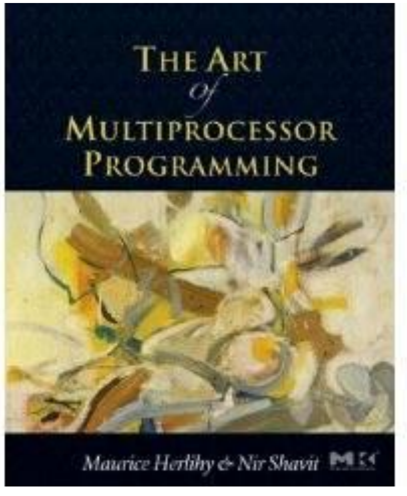
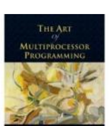
Concurrent Skip Lists



Companion slides for
The Art of Multiprocessor Programming
by Maurice Herlihy & Nir Shavit

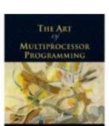
Set Object Interface

- Collection of elements
- No duplicates
- Methods
  - – add() a new element
  - – remove() an element
  - – contains() if element is present



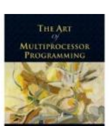
Many are Cold but Few are Frozen

- Typically high % of contains() calls
- Many fewer add() calls
- And even fewer remove() calls
  - 90% contains()
  - 9% add()
  - 1% remove()
- Folklore?
  - Yes but probably mostly true



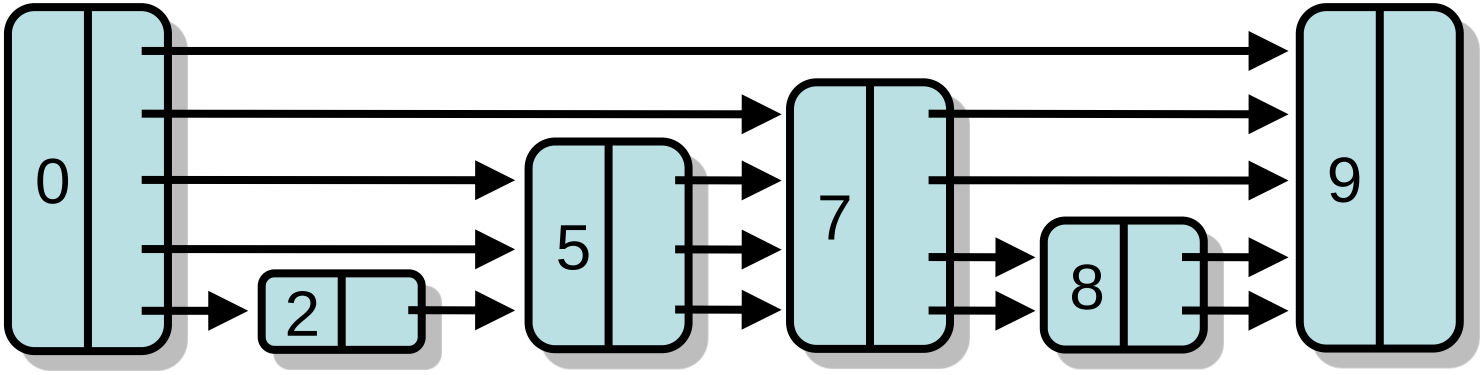
Concurrent Sets

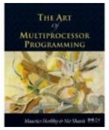
- Balanced Trees?
  - Red-Black trees, AVL trees, …
- Problem: no one does this well …
- … because **rebalancing** after add() or remove() is a global operation



Skip Lists

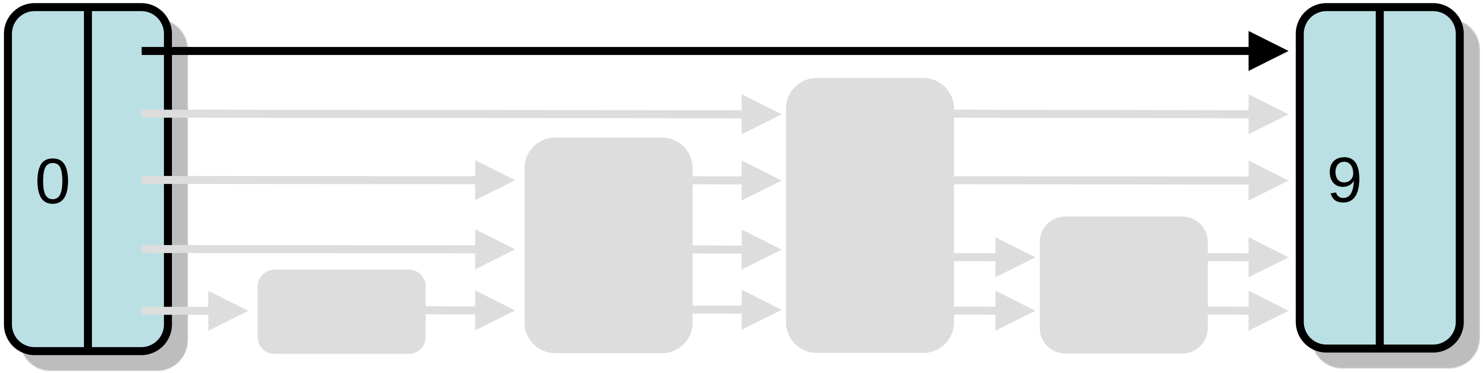
- Probabilistic Data Structure
- No global rebalancing
- Logarithmic-time search

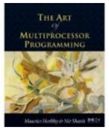




Skip List Property

- Each layer is sub-list of lower levels





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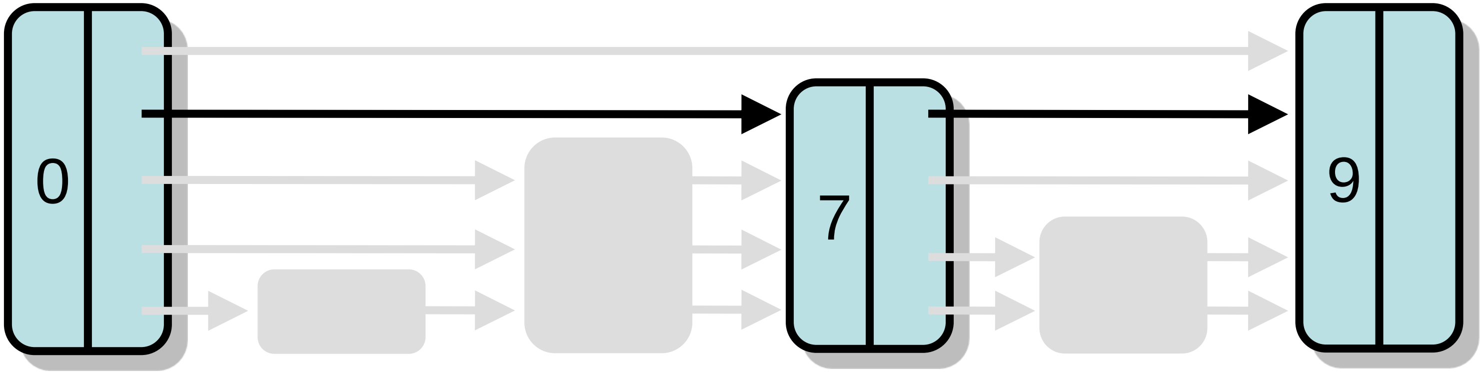

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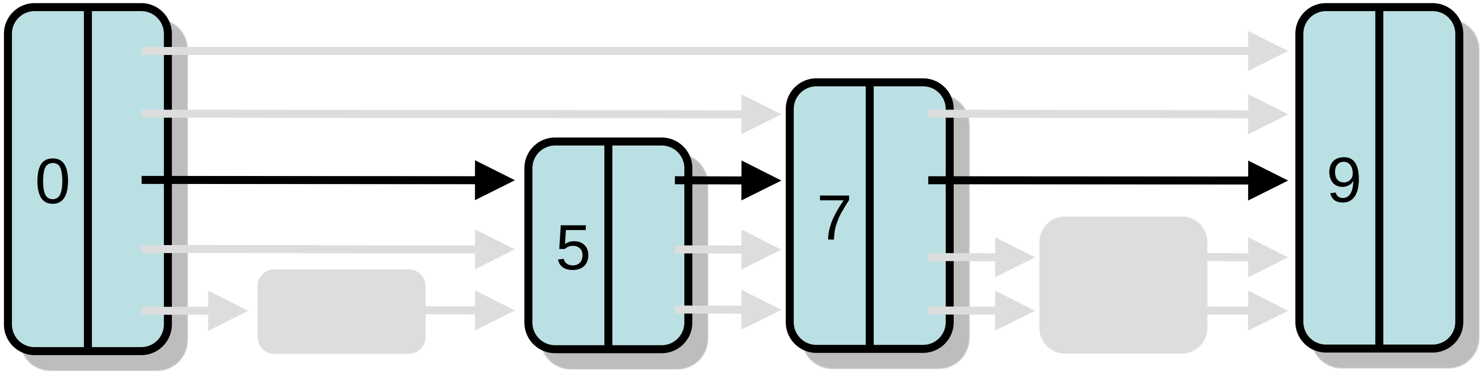

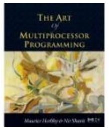

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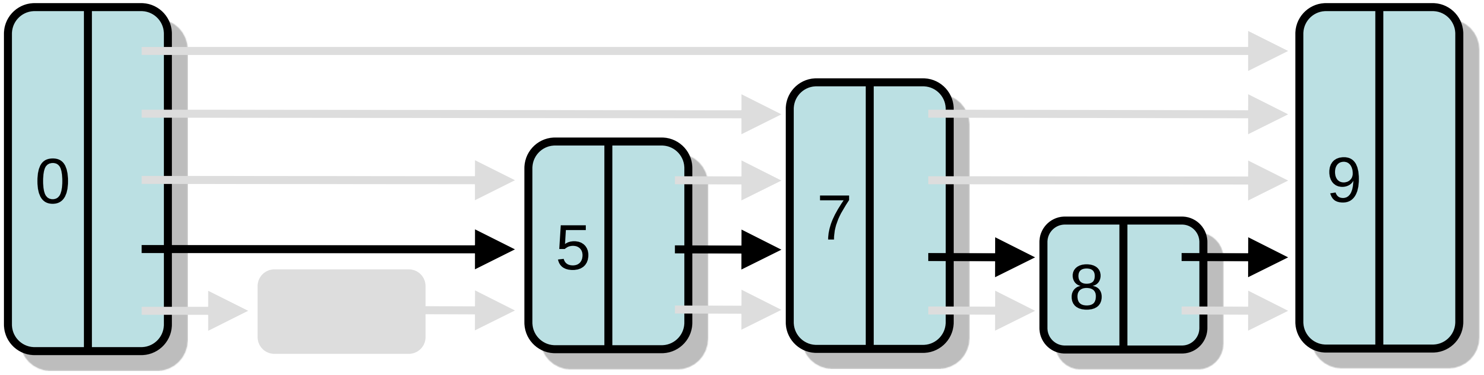
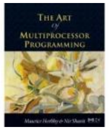
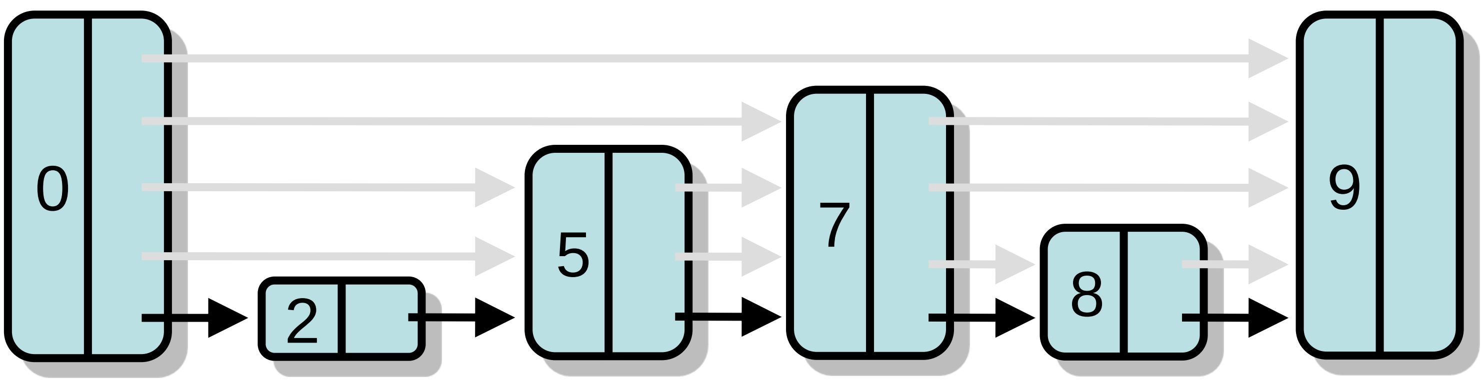
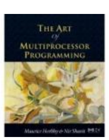




Skip List Property

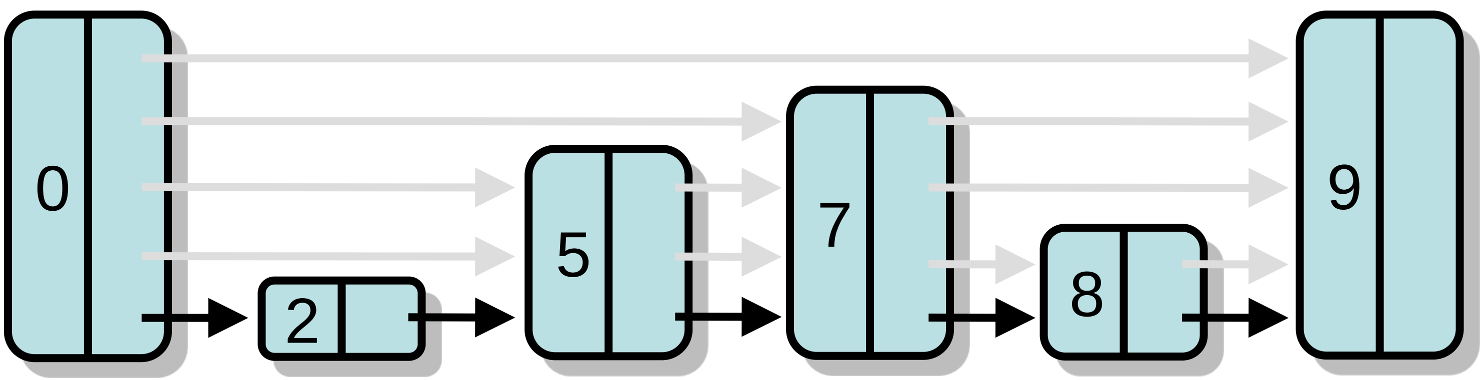
- Each layer is sub-list of lower levels
- Lowest level is entire list



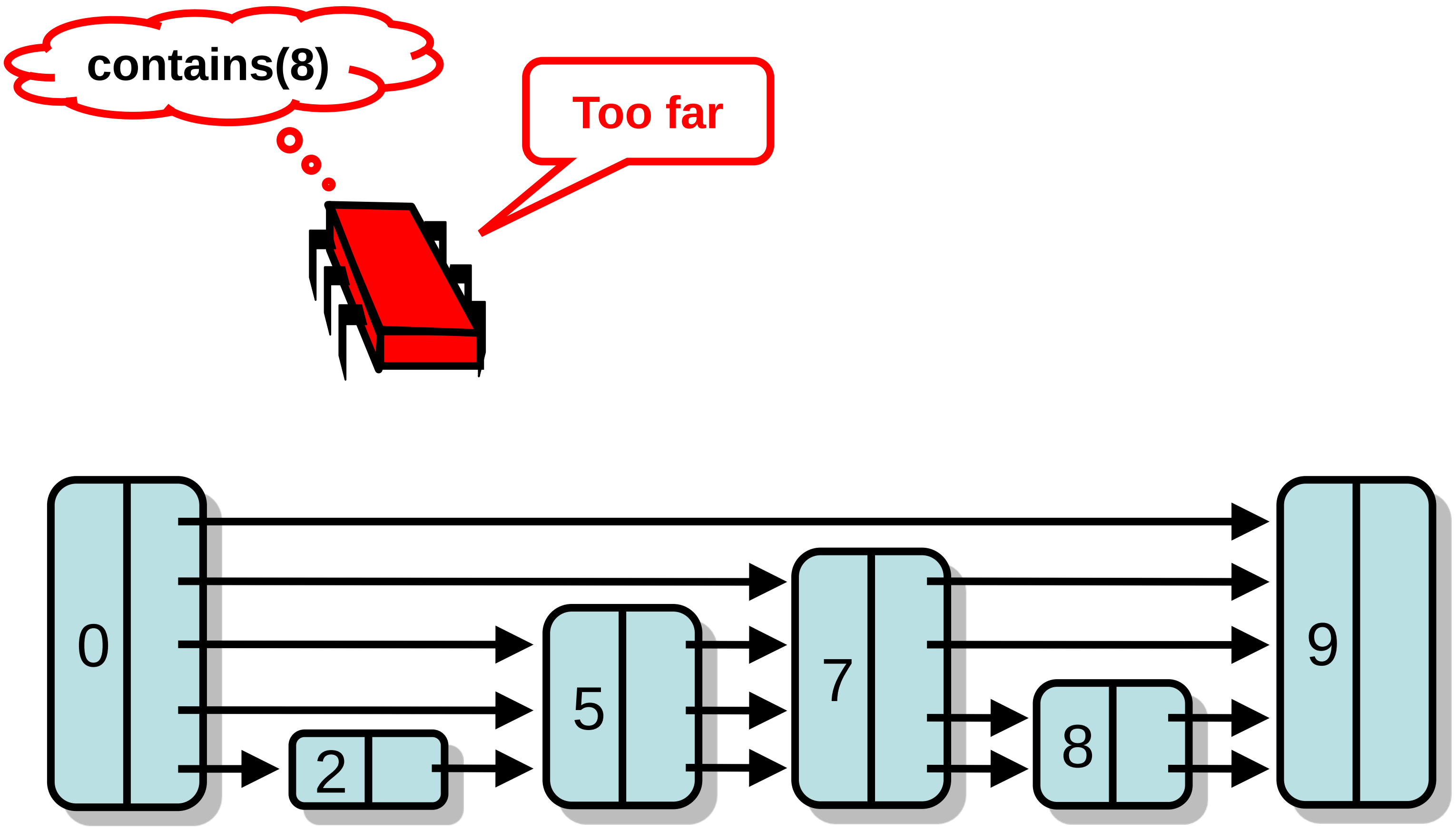


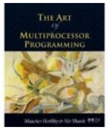
Skip List Property

- Each layer is sub-list of lower levels
- Not easy to preserve in concurrent implementations …



