Homework 1: Link Layer

Due: 11:59 PM, Sep 29, 2016

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1 Errors

Suppose your friend develops a scheme to add redundancy to a protocol. The idea is to add, to every two bits, one extra bit: a 1 if the 2-bit number is odd (i.e., 01, 11), a 0 if it is even (i.e., 00, 10). Assume errors can affect every bit with the same probability.

This is a simple diagram of the valid codewords produced by this scheme, among all of the possible 3-bit words. The valid words are marked with an *, and the edges of this graph connect words that are 1-bit-error apart.

```
     111* --  011*
    / |       / |
101 -+--  001  |
 |   |     |   |
 |   110  -+- 010
 | /       | /
100*  --  000*
```

1. Can this code detect any 1-bit error?

Yes, it will detect bit errors in the second bit.

2. Can this code detect all 1-bit errors? Why or why not? Explain using the Hamming distance argument.

No. It won't detect errors in the first bit. The smallest distance between two valid codewords, d, is 1, between the pairs 100-000 and 111-011. Thus, it won't detect all 1-bit errors, as d-1 is 0.

3. Can this code correct any 1-bit error?

No. Any 1-bit error in this code will either go to a valid codeword (if it is in the first bit), or be equidistant to two valid codewords (if the error is in the 2nd or 3rd bit), which will be ambiguous and, thus, not correctible.

4. Can this code correct all 1-bit errors? Why or why not? Explain using the Hamming distance argument.

No. The argument is the same as in the detection, and also follows from the previous question. This code can correct no errors.

5. Is there a pattern of errors that makes this code good? (Assume you control the Universe and can dictate how errors affect the bits in your transmissions.)

Yes. If you know that errors will **only** occur in the second bit, then you can basically ignore it and use the third bit in its place. This is fairly unrealistic, but in this case the code would make all errors correctible.

2 Reliability at multiple layers

Now suppose your friend designs a link layer that employs a retransmission scheme on each link to provide (bounded) reliability.

1. You read that it might be useful sometimes to provide some functionality in a lower layer, even if it has to be provided anyway in a higher layer. What is a potential reason?

A potential reason is if said functionality improves performance, or solves the common case much cheaper than when it is only performed at the upper layers.

Let's explore this with some numbers. Suppose that you have a path with 4 links, each link with a 10% chance of dropping a packet (you can assume that acknowledgments are not dropped).

2. What is the probability that a packet sent by the first node arrives at the 5th node (after the 4 links), if there are no retransmissions?

This is the event in which the packet doesn't fail in any link, or $p = (1 - 0.1)^4 = 0.6561$

3. Now let's think about one link in isolation. If you employ a scheme that re-sends each packet up to 3 times and waits for an acknowledgment, what is the probability that the packet is not dropped on this link? (It may be easier to think about 1 - the probability of the complementary event.)

The packet is dropped if it fails to get transmitted 3 times: $p(\text{drop}) = 0.1^3 = 0.001$. The complement of this is what we want: $p(\text{success}) = 1 - p(\text{drop}) = 0.999$.

The direct computation of this probability is a bit more laborsome, as we have to take into account the packet making into 1 transmission, 2 transmissions, or 3 transmissions:

$p(\text{success}) = 0.9 + 0.1 \times 0.9 + (0.1)^2 \times 0.9 = 0.999$.

4. If you assume that all four links employ the scheme in (3), what then is the probability that a packet sent by the first node arrives at the 5th node?

We can now redo the calculation in 2. above, but using the new improved $p(\text{success})$: $p = (0.999)^4 = 0.996$!

3 Modulation

Wifi standards can use various modulation schemes, from 2 levels (BPSK) up to 256 levels (256-QAM) modulation.

Suppose that a wireless client measures the noise floor of -90dBm, and the strength of the signal it receives from an access point is -70dBm. (dBm is a way to express power as a logarithmic ratio to a reference power of 1mW: p dBm $= 10 \log_{10} \frac{PmW}{1mW}$. Thus, $P(\text{mW}) = 10^{\frac{p\text{dBm}}{10}}$, and -50dBm = 0.00001mW). The SNR this client sees is 20dB, or a ratio of 100.

1. What is the power in mW of the noise in this scenario?

$-90\text{dBm} = 10 \log_{10} \frac{PmW}{1mW}$
$\log_{10} \frac{P\text{mW}}{1\text{mW}} = -9$
$P = 10^{-9}\text{mW}$.

2. How many levels of modulation (M) can the transmitter use in this scenario? (The answer should be a power of 2.)

The SNR is -70 - (-90)dBm = 20dBm, or, expressed as a ratio, $10^(20/10) = 100$.

Now $M \leq \sqrt{(1 + \text{SNR})} \approx 10.05$.

The largest power of 2 smaller than this is 8.

3. If the noise floor is kept the same, what would the receive signal strength have to be to allow the use of 256-QAM modulation (M=256)? Use the formulas given in the lecture, even though they are approximations and don't take some factors into account. Give the answer both in mW and in dBm.

We want $M = 256$.

$\sqrt{1 + S/N} \geq 256$
$1 + S/N \geq 256^2 = 2^{16}$
$S/N \geq 65535$.

The noise floor is -90dBm = 10^{-9}mW.
$S \geq 65535 \times 10^{-9}$mW
$S > 6.5 \times 10^{-5}$mW.

To convert this to dBm: $p = 10 \log_{10}(6.5 \times 10^{-5}) = 10(0.8129 - 5) = -41.871$dBm.

This is pretty bad, as it is almost 3 orders of magnitude more power than previously (which was -70 dBm or $10^{-7} mW$!

4 Medium Access

1. Give one advantage and one disadvantage of a partition-based medium access control.

Possible advantages:

- High utilization if demands are predictable as there is no overhead to deal with collisions or medium arbitration.
- Predictable service.
- (related) Easy to offer guarantees.

Possible disadvantages:

- Low utilization if the demands are unpredictable, as there will be many wasted slots.
- No graceful degradation as more entities demand service: users get busy signal.

2. Give one advantage and one disadvantage of a random-access medium access control.

Possible advantages:

- Can better utilize the medium when the demands are unknown in advance
- Graceful degradation: as more entities demand service, service quality degrades for everyone equally (this may be desirable)

Possible disadvantages:

- There is potentially more overhead for coordinating access to the medium (such as backoffs)
- There is the potential for collisions
- It is difficult to provide guarantees and/or predict the level of service.

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