Homework 5

Due: Thu 23 March 2017, 2:30pm

Please follow these guidelines while writing up your CS 155 homeworks.

A major goal of this class is that you learn how to mathematically analyze probabilistic processes and events. Hence make sure that your proofs are rigorous, e.g., that every step is adequately justified. Try to keep your proofs as simple and as concise as possible, keeping in mind that you will potentially have to examine multiple proof strategies in order to achieve this. Make sure to CLEARLY STATE YOUR ASSUMPTIONS at the beginning of any solution, when necessary. As we are doing probabilistic analysis, we shall require that you clearly define the probability space over which you are working with and identify the events within it that you will analyse.

You are allowed to discuss with other students about these problems but you have to leave any meeting with other students without any written material about the problems, and the write ups should be written by yourself without help from others. If you have any questions regarding how these guidelines apply to a particular problem or what they mean in general please post on the newsgroup or email the TAs.

Problem 1

Consider a range space \((\mathbb{R}^2, C)\) of all axis-aligned squares in \(\mathbb{R}^2\).

- Show that the VC dimension of \((\mathbb{R}^2, C)\) is equal to 3.
- Construct and analyze a a PAC learning algorithm for the concept class of all axis-aligned squares in \(\mathbb{R}^2\).

Problem 2

Exercise 14.8 from the text book.

Problem 3

Exercise 14.9 from the text book.
Problem 4

Use the bound of Exercise 14.9 to improve the result of Theorem 14.15 to show that there is an

\[ m = O \left( \frac{d}{\varepsilon^2} \ln \frac{1}{\varepsilon} + \frac{1}{\varepsilon^2} \ln \frac{1}{\delta} \right) \]  

such that a random sample from \( D \) of size greater or equal to \( m \) suffices to obtain an \( \varepsilon \)-sample with probability at least \( 1 - \delta \). (Hint: Use Lemma 14.3 with \( x = 1/\varepsilon^2 \) and \( y = 2m/d \)