Search





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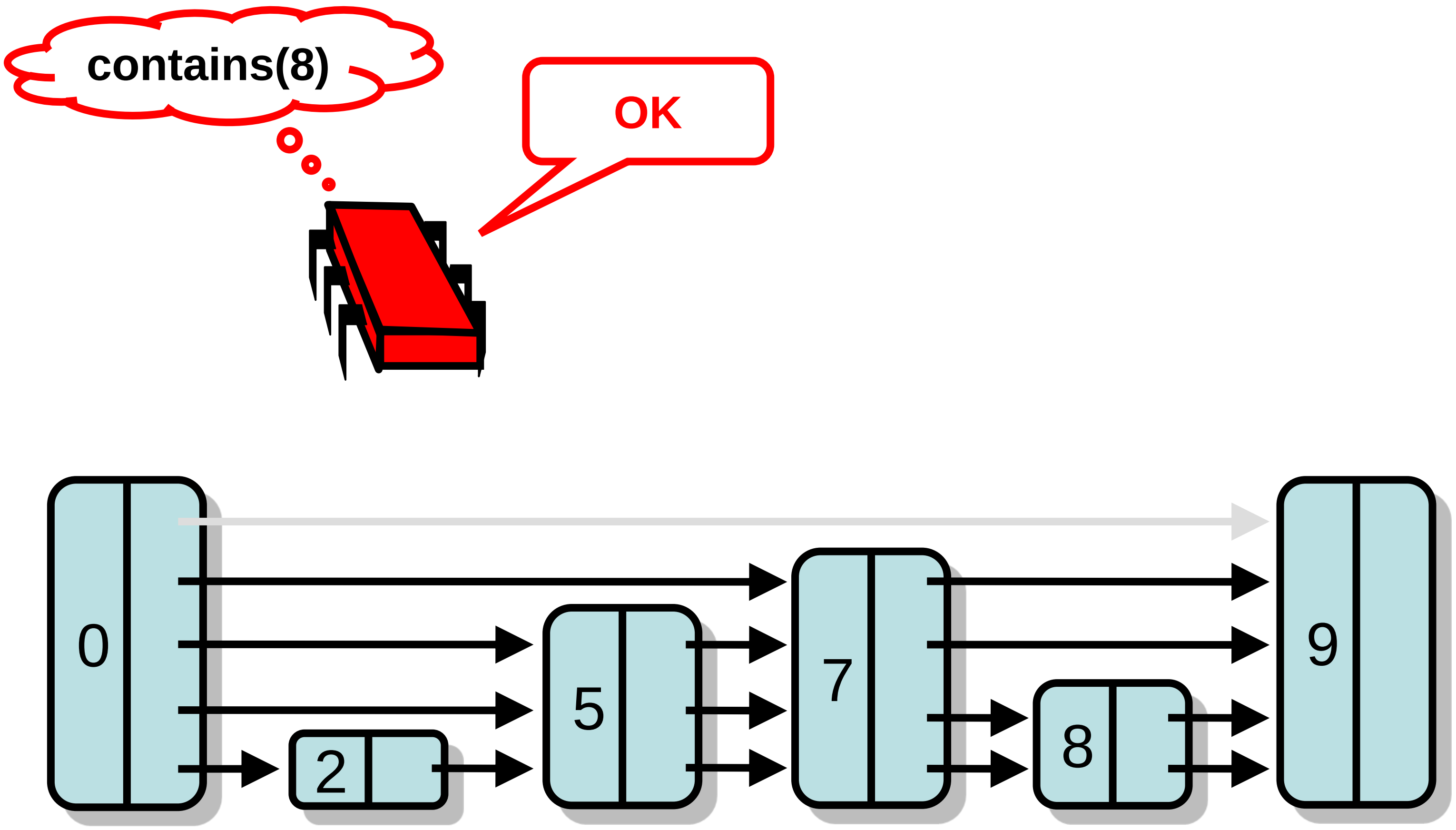

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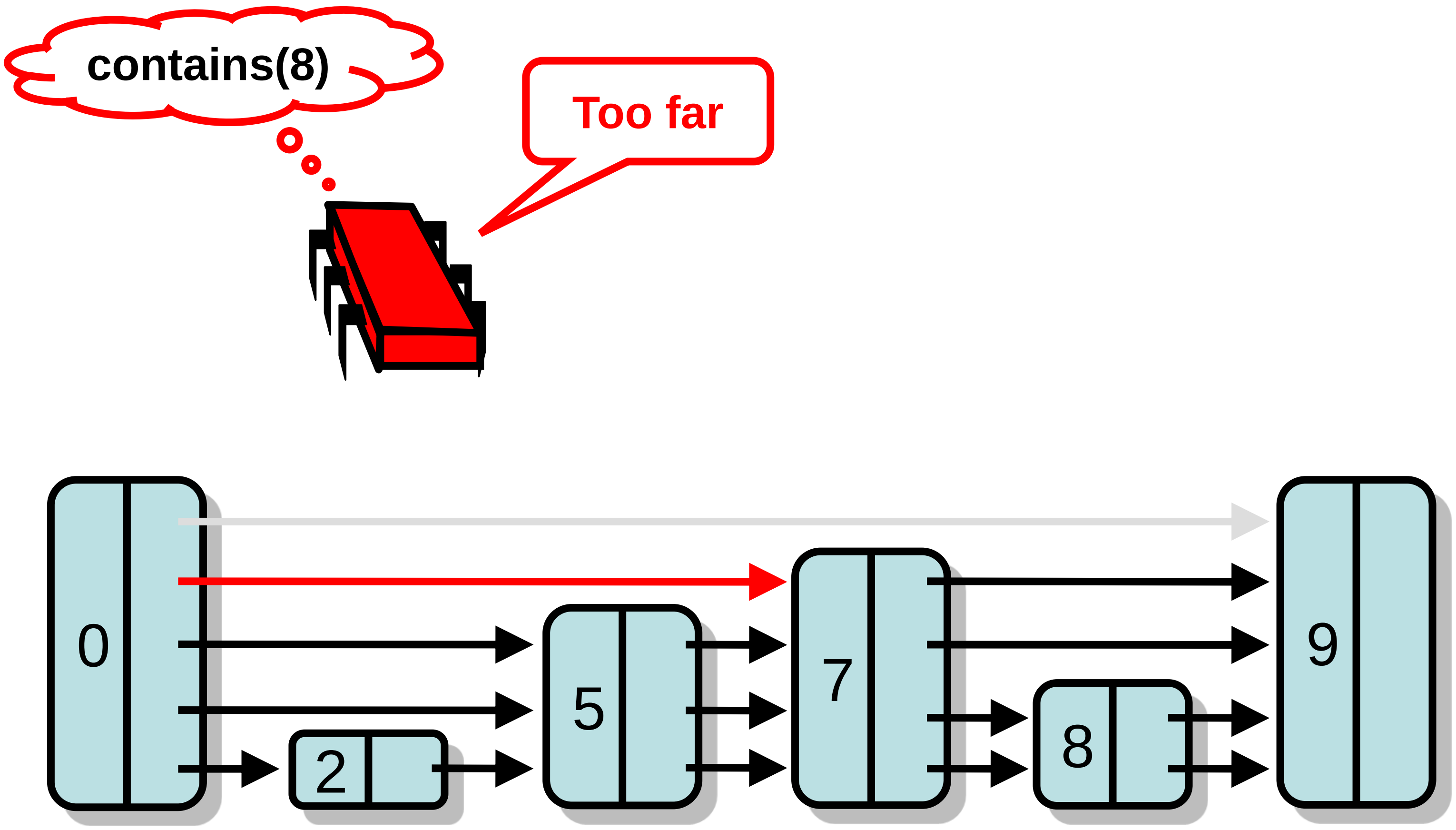

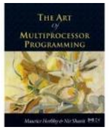

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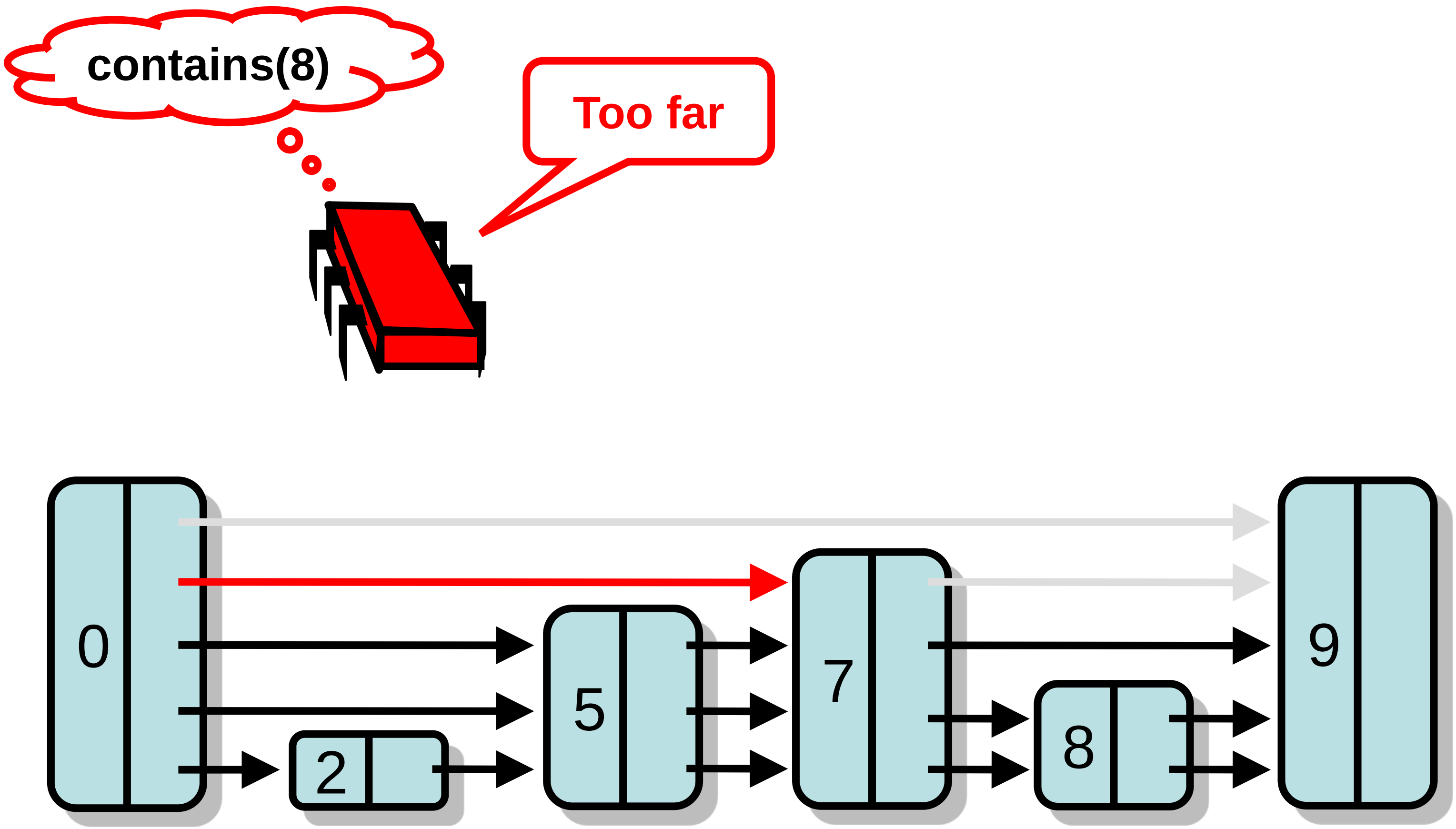

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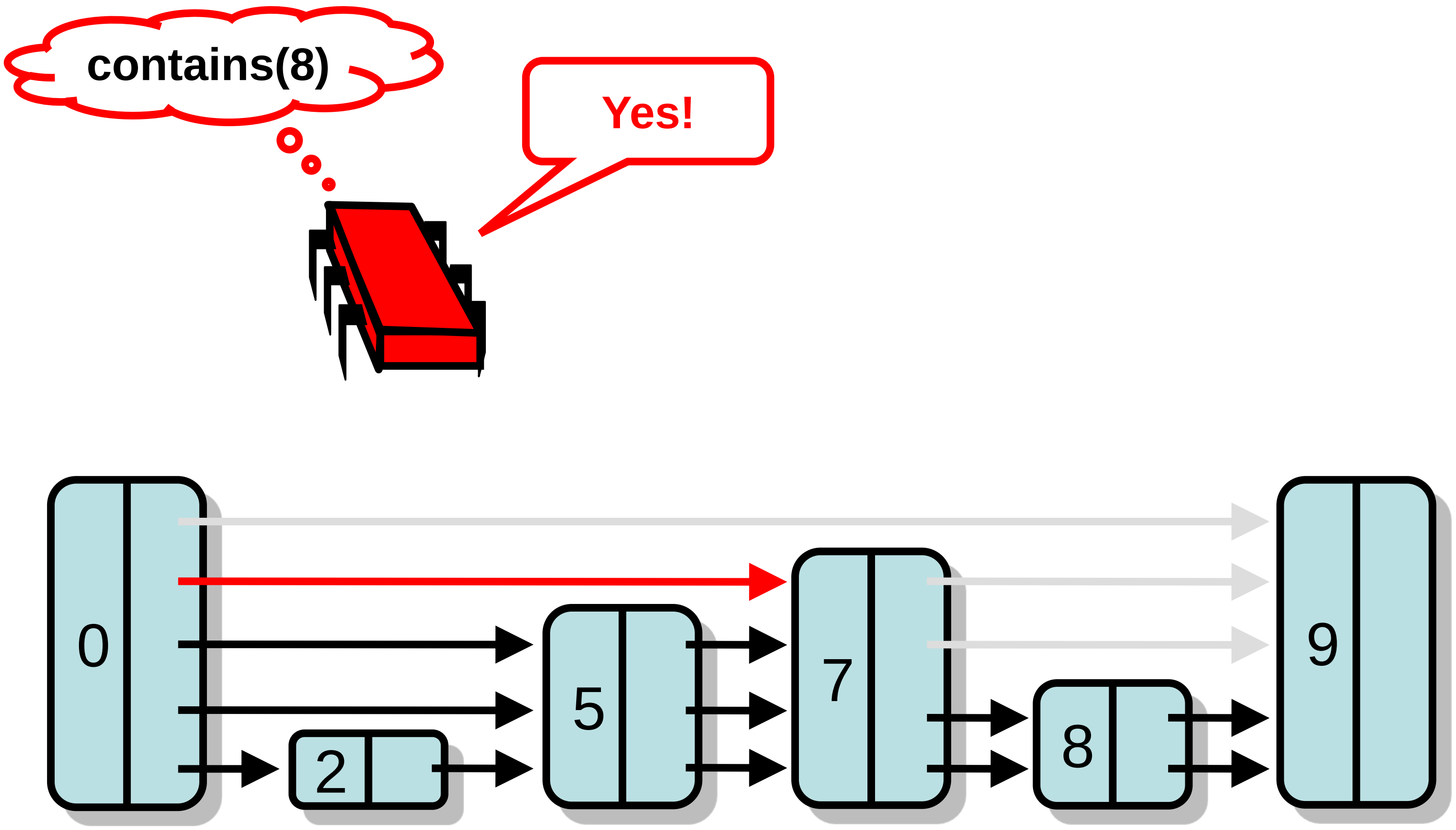

