Problem 1

Consider a range space \((X, C)\) where \(X = \{1, 2, \ldots, n\}\) and \(C\) is the set of all subsets of \(X\) of size \(k\) for some \(k < n\). What is the VC dimension of \(C\)?

Problem 2

Let \(S = (X, R)\) and \(S' = (X, R')\) be two range spaces. Prove that if \(R' \subseteq R\) then the VC dimension of \(S'\) is no larger than the VC dimension of \(S\).

Problem 3

Consider the range space \((X, R)\), where \(X\) is an interval \([a, b]\) and \(R\) is the collection of all intervals \([x, y] \subseteq [a, b]\).

Let \(S = \{x_1, \ldots, x_n\}\) be a finite set of points in \(X\), where \(x_1 < x_2 < \cdots < x_n\), and \(|S| = n\).

1. Which sets are in \(\Pi_R(S)\) (the projection of \(R\) on \(S)\)?

2. Compare the number of sets you found in part a to the bound obtained using Sauer’s Lemma.