borrowed in January from the line of credit;
- we receive the 0.03% on the January excess funds $e_1$;
It must hold $c_2 + p_2 + 1.003e_1 - 100 - 1.01c_1 = e_2$, i.e.,
\[ c_2 + p_2 + 1.003e_1 - 1.01c_1 - e_2 = 100. \]

**March:** Similarly as in February, it must hold:
\[ c_3 + p_3 + 1.003e_2 - 1.01c_2 - e_3 = -200. \]
Constraints (cont.)

**April:** We can no longer issue commercial paper, rather we must pay the 2% interest on the amount $p_1$ of commercial paper issued in January. It must hold:

$$c_4 - 1.02p_1 - 1.01c_3 + 1.003e_3 - e_4 = 200 .$$

**May:** Similarly to April, we must have

$$c_5 - 1.02p_2 - 1.01c_4 + 1.003e_4 - e_5 = -50 .$$

**June:** We can no longer request credit, and the excess funds at the end of June are the wealth $w$. It must hold

$$-1.02p_3 - 1.01c_5 + 1.003e_5 - w = -300 .$$
The complete formulation of our optimization problem is:

\[
\begin{align*}
\text{max } w \\
c_1 + p_1 - e_1 &= 150 \\
c_2 + p_2 + 1.003e_1 - 1.01c_1 - e_2 &= 100 \\
c_3 + p_3 + 1.003e_2 - 1.01c_2 - e_3 &= -200 \\
c_4 - 1.02p_1 - 1.01c_3 + 1.003e_3 - e_4 &= 200 \\
c_5 - 1.02p_2 - 1.01c_4 + 1.003e_4 - e_5 &= -50 \\
-1.02p_3 - 1.01c_5 + 1.003e_5 - w &= -300 \\
0 \leq c_i \leq 100, & 1 \leq i \leq 5 \\
p_i > 0, & 1 \leq i \leq 3 \\
e_i \geq 0, & 1 \leq i \leq 5
\end{align*}
\]
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Modeling the portfolio optimization problem

Settings:
- For each security $i, 1 \leq i \leq n$ we know the expectation $\mu_i$ and the variance $\sigma_i^2$ of its return.
- For each pair $(i, j)$ of securities, we know the correlation coefficient $\rho_{ij}$ of their returns ($\rho_{ii} = 1$).
- Let $\mu$ be the vector of the $\mu_i$, and let $Q$ be the $n \times n$ matrix with entries $Q_{ij} = \rho_{ij} \sigma_i \sigma_j$.
- Minimal expected return $R$

Decision variables: $x_i \geq 0, 1 \leq i \leq n$, denoting the fraction of $\$ allocated to security $i$.

Constraints:
- Allocate all $\$: $e^T x = 1$ where $e$ is vector of all 1 in $\mathbb{R}^n$.
- The expected return of the portfolio must be at least $R$. Using linearity of expectation, we want $\mu^T x \geq R$.

Objective: minimize the variance of the portfolio, $x^T Q x$. 
Modeling the portfolio optimization problem (cont.)

Formulation

\[
\begin{align*}
\min & \quad x^T Q x \\
\text{subject to} & \quad e^T x = 1 \\
& \quad \mu^T x \geq R \\
& \quad x \geq 0
\end{align*}
\]
Classes of optimization problems

Optimization problems are classified depending on:

- domains of the variables (e.g., continuous vs. discrete)
- types of constraints (e.g., linear, non-linear)
- the type of objective function (e.g., linear, non-linear, quadratic, convex, ...)

Example

The Cash Flow Matching problem has real variables, linear constraints, and a linear objective function: it is a Linear Programming (LP) problem.

Example

The Portfolio Optimization problem has real variables, linear constraints, and a quadratic objective function: it is a Quadratic Programming (QP) problem.

Different classes of optimization problems have different computational complexity (LP and QP are polynomial).