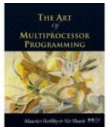

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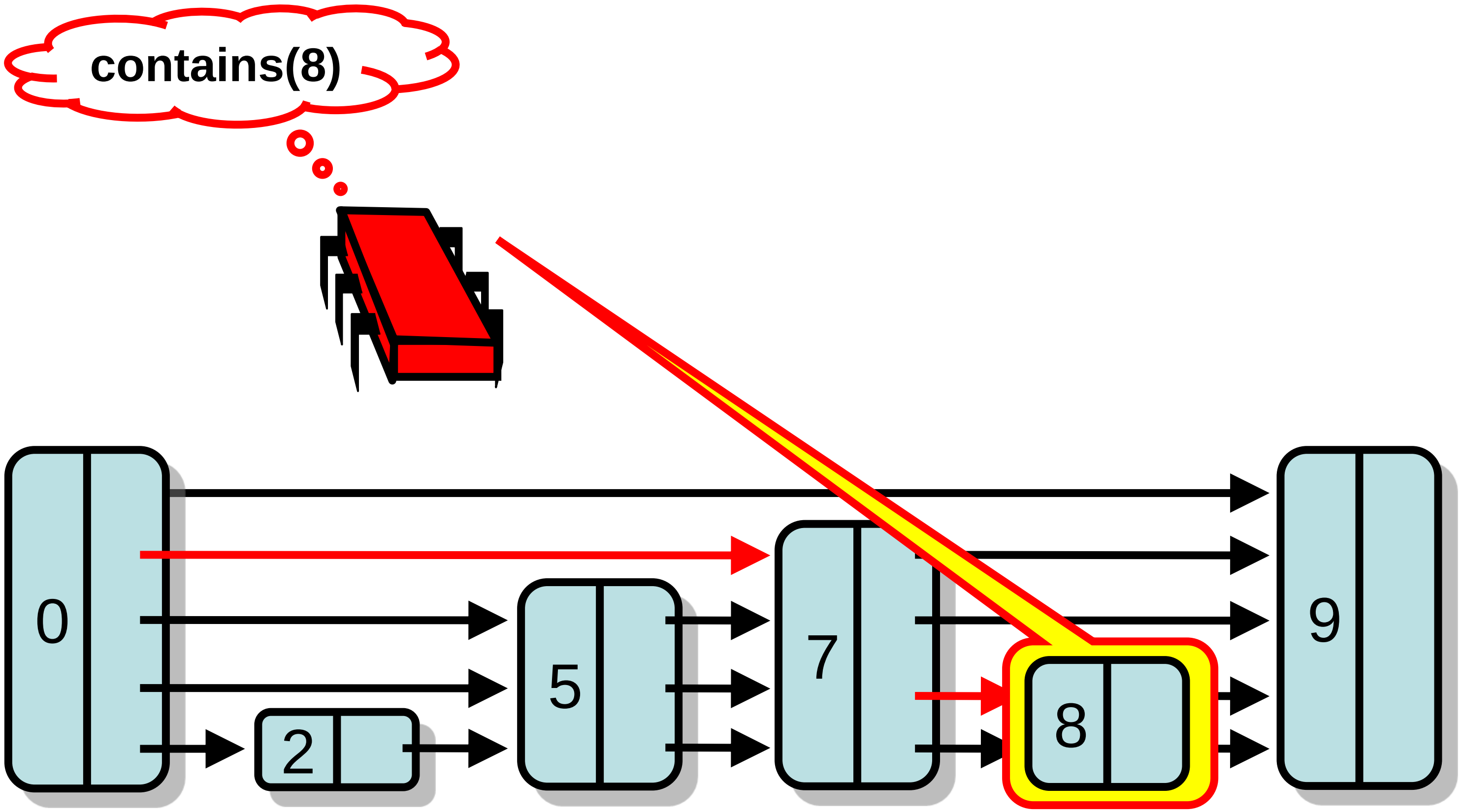
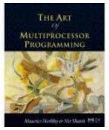
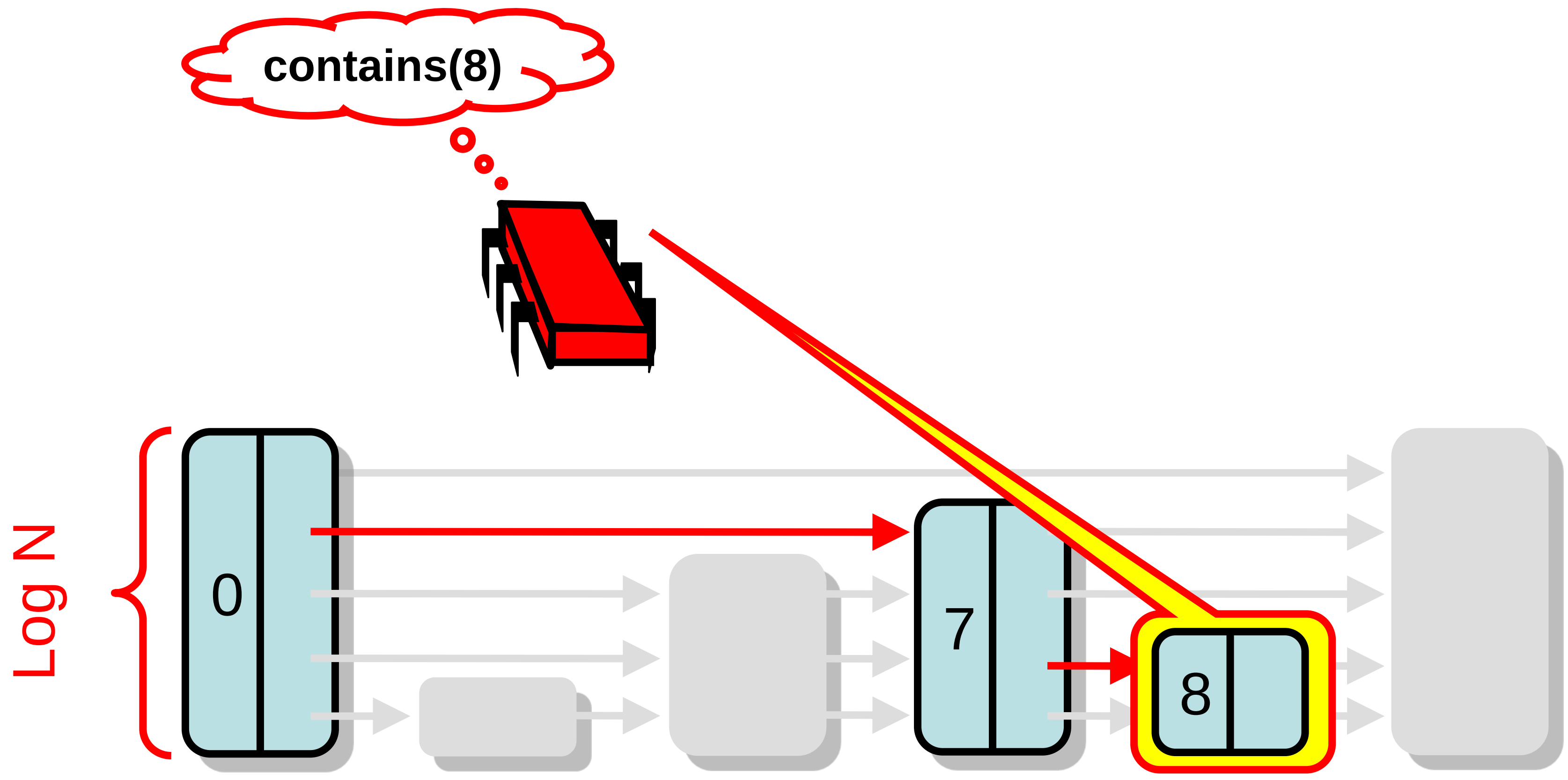


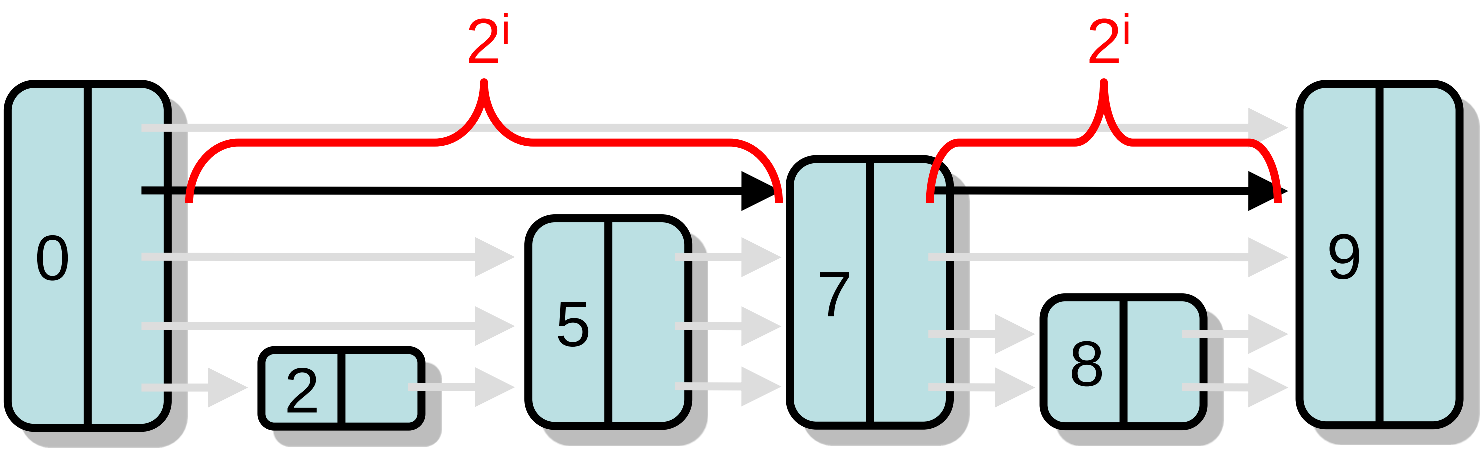


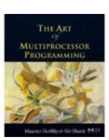
Logarithmic



Why Logarthimic

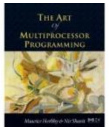
- Property: Each pointer at layer i jumps over roughly 2^i nodes
- Pick node heights randomly so property guaranteed probabilistically





Sequential Find

```
int find(T x, Node<T>[] preds, Node<T>[] succs) {
  …
  }
```



Sequential Find

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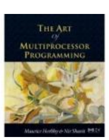
object height (-1 if not there)



Sequential Find

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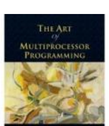

Sequential Find

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int find(T x, Node<T>[] preds, Node<T>[] succs) {
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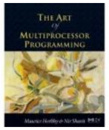
Object height (-1 if not there)

object sought

return predecessors



Sequential Find

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Object height (-1 if not there)

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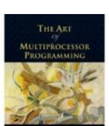
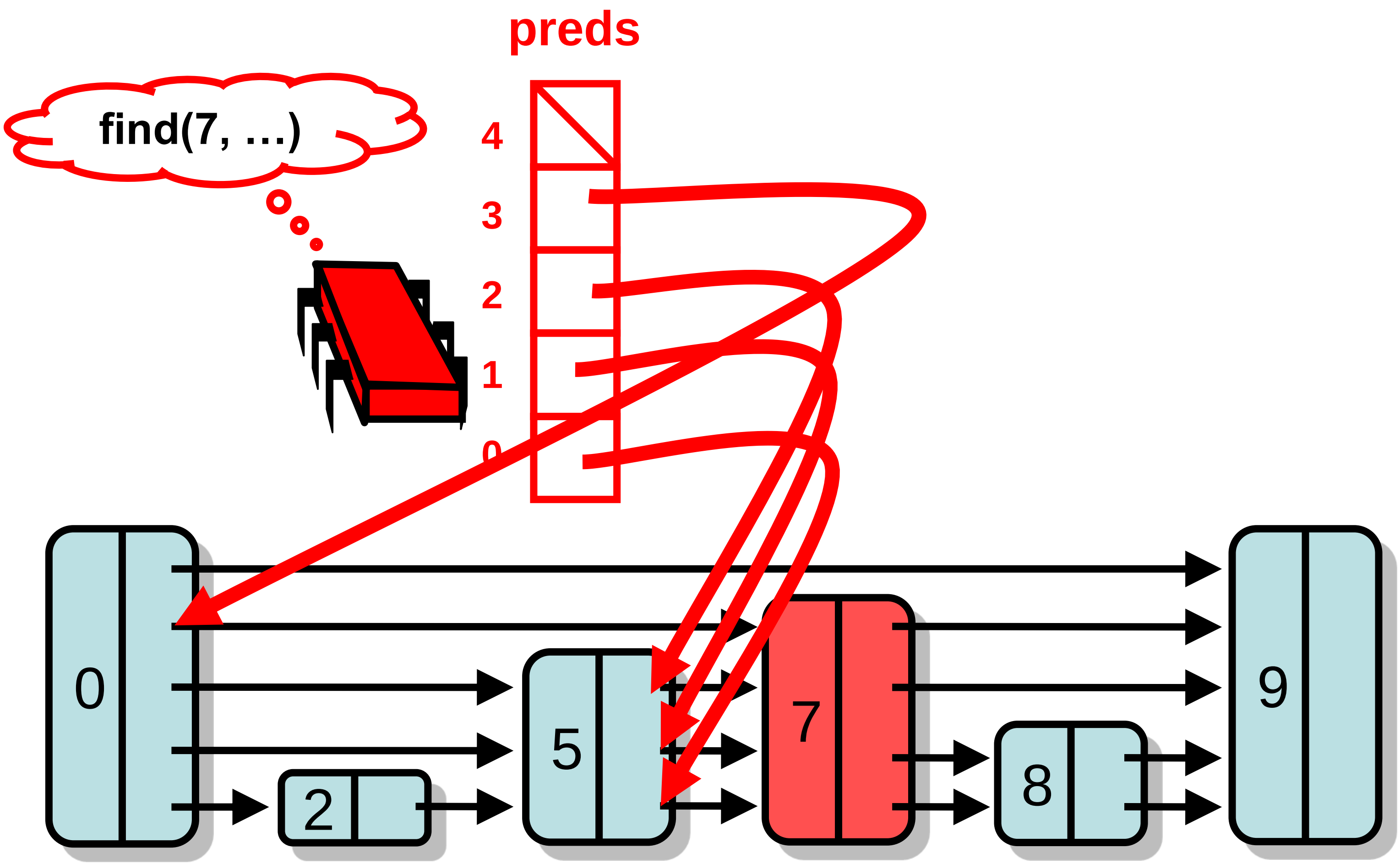
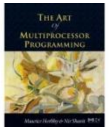
return predecessors

return successors



Successful Search





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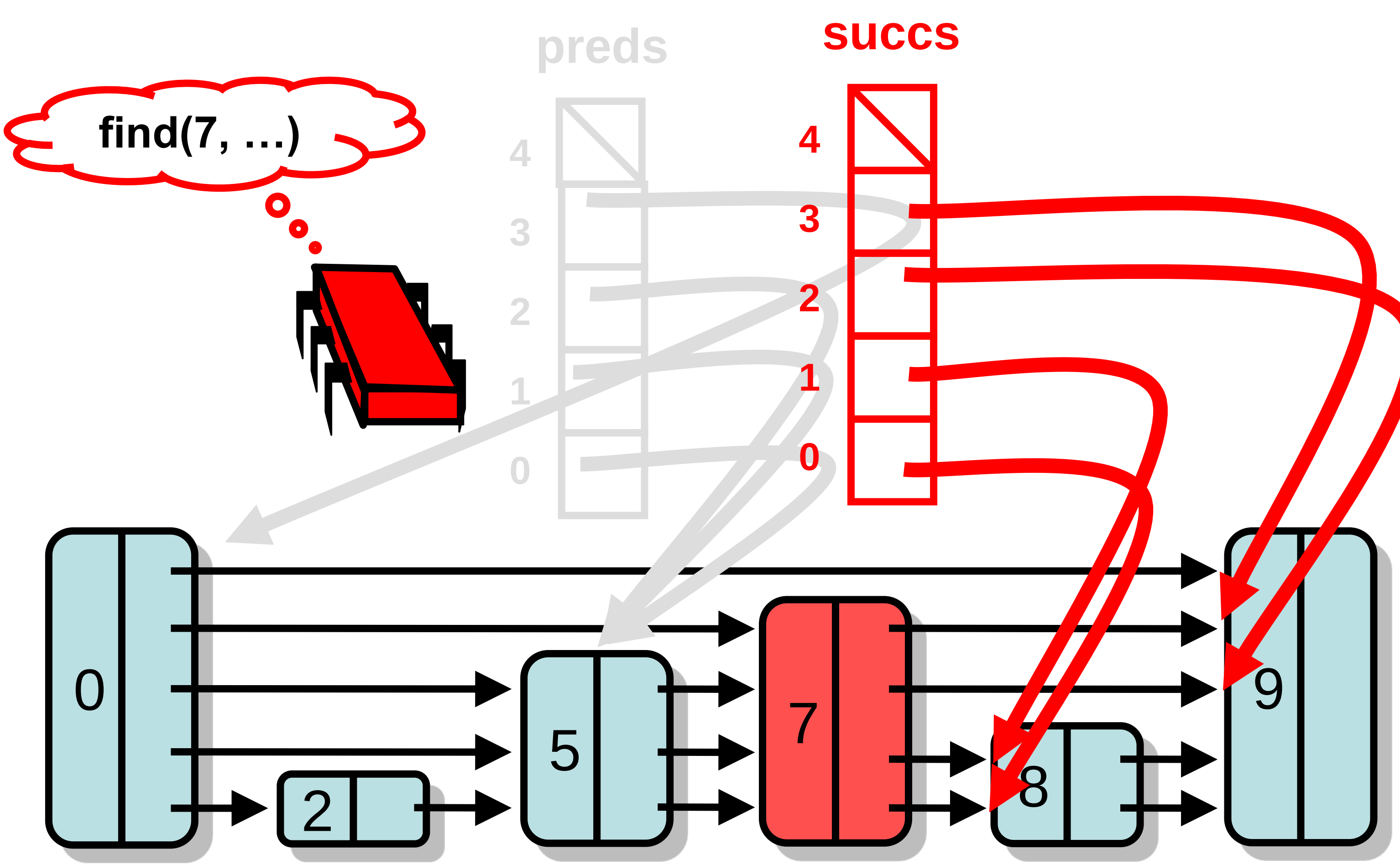
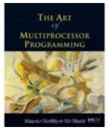
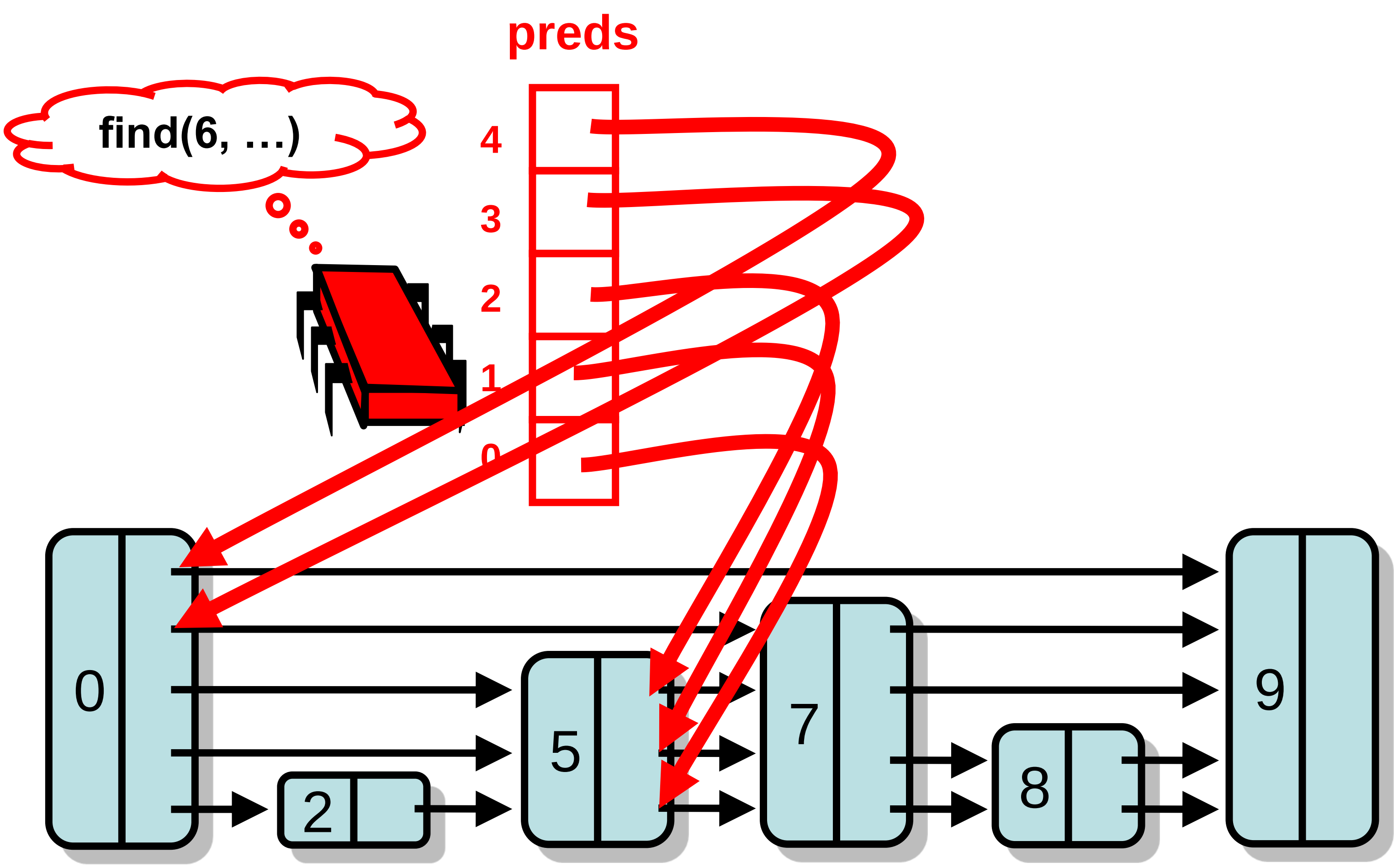
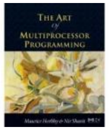




Unsuccessful Search





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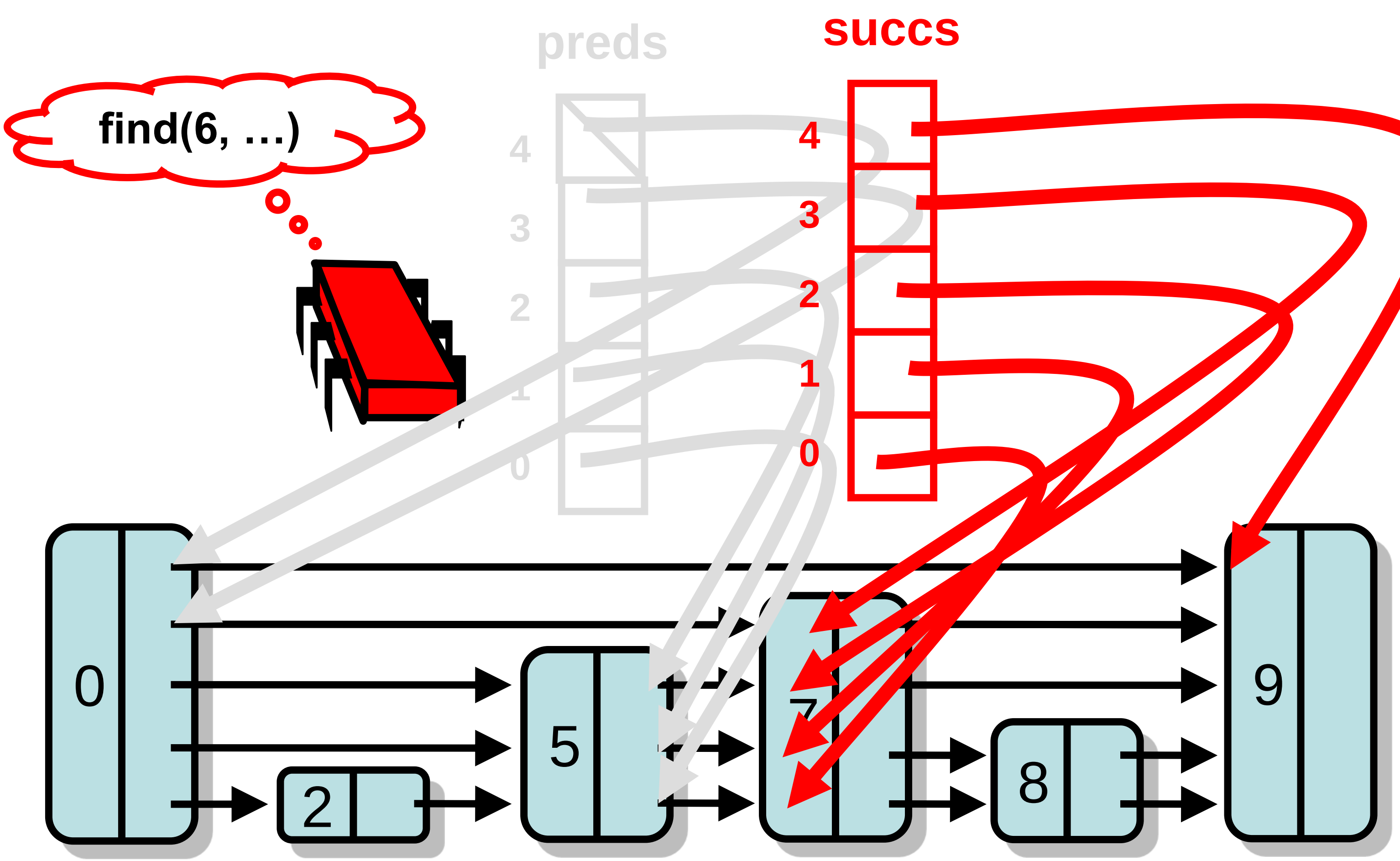
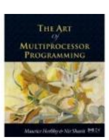
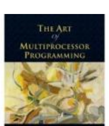


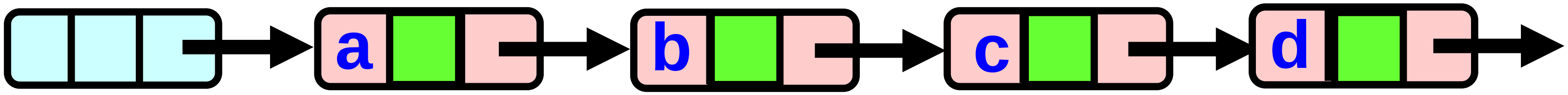


Lazy Skip List

- Mix blocking and non-blocking techniques:
  - Use optimistic-lazy locking for add() and remove()
  - Wait-free contains()
- Remember: typically lots of contains() calls but few add() and remove()



Review: Lazy List Remove



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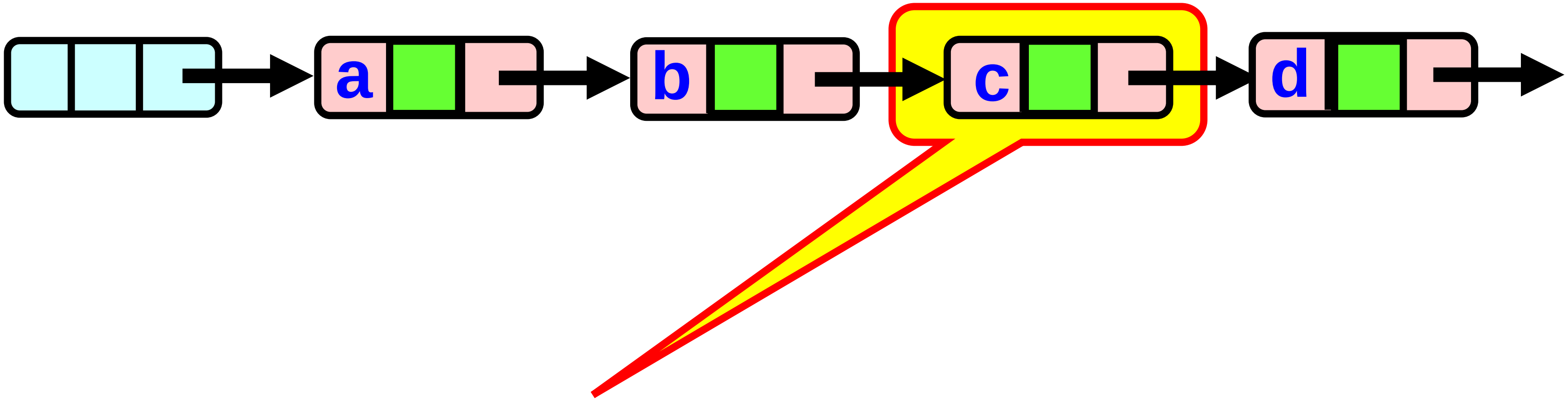
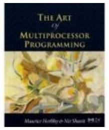
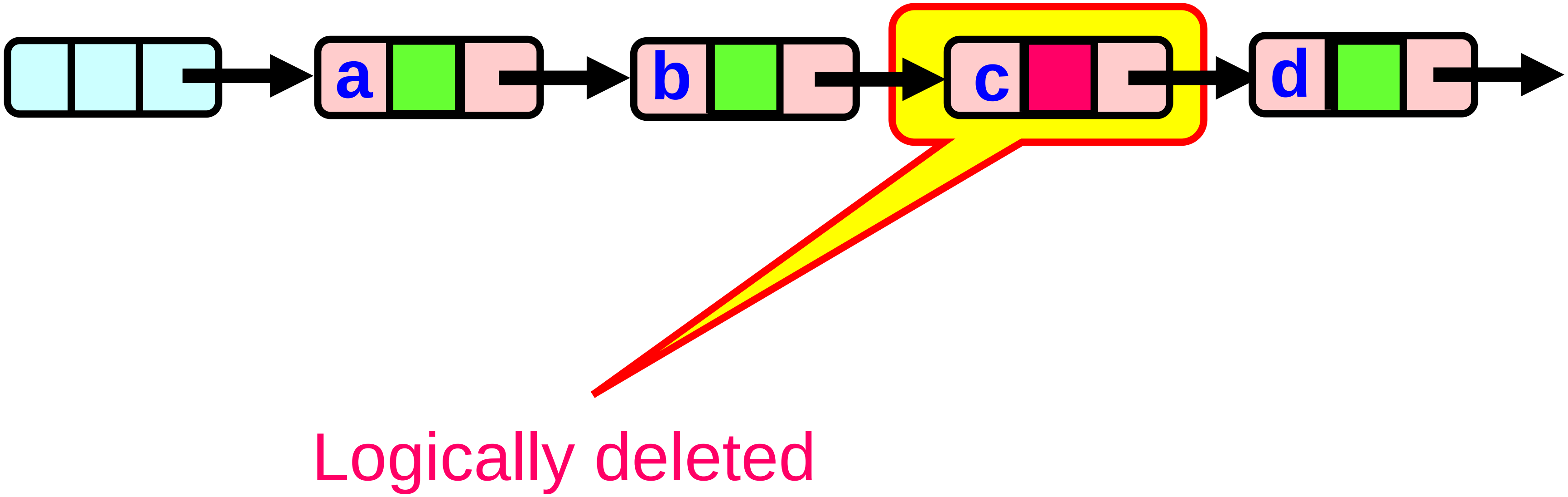
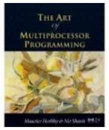


Present in list

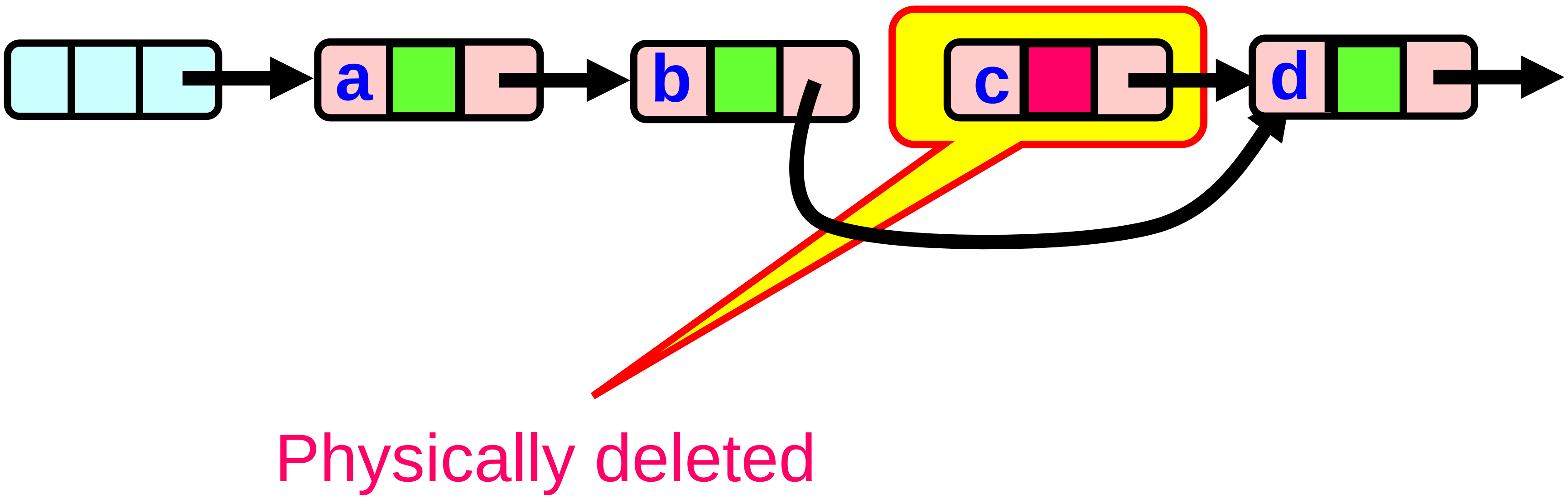


Review: Lazy List Remove



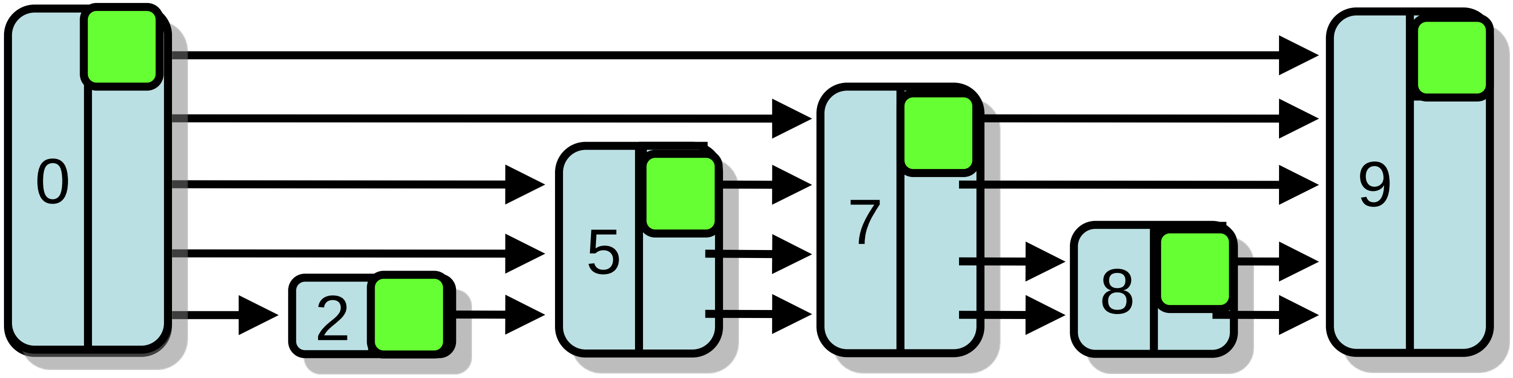


Review: Lazy List Remove



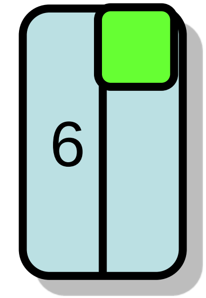
Lazy Skip Lists

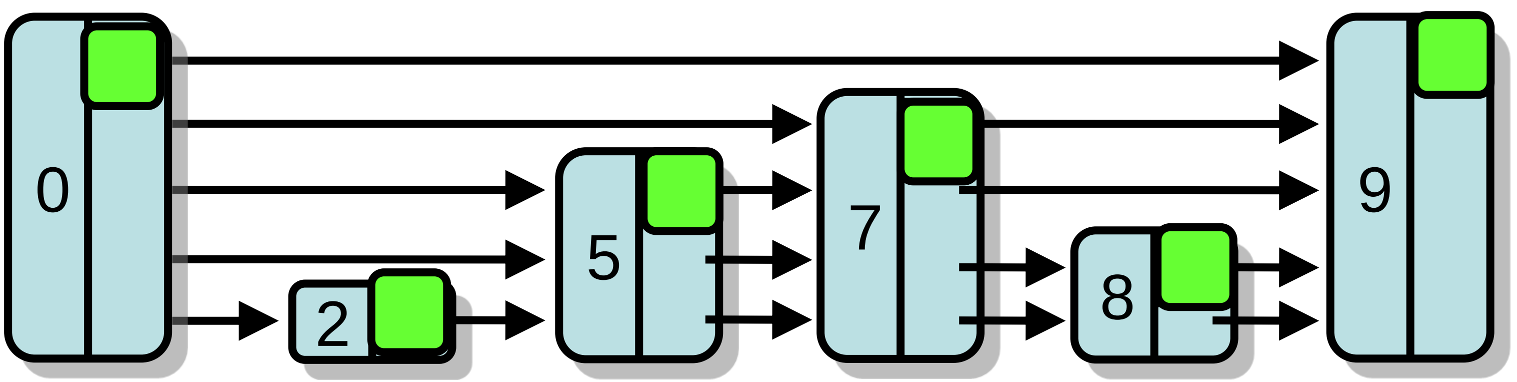
- Use a mark bit for logical deletion



add(6)

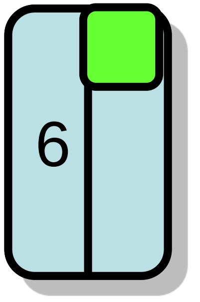
- Create node of (random) height 4

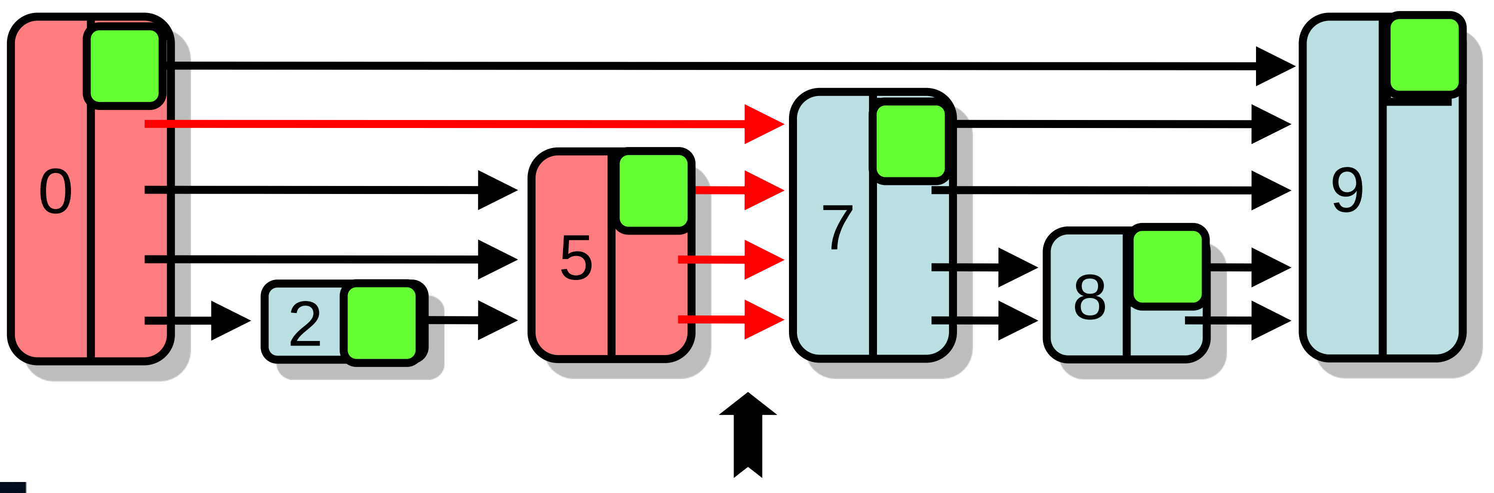


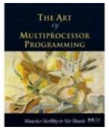


add(6)

- **find()** predecessors

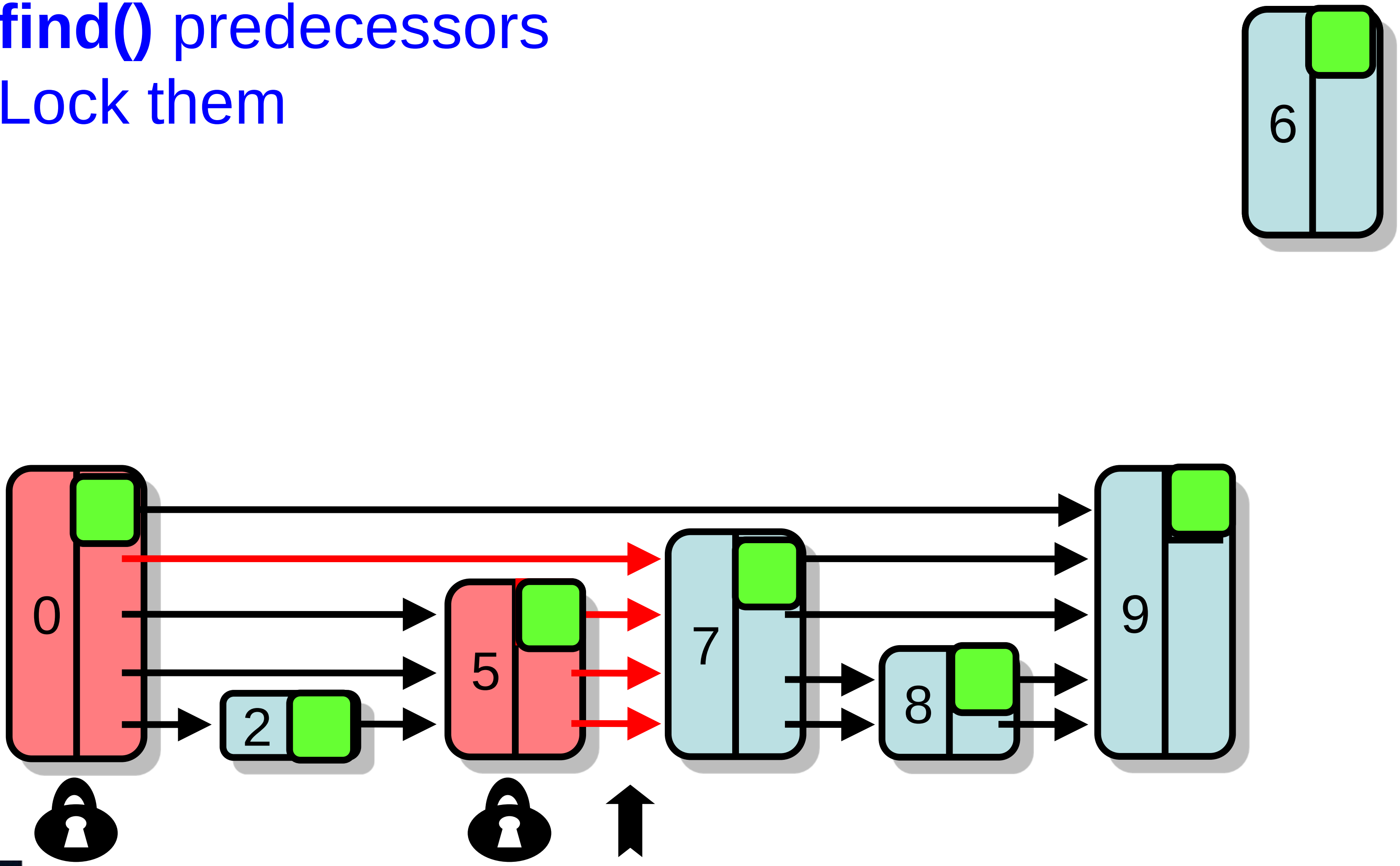


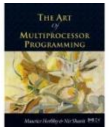




add(6)

- **find()** predecessors
- Lock them

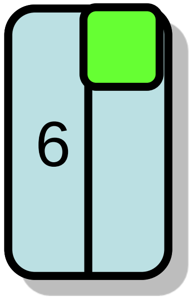


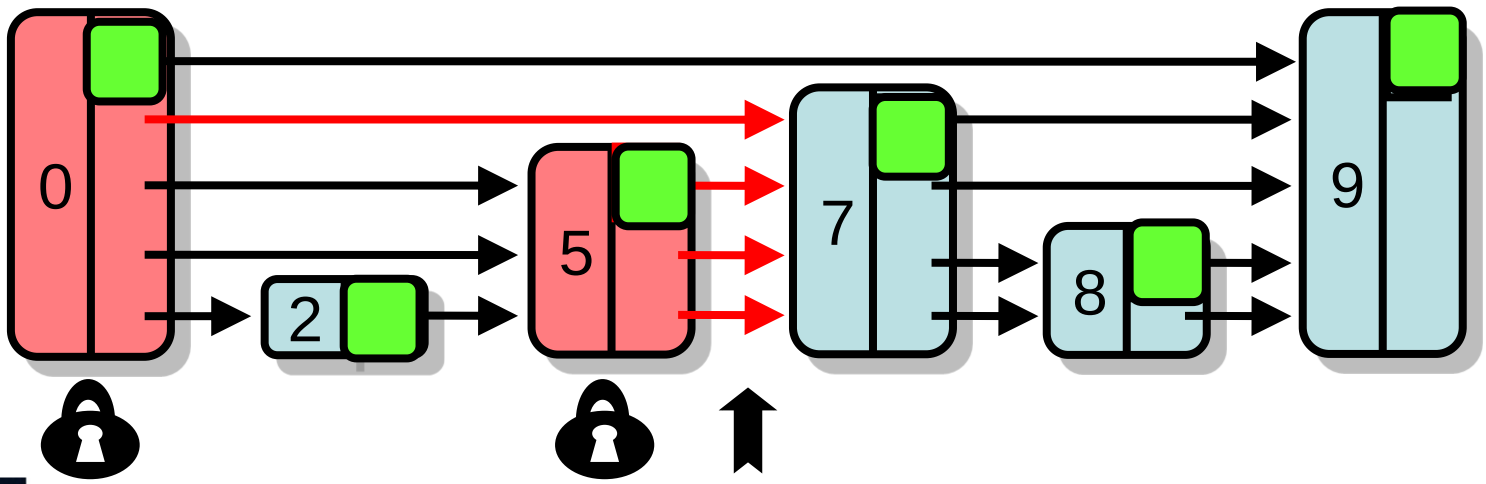


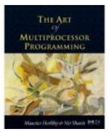
add(6)

- **find()** predecessors
- Lock them
- Validate

Optimistic approach

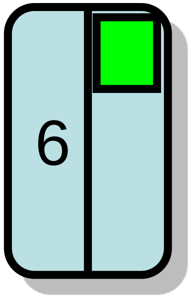


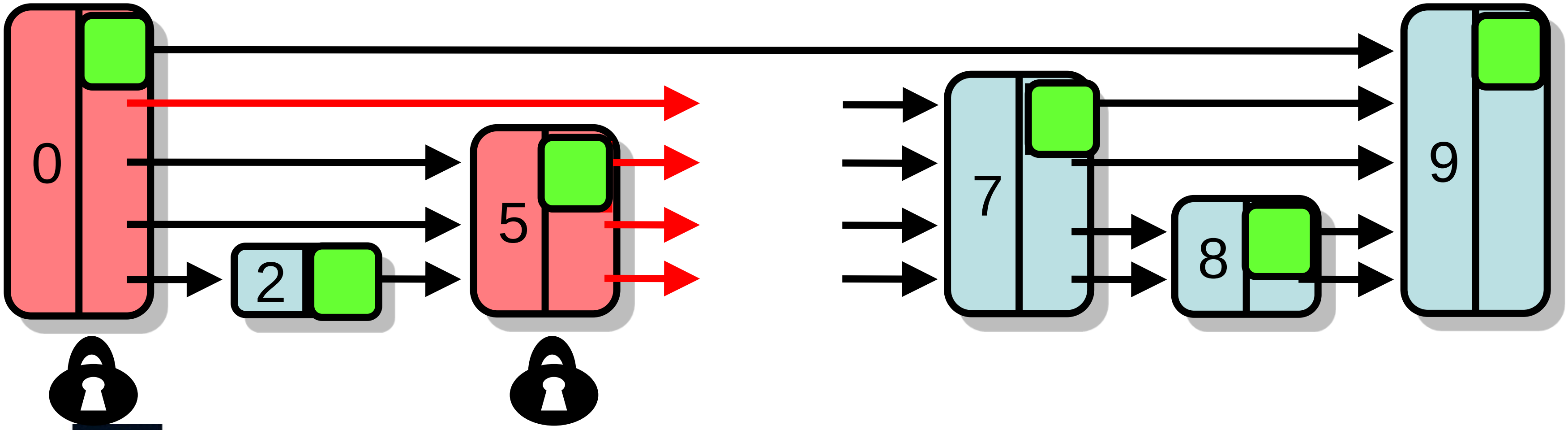


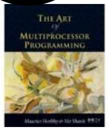


add(6)

- **find()** predecessors
- Lock them
- Validate
- Splice

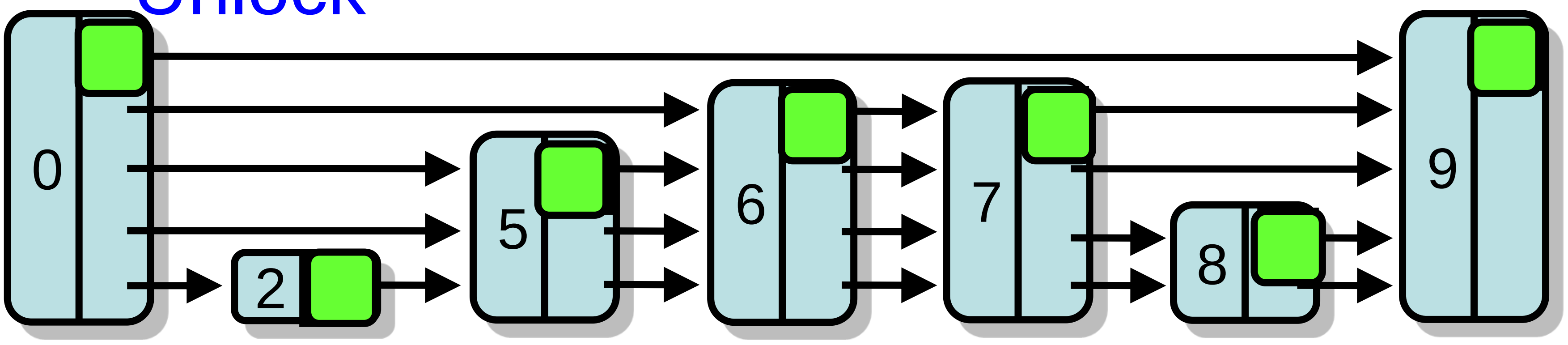




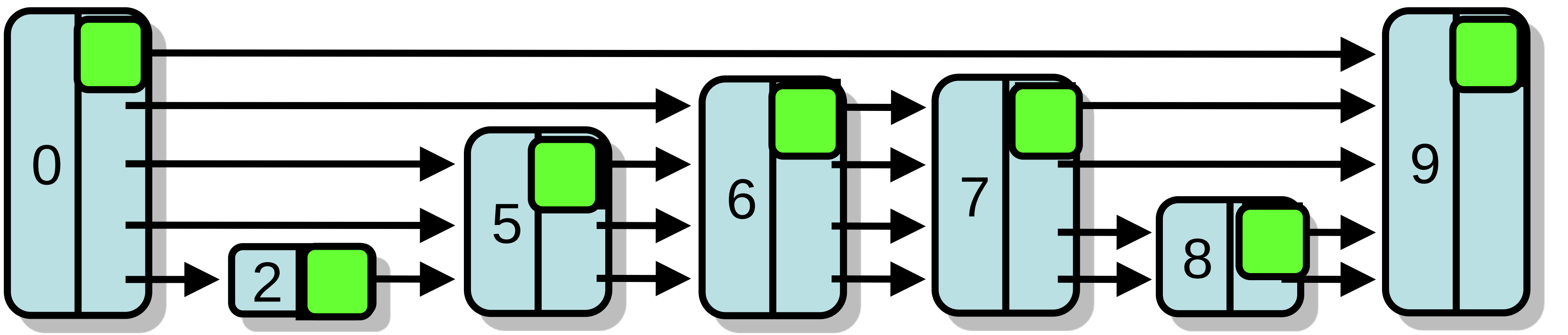


add(6)

- **find()** predecessors
- Lock them
- Validate
- Splice
- Unlock

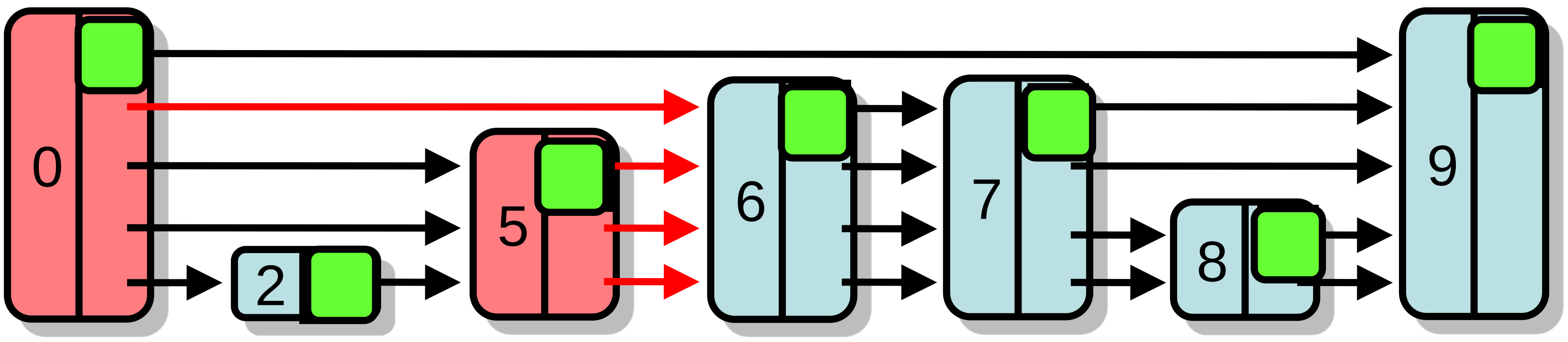


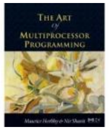
remove(6)



remove(6)

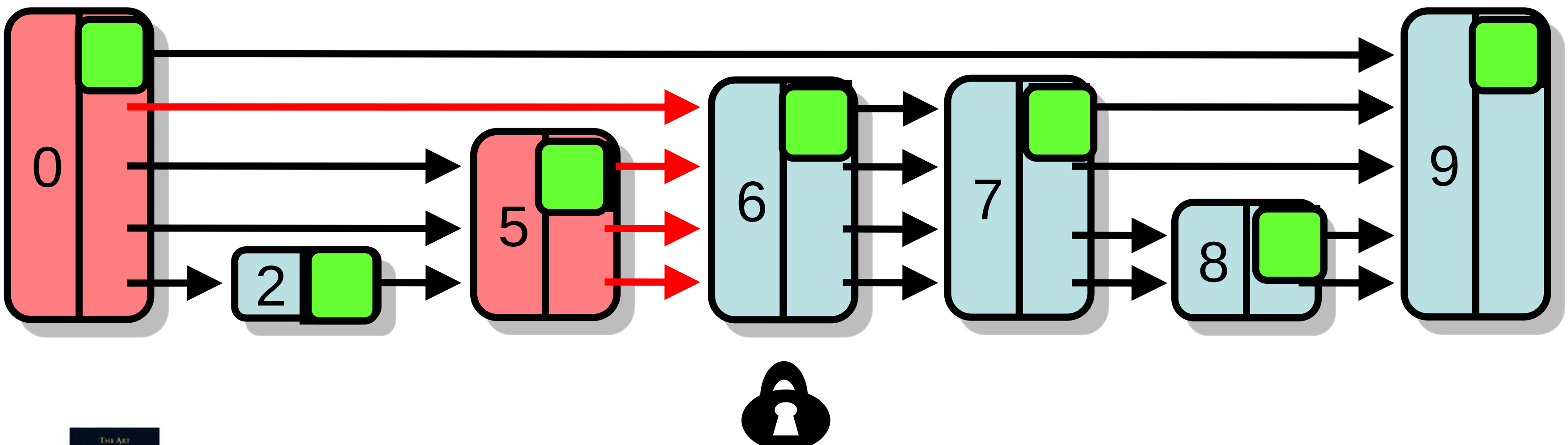
- **find()** predecessors

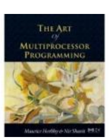




remove(6)

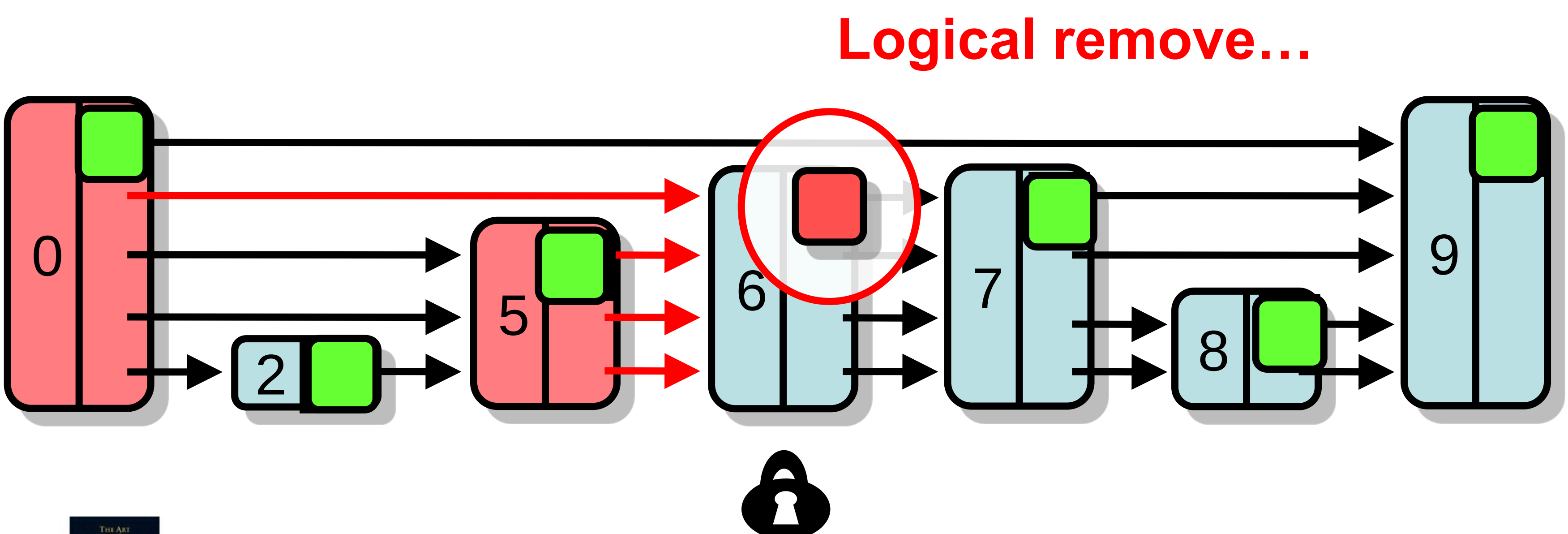
- **find()** predecessors
- Lock victim

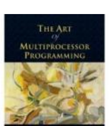




remove(6)

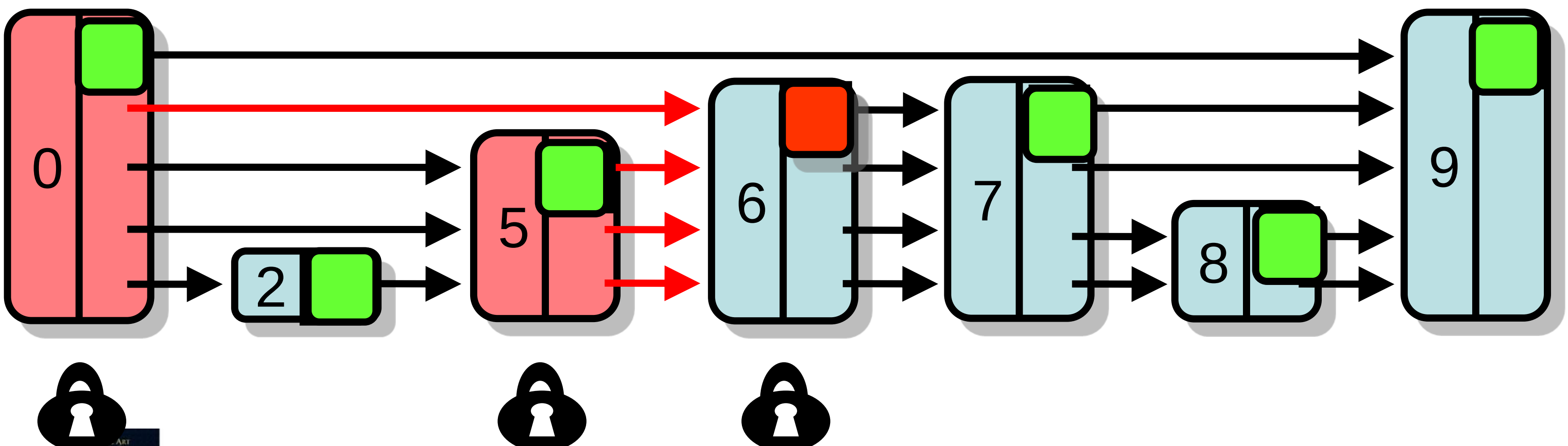
- **find()** predecessors
- Lock victim
- Set mark (if not already set)

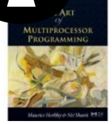




remove(6)

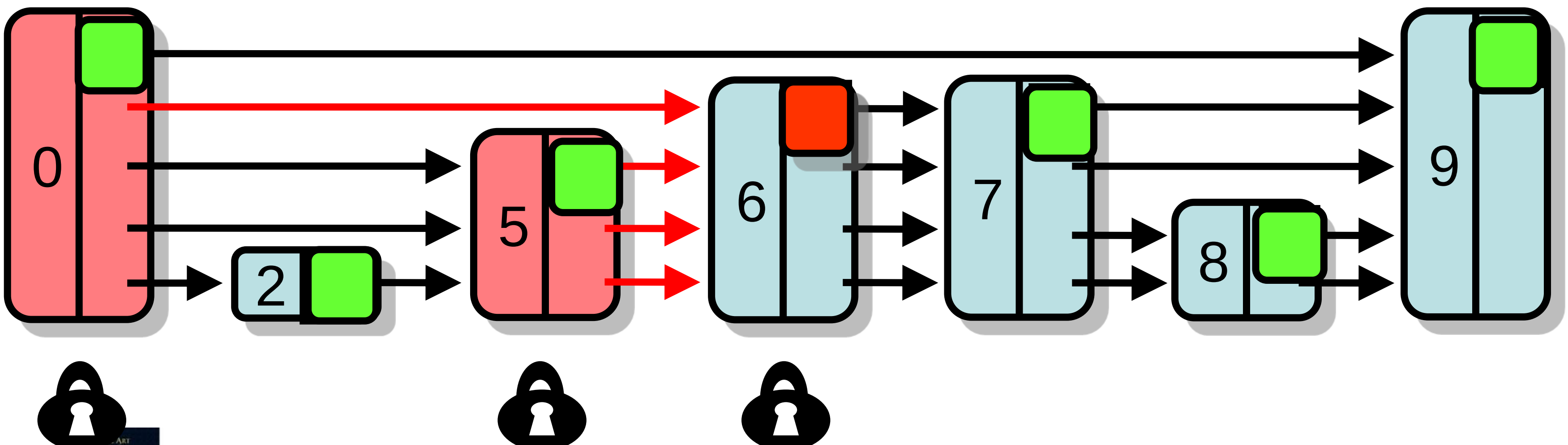
- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate

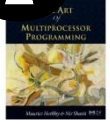




remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove





remove(6)

- **find()** predecessors
- Lock victim
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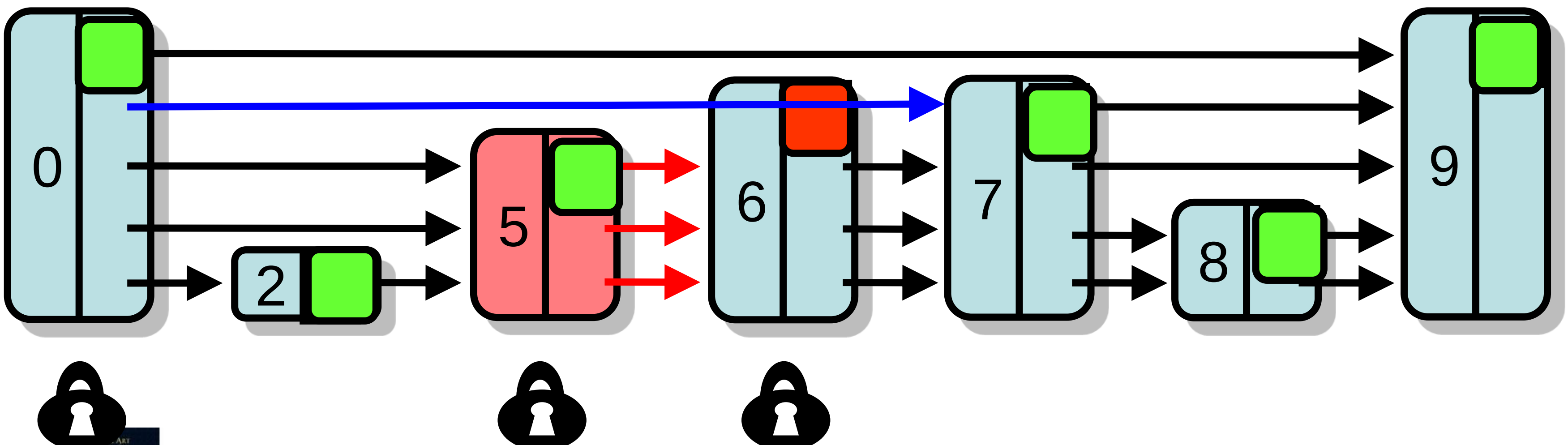
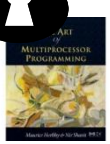
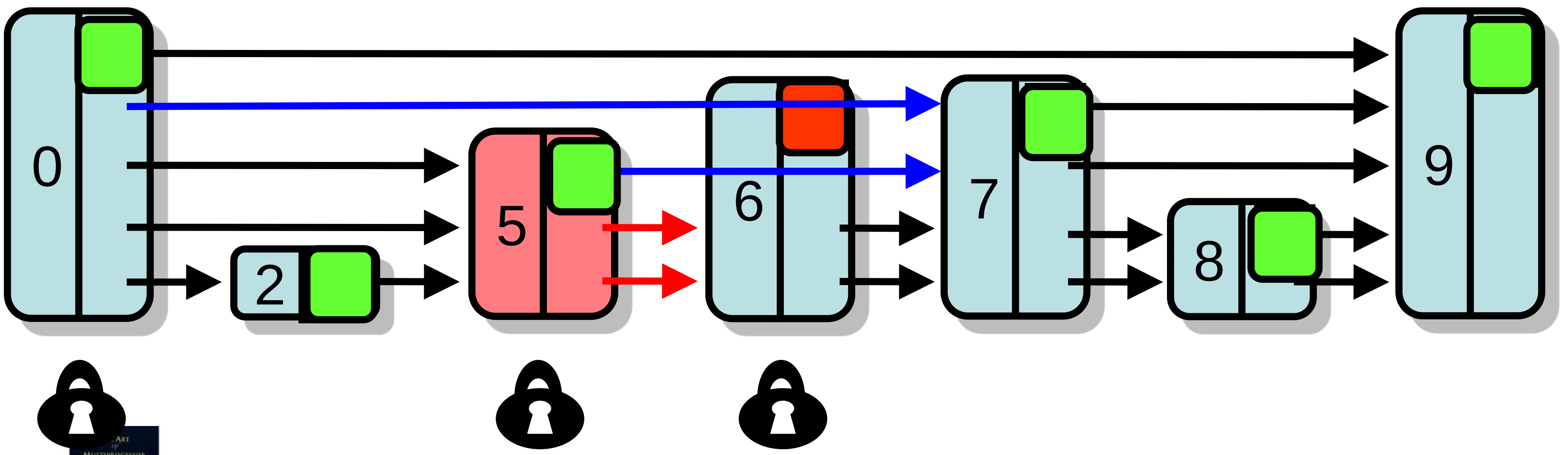
- Lock predecessors (ascending order) & validate
- Physically remove





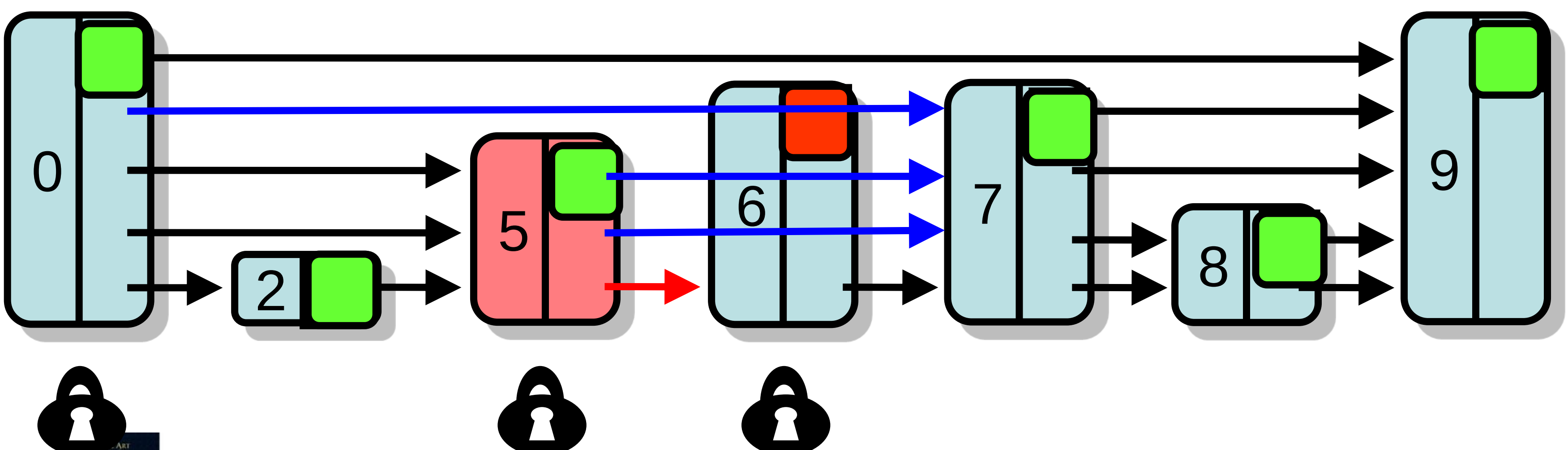
remove(6)

- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove



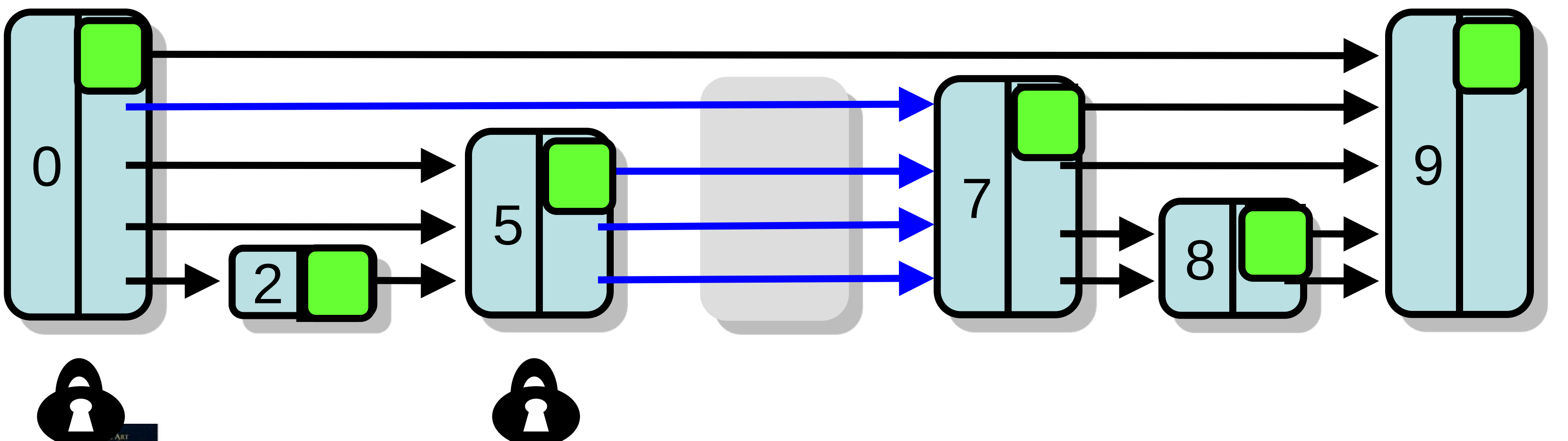
remove(6)

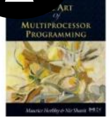
- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove



remove(6)

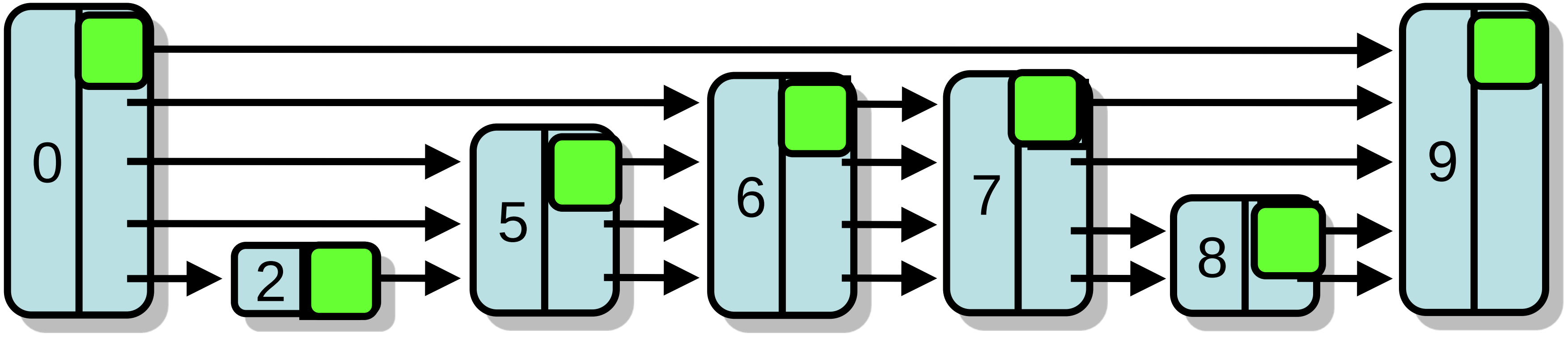
- **find()** predecessors
- Lock victim
- Set mark (if not already set)
- Lock predecessors (ascending order) & validate
- Physically remove

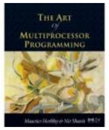




contains(8)

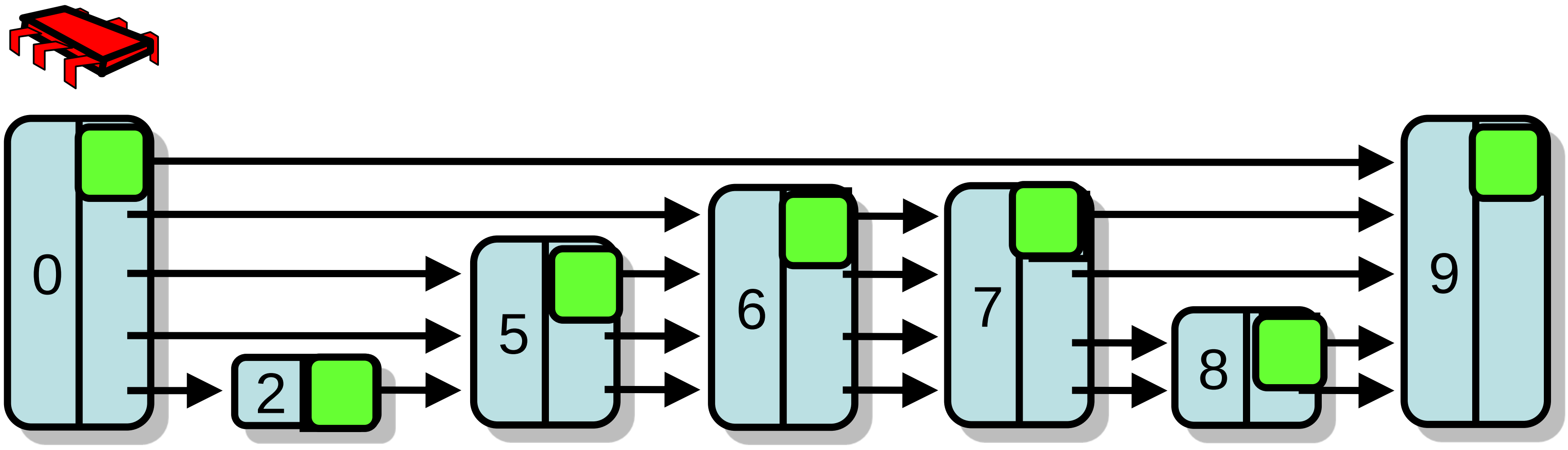
- **find()** & not marked





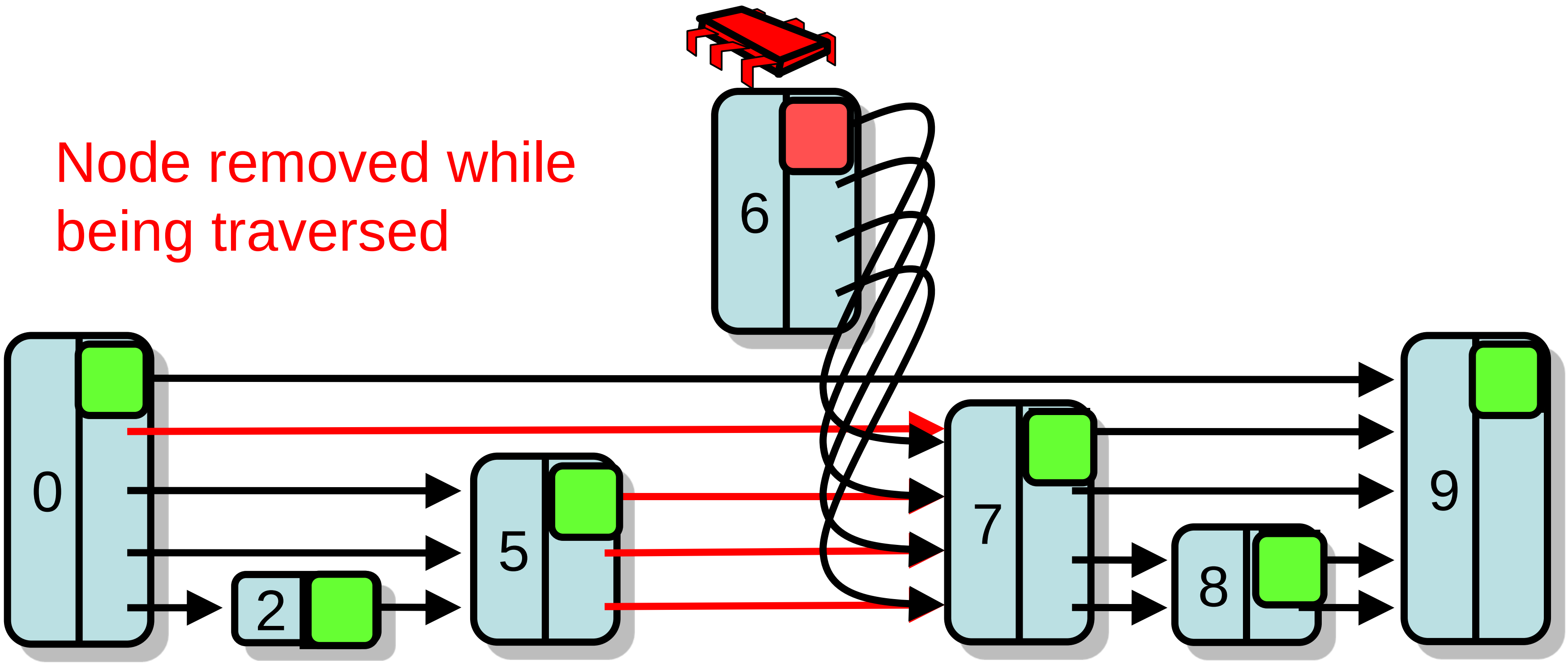
contains(8)

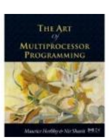
Node 6 removed while traversed



contains(8)

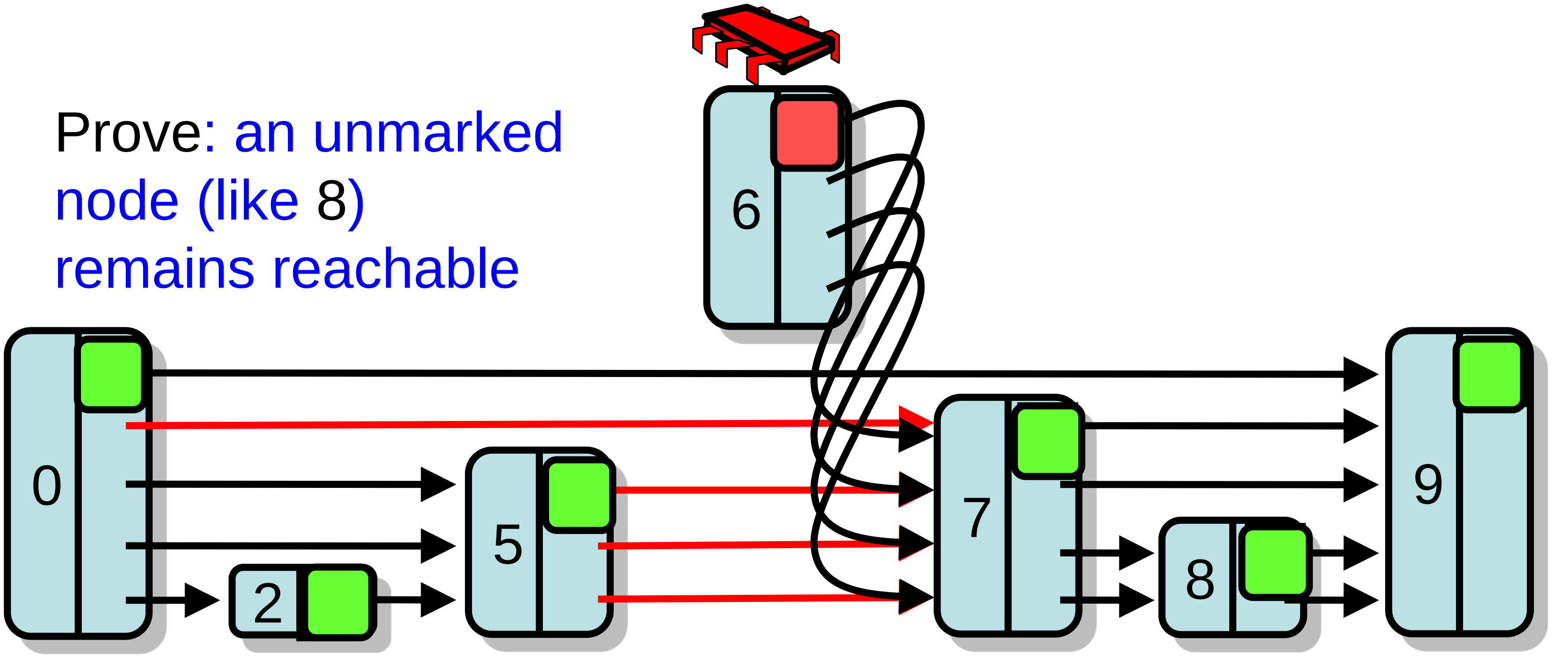
Node removed while being traversed





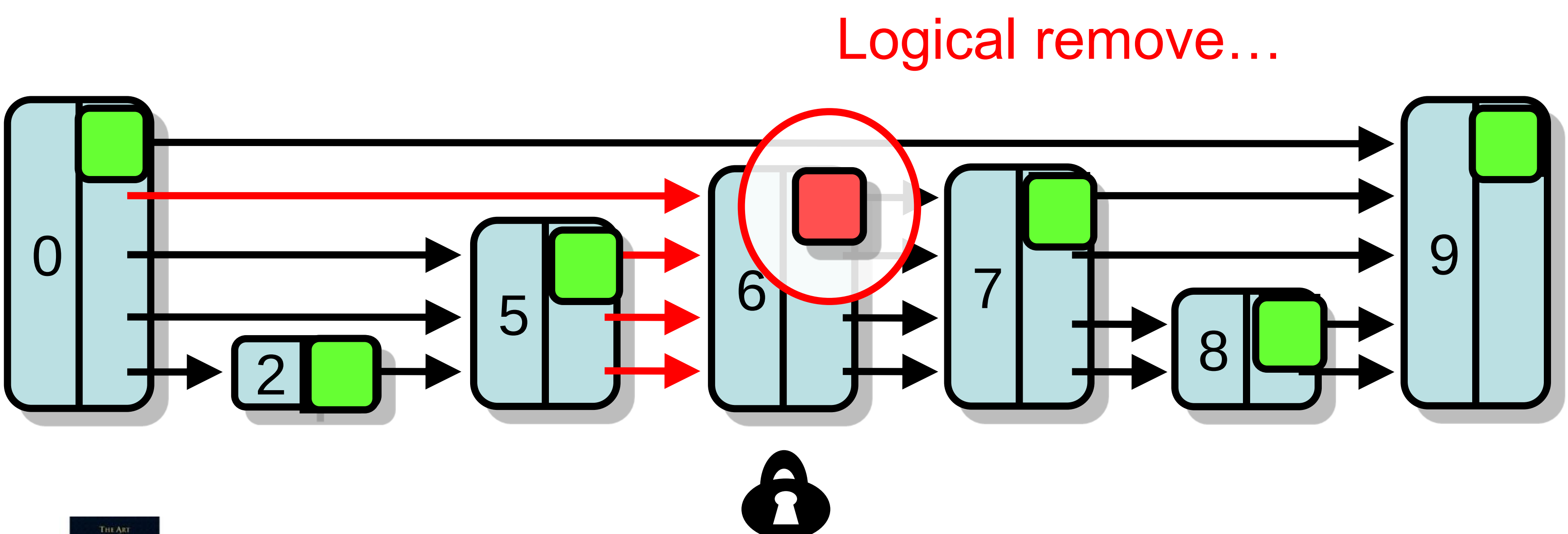
contains(8)

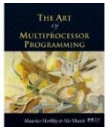
Prove: an unmarked node (like 8) remains reachable



remove(6): Linearization

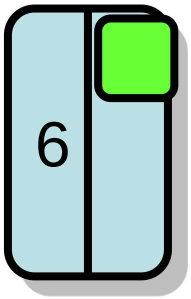
- Successful remove happens when bit is set

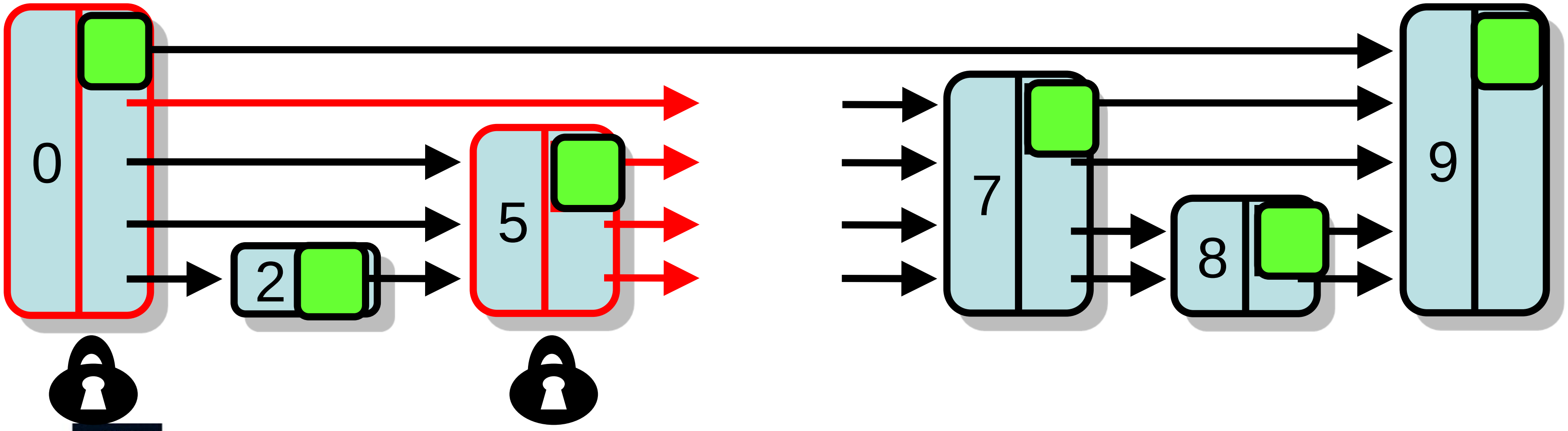




Add: Linearization

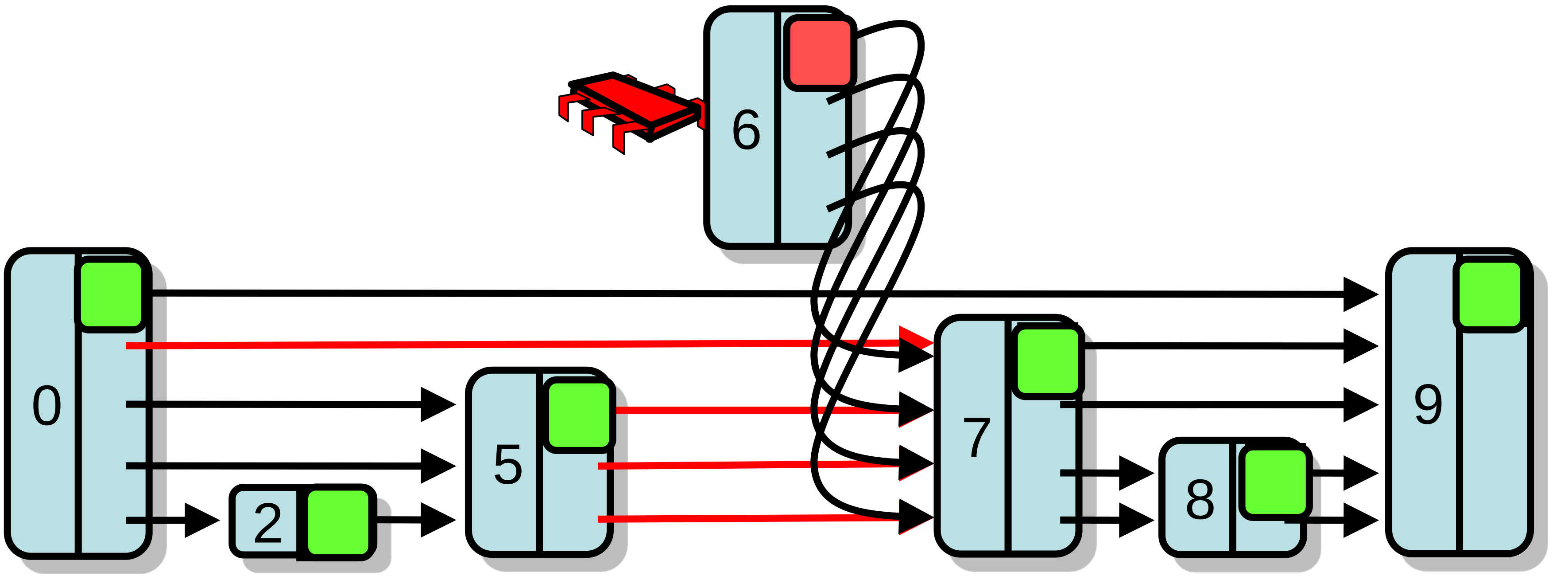
- Successful add() at point when fully linked
- Add fullyLinked bit to indicate this
- Bit tested by contains()

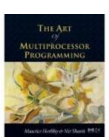




contains(7): Linearization

- When fully-linked unmarked node found
- Pause while fullyLinked bit unset

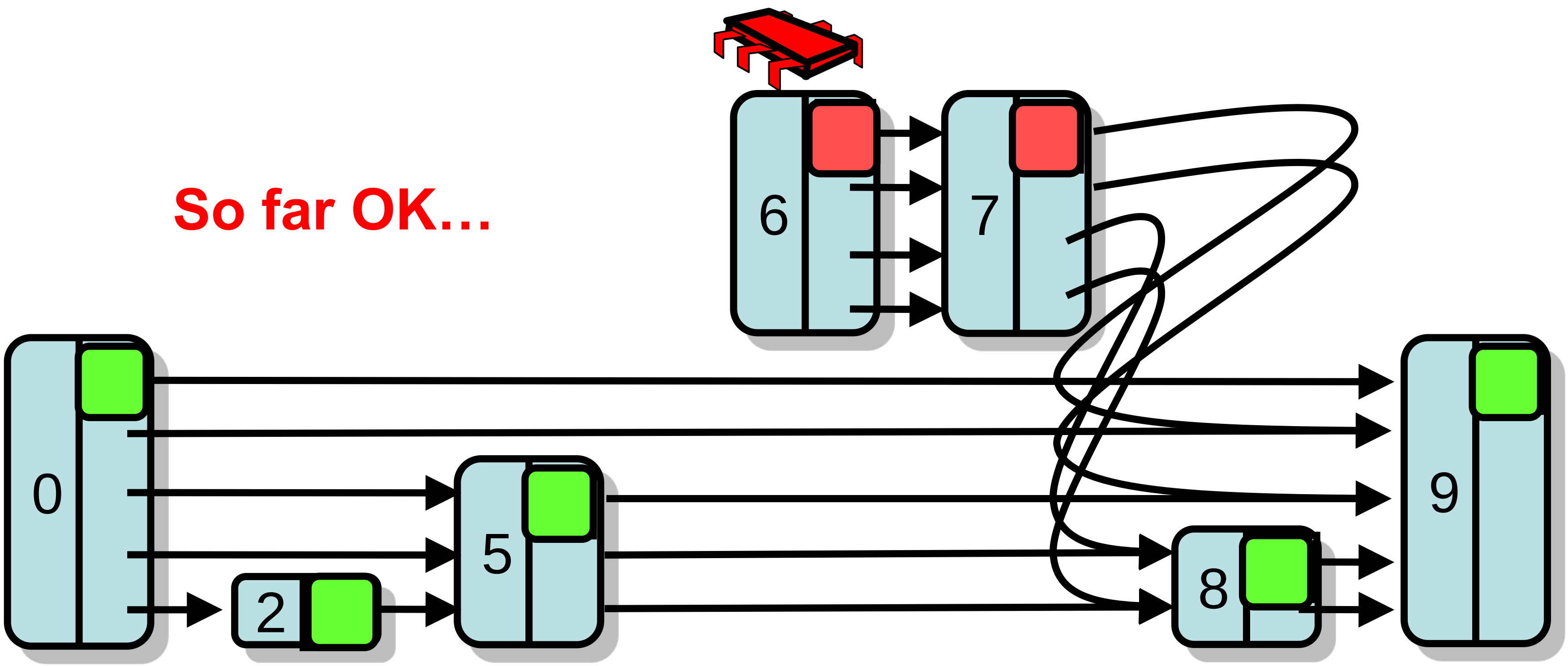


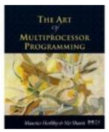


contains(7): Linearization

- When do we linearize unsuccessful Search?

So far OK…

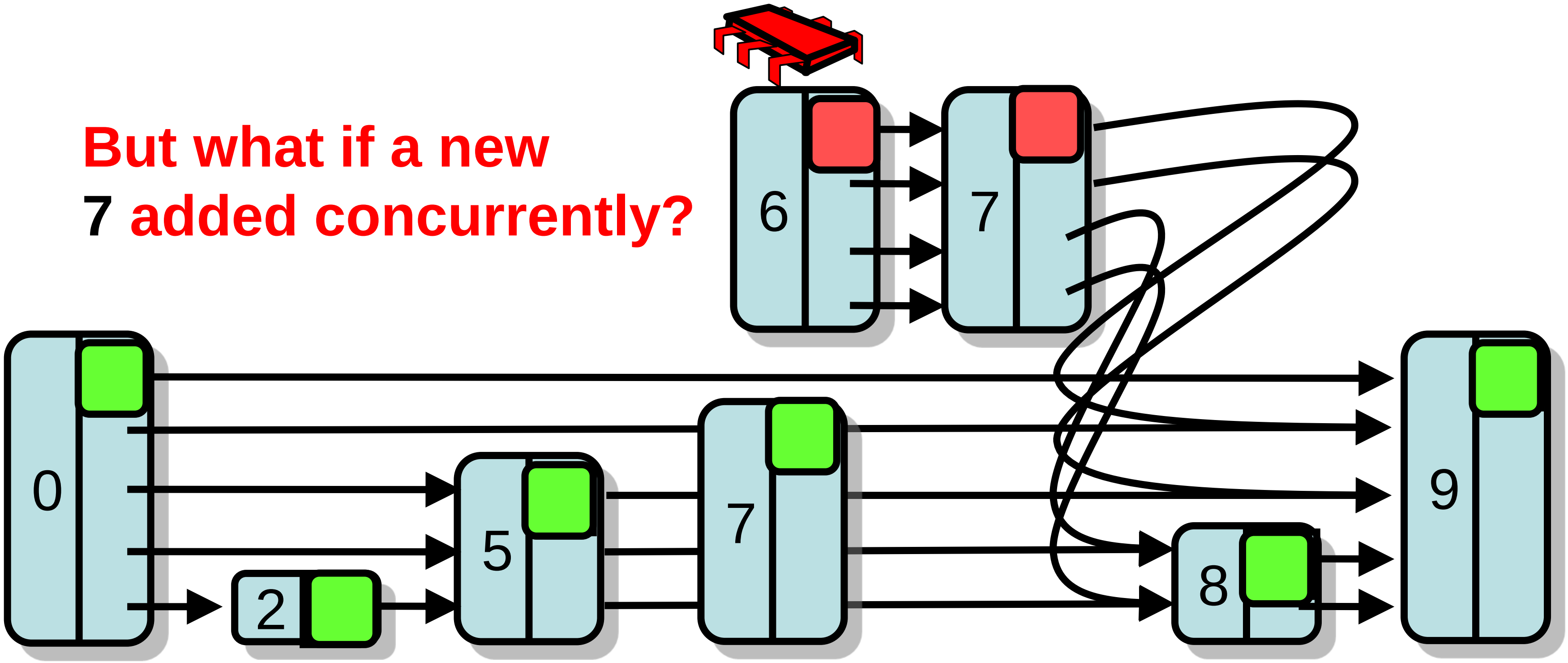


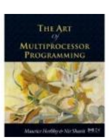


contains(7): Linearization

- When do we linearize unsuccessful Search?

But what if a new 7 added concurrently?

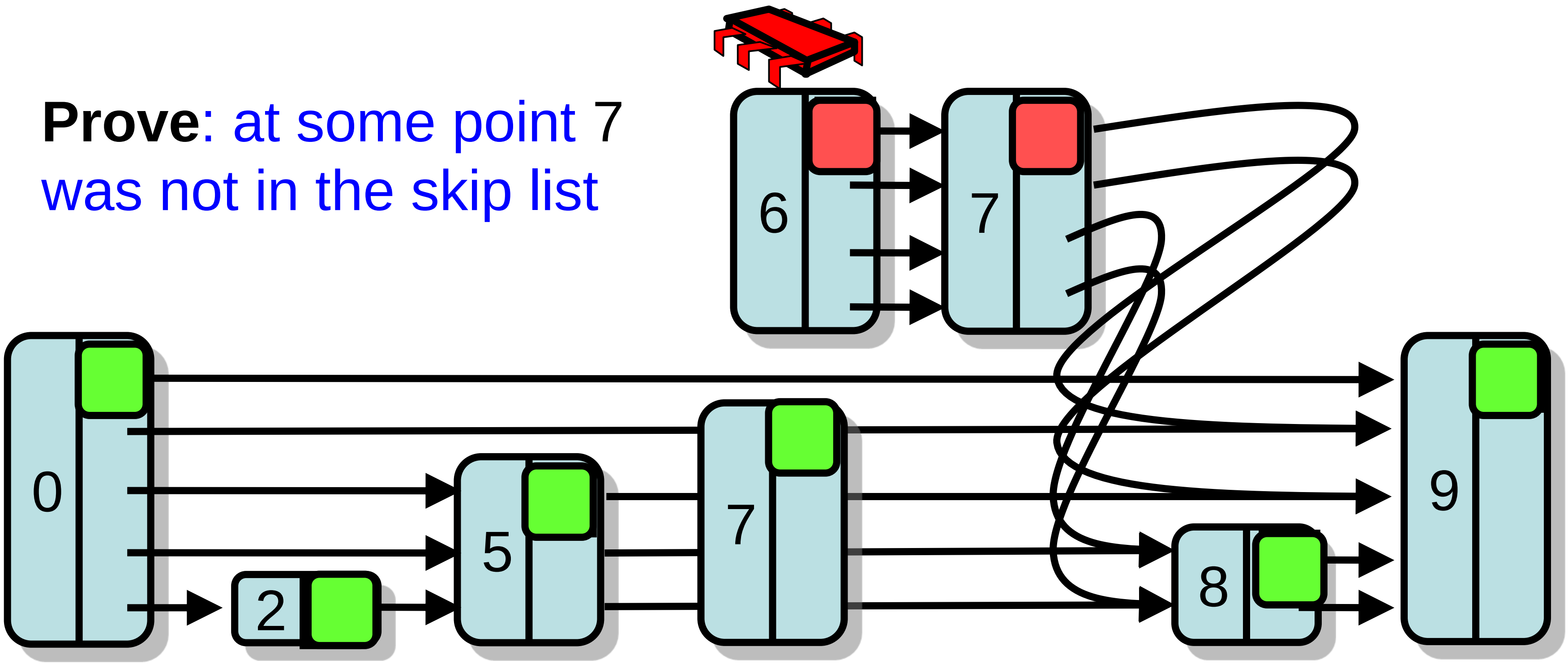


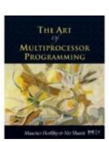


contains(7): Linearization

- When do we linearize unsuccessful Search?

Prove: at some point 7 was not in the skip list

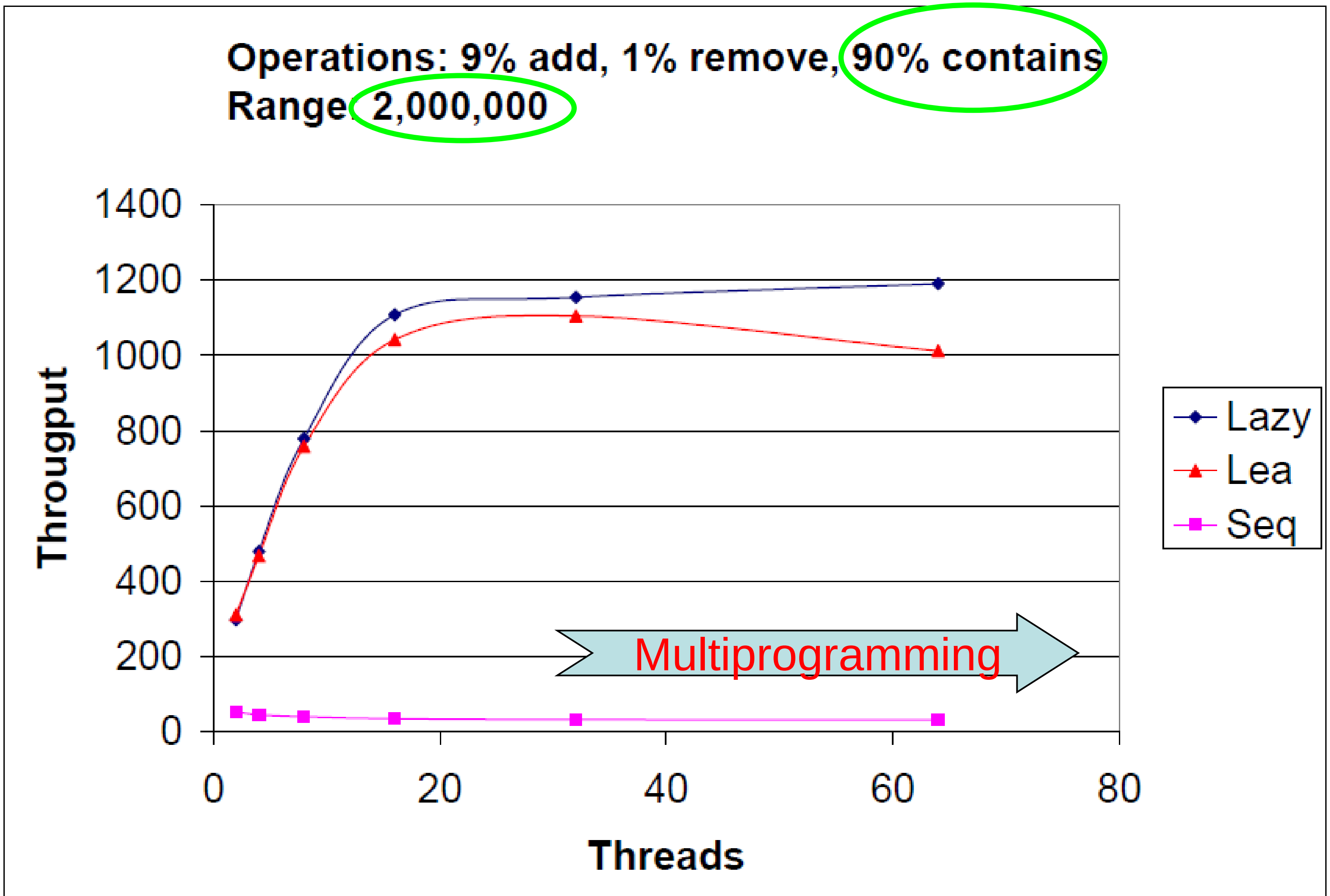


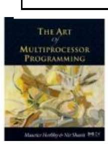


A Simple Experiment

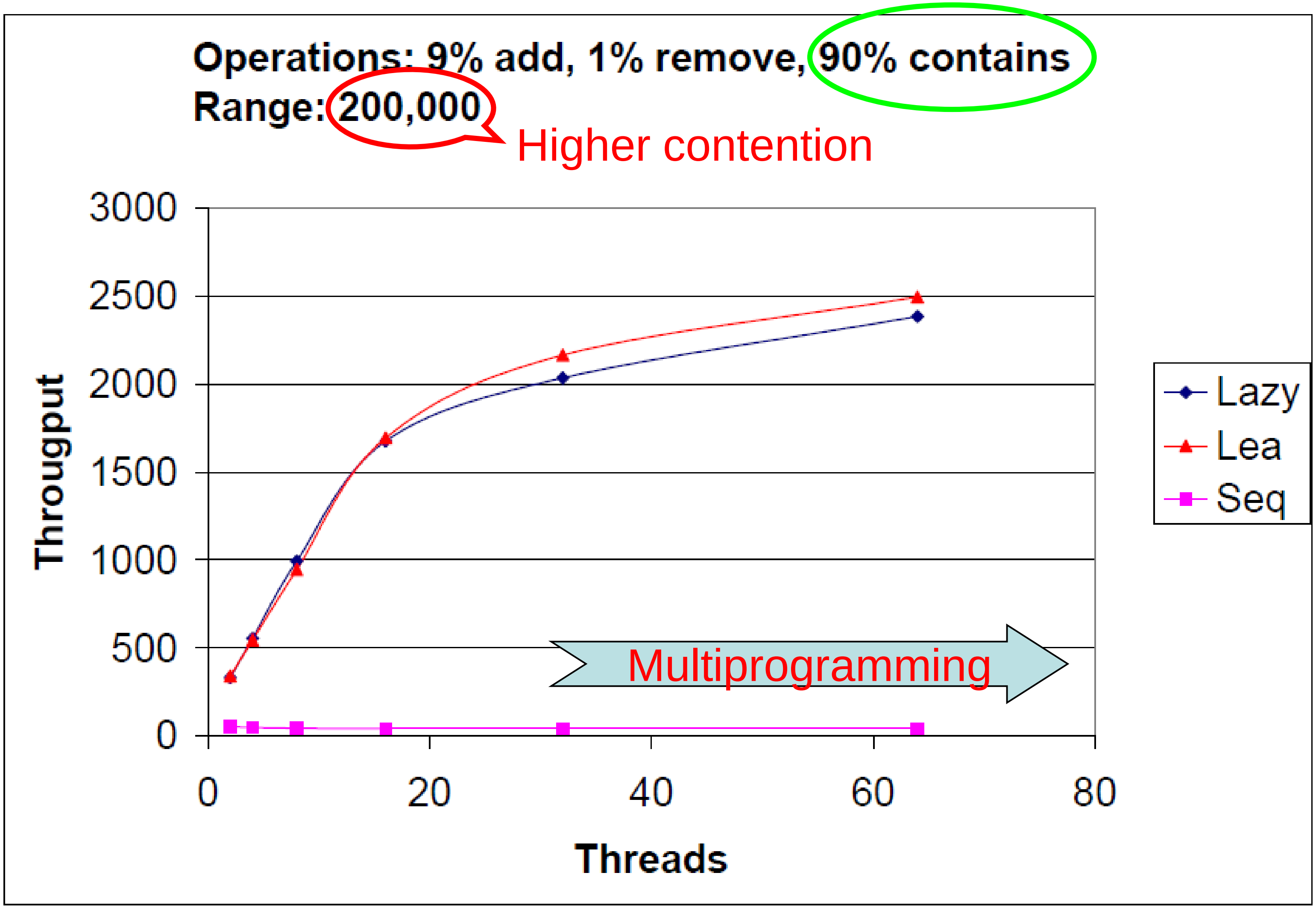
- Each thread runs 1 million iterations, each either:
  - **add()**
  - **remove()**
  - **contains()**
- Item and method chosen in random from some distribution

Lazy Skip List: Performance

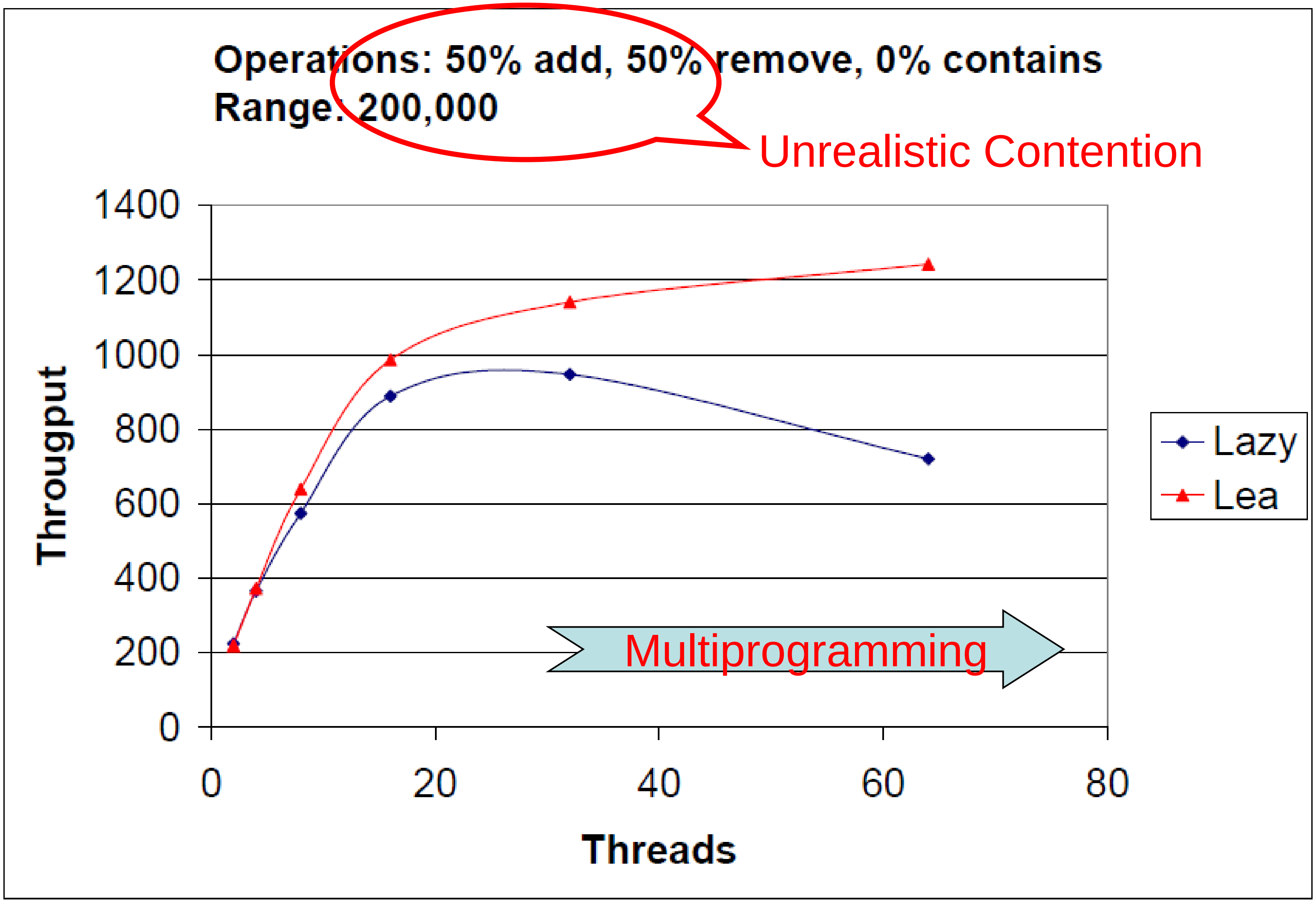


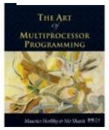


Lazy Skip List: Performance



Lazy Skip List: Performance





Summary

- Lazy Skip List
  - Optimistic fine-grained Locking
- Performs as well as the lock-free solution in "common" cases
- Simple